Please check the examination details below before entering your candidate information

Candidate surname	Other names

Centre Number | Candidate Number

Pearson Edexcel Level 3 GCE

Time 2 hours

Paper reference **9MA0/02**

Mathematics

Advanced

PAPER 2: Pure Mathematics 2

You must have:
Mathematical Formulae and Statistical Tables (Green), calculator

Total Marks

Candidates may use any calculator allowed by Pearson regulations. Calculators must not have the facility for symbolic algebra manipulation, differentiation and integration, or have retrievable mathematical formulae stored in them.

Instructions

- Use **black** ink or ball-point pen.
- If pencil is used for diagrams/sketches/graphs it must be dark (HB or B).
- **Fill in the boxes** at the top of this page with your name, centre number and candidate number.
- Answer **all** questions and ensure that your answers to parts of questions are clearly labelled.
- Answer the questions in the spaces provided – *there may be more space than you need.*
- You should show sufficient working to make your methods clear. Answers without working may not gain full credit.
- Inexact answers should be given to three significant figures unless otherwise stated.

Information

- A booklet 'Mathematical Formulae and Statistical Tables' is provided.
- There are 16 questions in this question paper. The total mark for this paper is 100.
- The marks for **each** question are shown in brackets – *use this as a guide as to how much time to spend on each question.*

Advice

- Read each question carefully before you start to answer it.
- Try to answer every question.
- Check your answers if you have time at the end.

Turn over

P69602A

Q:1/1/1/1/1/

P 6 9 6 0 2 A 0 1 4 8

Pearson

1. **In this question you must show all stages of your working.**

Solutions relying entirely on calculator technology are not acceptable.

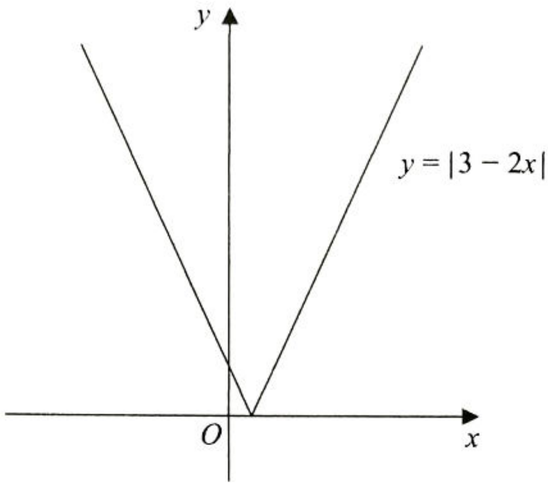


Figure 1

Figure 1 shows a sketch of the graph with equation $y = |3 - 2x|$

Solve

$$|3 - 2x| = 7 + x$$

(4)

$|3-2x| = 7+x$

$3 - 2x = 7+x$ or $2x - 3 = 7 + x$

$3 = 7 + 3x$ $\quad$ $x - 3 = 7$

$3x = -4$ $\quad$ $x = 10$

$x = -\frac{4}{3}$

2. (a) Sketch the curve with equation

$$y = 4^x$$

stating any points of intersection with the coordinate axes.

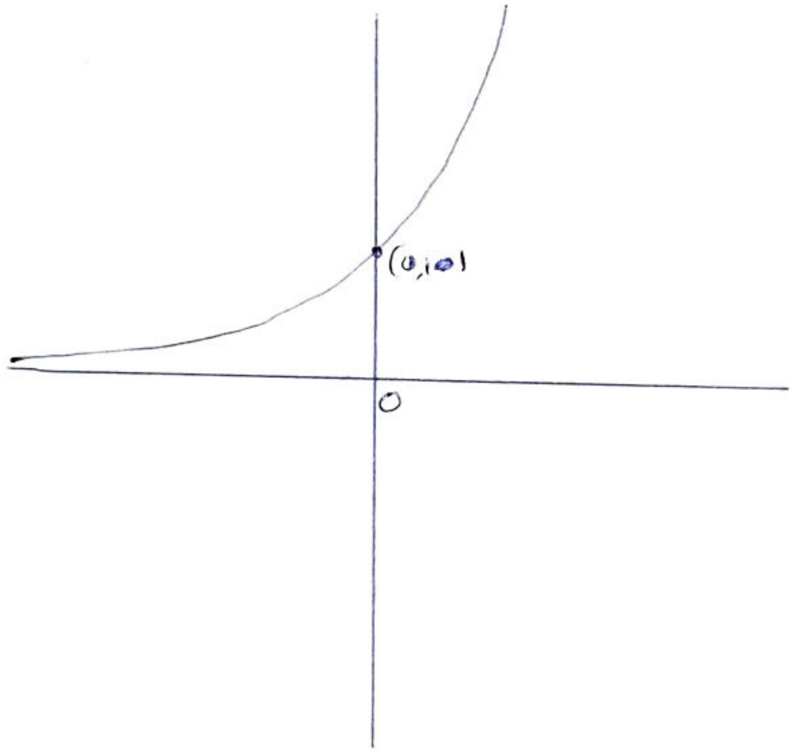
(2)

(b) Solve

$$4^x = 100$$

giving your answer to 2 decimal places.

(2)



Question 2 continued

$4^x = 100$

$\ln(4^x) = \ln(100)$

$x \ln(4) = \ln(100)$

$x = \dfrac{\ln(100)}{\ln(4)}$

$x = 3.32$ (2 decimal places)

(Total for Question 2 is 4 marks)

3. A sequence of terms $a_1, a_2, a_3, \ldots$ is defined by

$$a_1 = 3$$
$$a_{n+1} = 8 - a_n$$

(a) (i) Show that this sequence is periodic.

(ii) State the order of this periodic sequence.

(2)

(b) Find the value of

$$\sum_{n=1}^{85} a_n$$

(2)

a) i) $a_1 = 3$

$a_2 = 8 - 3 = 5$

$a_3 = 8 - 5 = 3$

ii) 2

b) $\sum_{n=1}^{85} a_n$

$a_n = 3$ for odd a

$a_n = 5$ for even a

There are 43 odd numbers and 42 even numbers up to 85.

$\sum_{n=1}^{85} a_n = 43 \times 3 + 42 \times 5 = 339$

4. Given that

$$y = 2x^2$$

use differentiation from first principles to show that

$$\frac{dy}{dx} = 4x$$

(3)

$y(x) = 2x^2$

$\frac{dy}{dx} = \lim_{h \to 0} \frac{y(x+h) - y(x)}{h}$

$\frac{dy}{dx} = \lim_{h \to 0} \frac{2(x+h)^2 - 2x^2}{h}$

$\frac{dy}{dx} = \lim_{h \to 0} \frac{2(x^2 + 2hx + h^2) - 2x^2}{h}$

$\frac{dy}{dx} = \lim_{h \to 0} \frac{2x^2 + 4hx + 2h^2 - 2x^2}{h}$

$\frac{dy}{dx} = \lim_{h \to 0} \frac{4hx + 2h^2}{h}$

$\frac{dy}{dx} = \lim_{h \to 0} 4x + 2h$

As $h \to 0$, $4x$ doesn't change, and $2h \to 0$.
Hence,

$\frac{dy}{dx} = 4x$

5. The table below shows corresponding values of x and y for $y = \log_3 2x$

The values of y are given to 2 decimal places as appropriate.

x	3	4.5	6	7.5	9
y	1.63	2	2.26	2.46	2.63

(a) Using the trapezium rule with all the values of y in the table, find an estimate for

$$\int_3^9 \log_3 2x \, dx$$

(3)

Using your answer to part (a) and making your method clear, estimate

(b) (i) $\int_3^9 \log_3 (2x)^{10} \, dx$

(ii) $\int_3^9 \log_3 18x \, dx$

(3)

a) $\int_3^9 \log_3(2x)\,dx = \frac{1.5}{2}\left(1.63 + 2(2 + 2.26 + 2.46) + 2.63\right)$

$\int_3^9 \log_3(2x)\,dx = \frac{531}{40}$

b) i) $\int_3^9 \log_3((2x)^{10})\,dx = \int_3^9 10\log_3(2x)\,dx$

$= 10\int_3^9 \log_3(2x)\,dx$

$= 10\left(\frac{531}{40}\right)$

$= \frac{531}{4}$

Question 5 continued

ii) $\int_3^9 \log_3(18x)\,dx = \int_3^9 \log_3(9 \times 2x)\,dx$

$= \int_3^9 \log_3(9) + \log_3(2x)\,dx$

$= \int_3^9 2 + \log_3(2x)\,dx$

$= \int_3^9 2\,dx + \int_3^9 \log_3(2x)\,dx$

$= \left[2x\right]_3^9 + \frac{531}{40}$

$= 2 \times 9 - 2 \times 3 + \frac{531}{40}$

$= 18 - 6 + \frac{531}{40}$

$= 12 + \frac{531}{40}$

$= \frac{1011}{40}$

(Total for Question 5 is 6 marks)

6.

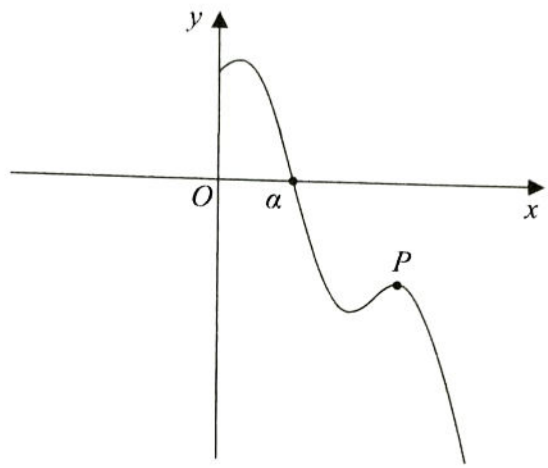


Figure 2

Figure 2 shows a sketch of part of the curve with equation $y = f(x)$ where

$$f(x) = 8\sin\left(\frac{1}{2}x\right) - 3x + 9 \qquad x > 0$$

and x is measured in radians.

The point P, shown in Figure 2, is a local maximum point on the curve.

Using calculus and the sketch in Figure 2,

(a) find the x coordinate of P, giving your answer to 3 significant figures.

(4)

The curve crosses the x-axis at $x = \alpha$, as shown in Figure 2.

Given that, to 3 decimal places, $f(4) = 4.274$ and $f(5) = -1.212$

(b) explain why α must lie in the interval $[4, 5]$

(1)

(c) Taking $x_0 = 5$ as a first approximation to α, apply the Newton-Raphson method once to $f(x)$ to obtain a second approximation to α.

Show your method and give your answer to 3 significant figures.

(2)

a) $f'(x) = 4\cos\left(\frac{1}{2}x\right) - 3$

$f'(x) = 0$ want the third time this occurs for $x > 0$.

$4\cos\left(\frac{1}{2}x\right) - 3 = 0$

$4\cos\left(\frac{1}{2}x\right) = 3$

$\cos\left(\frac{1}{2}x\right) = \frac{3}{4}$

$\frac{1}{2}x = 0.723, 5.560, 7.006$

Question 6 continued

Take third occurrence only:

$\frac{1}{2}x = 7.006$

$x = 14.0 \quad (3\,sf)$

b) $f(4) > 0$ and $f(5) < 0$.

Since the function is continuous, there must exist a value x where $4 < x < 5$ and $f(x) = 0$.

c) $x_{n+1} = x_n - \dfrac{f(x_n)}{f'(x_n)} = x_n - \dfrac{8\sin(\frac{1}{2}x_n) - 3x_n + 9}{4\cos(\frac{1}{2}x_n) - 3}$

$x_1 = 5$

$x_2 = 5 - \dfrac{8\sin(\frac{5}{2}) - 3\times 5 + 9}{4\cos(\frac{5}{2}) - 3} = 4.80 \quad (3\,sf)$

7. (a) Find the first four terms, in ascending powers of x, of the binomial expansion of

$$\sqrt{4-9x}$$

writing each term in simplest form.

(4)

A student uses this expansion with $x = \frac{1}{9}$ to find an approximation for $\sqrt{3}$

Using the answer to part (a) and without doing any calculations,

(b) state whether this approximation will be an overestimate or an underestimate of $\sqrt{3}$ giving a brief reason for your answer.

(1)

a) $\sqrt{4-9x} = 2\sqrt{1-\frac{9}{4}x}$

$= 2\left(1-\frac{9}{4}x\right)^{\frac{1}{2}}$

$= 2\left(1 + \frac{\frac{1}{2}\times\left(-\frac{9}{4}x\right)}{1!} + \frac{\frac{1}{2}\times\left(-\frac{1}{2}\right)\times\left(-\frac{9}{4}x\right)^2}{2!} + \frac{\frac{1}{2}\times\left(-\frac{1}{2}\right)\times\left(-\frac{3}{2}\right)\times\left(-\frac{9}{4}x\right)^3}{3!} + \dots\right)$

$= 2\left(1 - \frac{9}{8}x + \frac{-\frac{1}{4}\times\frac{81}{16}x^2}{2} - \frac{\frac{3}{8}\times\frac{729}{64}x^3}{6} + \dots\right)$

$= 2\left(1 - \frac{9}{8}x - \frac{81}{128}x^2 - \frac{729}{1024}x^3 + \dots\right)$

$= 2 - \frac{9}{4}x - \frac{81}{64}x^2 - \frac{729}{512}x^3 + \dots$

b) This will be an overestimate because all terms after the first in the expansion are negative.

8.

In this question you must show all stages of your working.

Solutions relying on calculator technology are not acceptable.

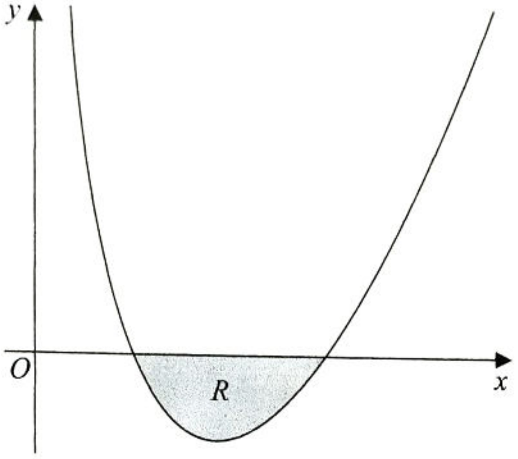


Figure 3

Figure 3 shows a sketch of part of a curve with equation

$$y = \frac{(x-2)(x-4)}{4\sqrt{x}} \qquad x > 0$$

The region R, shown shaded in Figure 3, is bounded by the curve and the x-axis.

Find the exact area of R, writing your answer in the form $a\sqrt{2} + b$, where a and b are constants to be found.

(6)

$$y = \frac{(x-2)(x-4)}{4\sqrt{x}}$$

$y = 0$ at $x = 2,$ $x = 4$.

$$R = \int_2^4 y \, dx$$

$$R = \int_2^4 \frac{(x-2)(x-4)}{4\sqrt{x}} \, dx$$

$$R = \int_2^4 \frac{x^2 - 6x + 8}{4\sqrt{x}} \, dx$$

$$R = \int_2^4 \frac{1}{4}x^{\frac{3}{2}} - \frac{3}{2}x^{\frac{1}{2}} + 2x^{-\frac{1}{2}} \, dx$$

$$R = \left[\frac{1}{10}x^{\frac{5}{2}} - x^{\frac{3}{2}} + 4x^{\frac{1}{2}}\right]_2^4$$

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Question 8 continued

$R = \frac{1}{10}(4)^{\frac{5}{2}} - 4^{\frac{3}{2}} + 4(4)^{\frac{1}{2}} - \frac{1}{10}(2)^{\frac{5}{2}} - 2^{\frac{3}{2}} + 4(2)^{\frac{1}{2}}$

$R = \frac{1}{10} \times 32 - 8 + 4 \times 2 - \frac{1}{10} \times 4 \times \sqrt{2} - 2 \times \sqrt{2} + 4 \times \sqrt{2}$

$R = \frac{16}{5} - 8 + 8 - \frac{2}{5}\sqrt{2} - 2\sqrt{2} + 4\sqrt{2}$

$R = \frac{16}{5} - \frac{12}{5}\sqrt{2}$

Note that $R < 0$ and we want positive area.

$R = \frac{12}{5}\sqrt{2} - \frac{16}{5}$

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9.

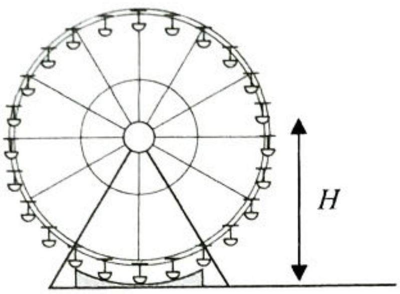

Figure 4

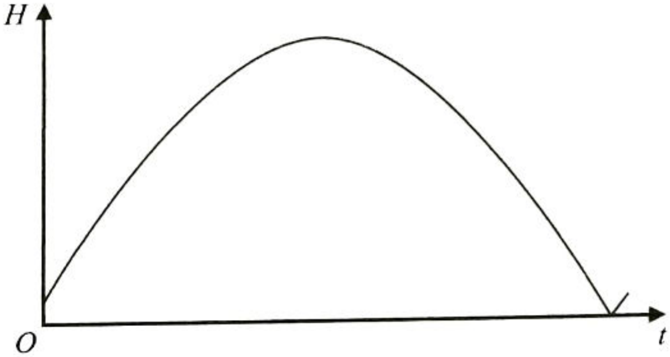

Figure 5

Figure 4 shows a sketch of a Ferris wheel.

The height above the ground, H m, of a passenger on the Ferris wheel, t seconds after the wheel starts turning, is modelled by the equation

$$H = |A\sin(bt + \alpha)°|$$

where A, b and α are constants.

Figure 5 shows a sketch of the graph of H against t, for one revolution of the wheel.

Given that

- the maximum height of the passenger above the ground is 50 m
- the passenger is 1 m above the ground when the wheel starts turning
- the wheel takes 720 seconds to complete one revolution

(a) find a complete equation for the model, giving the exact value of A, the exact value of b and the value of α to 3 significant figures.

(4)

(b) Explain why an equation of the form

$$H = |A\sin(bt + \alpha)°| + d$$

where d is a positive constant, would be a more appropriate model.

(1)

a) $H = |A\sin(bt + \alpha)|$

At max, $H = 50$

$\Rightarrow A = 50$

1 revolution $= 720$ s, so $b = \frac{180}{720} = \frac{1}{4}$.

180 used as one half turn in eqn corresponds to 1 full turn of the wheel because of modulus signs.

At $t=0$, $H=1$

$1 = 50 \sin(\alpha)$

$\sin(\alpha) = \frac{1}{50}$

$\alpha = 1.15°$.

$H = |50 \sin(\frac{1}{4}t + 1.15)|$

b) Since the ferris wheel does not scrape the ground the vertical height is always > 0. d is this minimum vertical height.

(Total for Question 9 is 5 marks)

10. The function f is defined by

$$f(x) = \frac{8x+5}{2x+3} \qquad x > -\frac{3}{2}$$

(a) Find $f^{-1}\left(\frac{3}{2}\right)$

(2)

(b) Show that

$$f(x) = A + \frac{B}{2x+3}$$

where A and B are constants to be found.

(2)

The function g is defined by

$$g(x) = 16 - x^2 \qquad 0 \leqslant x \leqslant 4$$

(c) State the range of g^{-1}

(1)

(d) Find the range of $f g^{-1}$

(3)

10) a) $f(x) = \frac{8x+5}{2x+3}$

$\frac{3}{2} = \frac{8x+5}{2x+3}$

$3(2x+3) = 2(8x+5)$

$6x+9 = 16x+10$

$10x = -1$

$x = -\frac{1}{10}$

b) $f(x) = \frac{8x+5}{2x+3}$

$f(x) = \frac{8x+12-7}{2x+3}$

$f(x) = \frac{8x+12}{2x+3} - \frac{7}{2x+3}$

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P 6 9 6 0 2 A 0 2 4 4 8

Question 10 continued

$$g(x) = \frac{4(2x+3)}{2x+3} - \frac{7}{2x+3}$$

$$g(x) = 4 - \frac{7}{2x+3}$$

$A = 4$
$B = -7$

c) $0 \leq g^{-1}(x) \leq 4$

d) g is always increasing, so only need to try 0 and 4

$$g(0) = \frac{8 \times 0 + 5}{2 \times 0 + 3}$$

$$g(0) = \frac{5}{3}$$

$$g(4) = \frac{8 \times 4 + 5}{2 \times 4 + 3}$$

$$g(4) = \frac{32+5}{8+3}$$

$$g(4) = \frac{37}{11}$$

$$\frac{5}{3} \leq gg^{-1}(x) \leq \frac{37}{11}$$

P 6 9 6 0 2 A 0 2 5 4 8

11. Prove, using algebra, that

$$n(n^2 + 5)$$

is even for all $n \in \mathbb{N}$.

(4)

11) Case 1 : n is even

$n = 2k$ where k is an integer

$2k((2k)^2 + 5) =$

$2k(4k^2 + 5) =$

$2(k(4k^2+5))$

$k(4k^2+5)$ is an integer

so $2(k(4k^2+5))$ is even

Case 2 : n is odd

$n = 2k+1$ where k is an integer

$(2k+1)((2k+1)^2+5) =$

$(2k+1)(4k^2+4k+1+5) =$

$(2k+1)(4k^2+4k+6) =$

$2(2k+1)(2k^2+2k+3)$

$(2k+1)(2k^2+2k+3)$ is an integer

so $2(2k+1)(2k^2+2k+3)$ is even.

Even in both cases hence even for all integers.

12. The function f is defined by

$$f(x) = \frac{e^{3x}}{4x^2 + k}$$

where k is a positive constant.

(a) Show that

$$f'(x) = (12x^2 - 8x + 3k)g(x)$$

where $g(x)$ is a function to be found.

(3)

Given that the curve with equation $y = f(x)$ has at least one stationary point,

(b) find the range of possible values of k.

(3)

12) a) $f(x) = \dfrac{e^{3x}}{4x^2+k}$

$f'(x) = \dfrac{(4x^2+k)3e^{3x} - e^{3x}(8x)}{(4x^2+k)^2}$

$f'(x) = \left(3(4x^2+k) - 8x\right)\dfrac{e^{3x}}{(4x^2+k)^2}$

$f'(x) = (12x^2 - 8x + 3k)\underbrace{\dfrac{e^{3x}}{(4x^2+k)^2}}_{g(x)}$

b) Stationary point at $f'(x) = 0$

First note $g(x) > 0$ always.

So $f'(x) = 0$ only happens when

$12x^2 - 8x + 3k = 0$

For 1 or more solution, need:

$b^2 - 4ac \geq 0$

$(-8)^2 - 4 \times 12 \times 3k \geq 0$

$64 - 144k \geq 0$

$144k \leq 64$

$k \leq \frac{64}{144}$ (k positive so $k > 0$)

$0 < k \leq \frac{4}{9}$

13. Relative to a fixed origin O

- the point A has position vector $4\mathbf{i} - 3\mathbf{j} + 5\mathbf{k}$ $= \underline{a}$
- the point B has position vector $4\mathbf{j} + 6\mathbf{k}$ $= \underline{b}$
- the point C has position vector $-16\mathbf{i} + p\mathbf{j} + 10\mathbf{k}$ $= \underline{c}$

where p is a constant.

Given that A, B and C lie on a straight line,

(a) find the value of p.

(3)

The line segment OB is extended to a point D so that $\overrightarrow{CD}$ is parallel to $\overrightarrow{OA}$

(b) Find $|\overrightarrow{OD}|$, writing your answer as a fully simplified surd.

(3)

a) A, B and C on straight line means $\overrightarrow{AB}$ and $\overrightarrow{AC}$ are parallel

$\overrightarrow{AB} = \underline{b} - \underline{a}$
$\overrightarrow{AB} = 4\underline{j} + 6\underline{k} - (4\underline{i} - 3\underline{j} + 5\underline{k})$
$\overrightarrow{AB} = 4\underline{j} + 6\underline{k} - 4\underline{i} + 3\underline{j} - 5\underline{k}$
$\overrightarrow{AB} = -4\underline{i} + 7\underline{j} + \underline{k}$

$\overrightarrow{AC} = \underline{c} - \underline{a}$
$\overrightarrow{AC} = -16\underline{i} + p\underline{j} + 10\underline{k} - (4\underline{i} - 3\underline{j} + 5\underline{k})$
$\overrightarrow{AC} = -16\underline{i} + p\underline{j} + 10\underline{k} - 4\underline{i} + 3\underline{j} - 5\underline{k}$
$\overrightarrow{AC} = -20\underline{i} + (p+3)\underline{j} + 5\underline{k}$

By comparing $\underline{i}$ components, can see that
$\overrightarrow{AC} = 5\overrightarrow{AB}$
Hence, by $\underline{j}$ components
$p+3 = 5 \times 7$
$p+3 = 35$
$p = 32$

b) $\overrightarrow{OD} = \lambda \overrightarrow{OB}$
$\overrightarrow{OD} = \lambda(4\underline{j} + 6\underline{k})$
$\overrightarrow{OD} = 4\lambda\underline{j} + 6\lambda\underline{k}$

Question 13 continued

$\overrightarrow{CD} = 4\lambda\mathbf{j} + 6\lambda\mathbf{k} - (-16\mathbf{i} + 32\mathbf{j} + 10\mathbf{k})$
$\overrightarrow{CD} = 4\lambda\mathbf{j} + 6\lambda\mathbf{k} + 16\mathbf{i} - 32\mathbf{j} - 10\mathbf{k}$
$\overrightarrow{CD} = 16\mathbf{i} + (4\lambda - 32)\mathbf{j} + (6\lambda - 10)\mathbf{k}$

This is parallel to $\overrightarrow{OA} = 4\mathbf{i} - 3\mathbf{j} + 5\mathbf{k}$
By comparing $\mathbf{i}$ components, we have
$\overrightarrow{CD} = 4\,\overrightarrow{OA}$
Now compare $\mathbf{j}$ components:
$4\lambda - 32 = 4 \times (-3)$
$4\lambda - 32 = -12$
$4\lambda = 20$
$\lambda = 5$

$\overrightarrow{OD} = 4\lambda\mathbf{j} + 6\lambda\mathbf{k}$
$\overrightarrow{OD} = 4 \times 5\,\mathbf{j} + 6 \times 5\,\mathbf{k}$
$\overrightarrow{OD} = 20\mathbf{j} + 30\mathbf{k}$
$|\overrightarrow{OD}| = \sqrt{20^2 + 30^2}$
$|\overrightarrow{OD}| = \sqrt{400 + 900}$
$|\overrightarrow{OD}| = \sqrt{1300}$
$|\overrightarrow{OD}| = \sqrt{13 \times 100}$
$|\overrightarrow{OD}| = \sqrt{100} \times \sqrt{13}$
$|\overrightarrow{OD}| = 10\sqrt{13}$.

14. (a) Express $\dfrac{3}{(2x-1)(x+1)}$ in partial fractions.

(3)

When chemical A and chemical B are mixed, oxygen is produced.

A scientist mixed these two chemicals and measured the total volume of oxygen produced over a period of time.

The total volume of oxygen produced, $V\,\text{m}^3$, t hours after the chemicals were mixed, is modelled by the differential equation

$$\frac{dV}{dt} = \frac{3V}{(2t-1)(t+1)} \qquad V \geqslant 0 \qquad t \geqslant k$$

where k is a constant.

Given that exactly 2 hours after the chemicals were mixed, a total volume of $3\,\text{m}^3$ of oxygen had been produced,

(b) solve the differential equation to show that

$$V = \frac{3(2t-1)}{(t+1)}$$

(5)

The scientist noticed that

- there was a **time delay** between the chemicals being mixed and oxygen being produced
- there was a **limit** to the total volume of oxygen produced

Deduce from the model

(c) (i) the **time delay** giving your answer in minutes,

(ii) the **limit** giving your answer in m^3

(2)

a) $\dfrac{3}{(2x-1)(x+1)} = \dfrac{A}{2x-1} + \dfrac{B}{x+1}$

$3 = A(x+1) + B(2x-1)$

$3 = Ax + A + 2Bx - B$

$3 = x(A+2B) + A - B$

$A+2B = 0$ ① $A - B = 3$ ②

① − ② $3B = -3$

$B = -1$

$A - (-1) = 3$

$A + 1 = 3$

Question 14 continued

$A = 2$

Hence,

$$\frac{3}{(2x-1)(x+1)} = \frac{2}{2x-1} - \frac{1}{x+1}$$

b)
$$\frac{dV}{dt} = \frac{3V}{(2t-1)(t+1)}$$
$$\frac{1}{V}\frac{dV}{dt} = \frac{3}{(2t-1)(t+1)}$$
$$\frac{1}{V}\frac{dV}{dt} = \frac{2}{2t-1} - \frac{1}{t+1}$$
$$\frac{1}{V}dV = \left(\frac{2}{2t-1} - \frac{1}{t+1}\right) dt$$
$$\int \frac{1}{V} dV = \int \left(\frac{2}{2t-1} - \frac{1}{t+1}\right) dt$$
$$\ln(V) = \ln(2t-1) - \ln(t+1) + c$$

At $t = 2$, $V = 3$

$$\ln(3) = \ln(2\times2-1) - \ln(2+1) + c$$
$$\ln(3) = \ln(4-1) - \ln(3) + c$$
$$\ln(3) = \ln(3) - \ln(3) + c$$
$$c = \ln(3)$$
$$\ln(V) = \ln(2t-1) - \ln(t+1) + \ln(3)$$
$$\ln(V) = \ln\left(\frac{2t-1}{t+1}\right) + \ln(3)$$
$$\ln(V) = \ln\left(\frac{3(2t-1)}{t+1}\right)$$
$$V = \frac{3(2t-1)}{t+1}$$

c) i) 30 mins

ii) $6m^3$

15. **In this question you must show all stages of your working.**

Solutions relying on calculator technology are not acceptable.

Given that the first three terms of a geometric series are

$$12\cos\theta \qquad 5+2\sin\theta \qquad \text{and} \qquad 6\tan\theta$$

(a) show that

$$4\sin^2\theta - 52\sin\theta + 25 = 0$$

(3)

Given that θ is an obtuse angle measured in radians,

(b) solve the equation in part (a) to find the exact value of θ

(2)

(c) show that the sum to infinity of the series can be expressed in the form

$$k(1-\sqrt{3})$$

where k is a constant to be found.

(5)

a) $\dfrac{5+2\sin(\theta)}{12\cos(\theta)} = \dfrac{6\tan(\theta)}{5+2\sin(\theta)}$

$(5+2\sin(\theta))^2 = 6\tan(\theta) \times 12\cos(\theta)$

$25 + 20\sin(\theta) + 4\sin^2(\theta) = 72\cos(\theta)\tan(\theta)$

$25 + 20\sin(\theta) + 4\sin^2(\theta) = 72\sin(\theta)$

$4\sin^2(\theta) - 52\sin(\theta) + 25 = 0$.

b) $4\sin^2(\theta) - 52\sin(\theta) + 25 = 0$

$(2\sin(\theta) - 1)(2\sin(\theta) - 25) = 0$

$\sin(\theta) = \frac{1}{2}$ $\qquad$ $\sin(\theta) = \frac{25}{2} \leftarrow$ No real solutions

$\theta = \frac{5\pi}{6}$

c) $12\cos\left(\frac{5\pi}{6}\right) = -6\sqrt{3}$

$5+2\sin\left(\frac{5\pi}{6}\right) = 6$ $\qquad$ $r = \dfrac{6}{-6\sqrt{3}} = -\dfrac{1}{\sqrt{3}}$ $\qquad$ $a = -6\sqrt{3}$

$6\tan\left(\frac{5\pi}{6}\right) = -2\sqrt{3}$

$S_\infty = \dfrac{a}{1-r}$

Question 15 continued

$$S_\infty = \frac{-6\sqrt{3}}{1-\left(-\frac{1}{\sqrt{3}}\right)}$$

$$S_\infty = \frac{-6\sqrt{3}}{1+\frac{1}{\sqrt{3}}}$$

$$S_\infty = \frac{-18}{\sqrt{3}+1}$$

$$S_\infty = \frac{-18(\sqrt{3}-1)}{(\sqrt{3}+1)(\sqrt{3}-1)}$$

$$S_\infty = \frac{18(1-\sqrt{3})}{3-1}$$

$$S_\infty = \frac{18(1-\sqrt{3})}{2}$$

$$S_\infty = 9(1-\sqrt{3})$$

16.

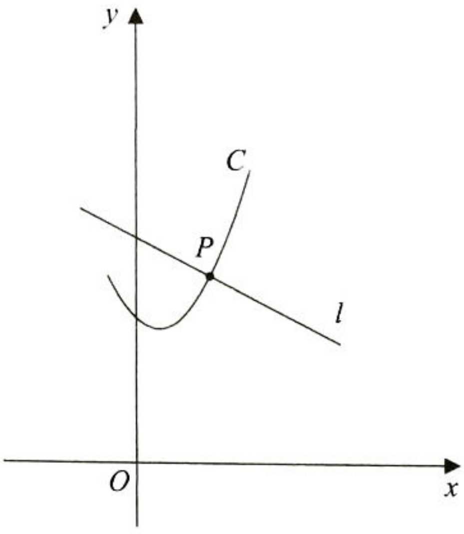

Figure 6

Figure 6 shows a sketch of the curve C with parametric equations

$$x = 2\tan t + 1 \qquad y = 2\sec^2 t + 3 \qquad -\frac{\pi}{4} \leqslant t \leqslant \frac{\pi}{3}$$

The line l is the normal to C at the point P where $t = \frac{\pi}{4}$

(a) Using parametric differentiation, show that an equation for l is

$$y = -\frac{1}{2}x + \frac{17}{2}$$

(5)

(b) Show that all points on C satisfy the equation

$$y = \frac{1}{2}(x-1)^2 + 5$$

(2)

The straight line with equation

$$y = -\frac{1}{2}x + k \qquad \text{where } k \text{ is a constant}$$

intersects C at two distinct points.

(c) Find the range of possible values for k.

(5)

a) $\frac{dy}{dx} = \frac{\frac{dy}{dt}}{\frac{dx}{dt}}$

P 6 9 6 0 2 A 0 4 4 4 8

Question 16 continued

$$\frac{dy}{dt} = 4\tan(t)\sec^2(t)$$

$$\frac{dx}{dt} = 2\sec^2(t)$$

$$\frac{dy}{dx} = \frac{4\tan(t)\sec^2(t)}{2\sec^2(t)}$$

$$\frac{dy}{dx} = 2\tan(t)$$

At $t = \frac{\pi}{4}$

$\frac{dy}{dx} = 2\tan\left(\frac{\pi}{4}\right) = 2 \Rightarrow$ Gradient $= -\frac{1}{2}$.

$y = 2\sec^2\left(\frac{\pi}{4}\right) + 3 = 7$

$x = 2\tan\left(\frac{\pi}{4}\right) + 1 = 3$

$$y - y_1 = m(x - x_1)$$

$$y - 7 = -\frac{1}{2}(x - 3)$$

$$y - 7 = -\frac{1}{2}x + \frac{3}{2}$$

$$y = -\frac{1}{2}x + \frac{17}{2}$$

b) $y = 2\sec^2(t) + 3$

$y = 2(\tan^2(t) + 1) + 3$

$x = 2\tan(t) + 1$

$x - 1 = 2\tan(t)$

$\tan(t) = \frac{x-1}{2}$

$y = 2\left(\left(\frac{x-1}{2}\right)^2 + 1\right) + 3$

Question 16 continued

$$y = 2\left(\frac{(x-1)^2}{4} + 1\right) + 3$$

$$y = \frac{(x-1)^2}{2} + 2 + 3$$

$$y = \frac{1}{2}(x-1)^2 + 5$$

c) $-\frac{1}{2}x + k = \frac{1}{2}(x-1)^2 + 5$

$$-x + 2k = (x-1)^2 + 10$$

$$-x + 2k = x^2 - 2x + 1 + 10$$

$$-x + 2k = x^2 - 2x + 11$$

$$x^2 - x + 11 - 2k = 0$$

For intersection at two distinct points,

$$b^2 - 4ac > 0$$

$$(-1)^2 - 4 \times 1 \times (11 - 2k) > 0$$

$$1 - 4(11 - 2k) > 0$$

$$1 - 44 + 8k > 0$$

$$-43 + 8k > 0$$

$$8k > 43$$

$$k > \frac{43}{8}$$

Lower bound is $\frac{43}{8}$.

Upper bound means line passes through the curve at $t = -\frac{\pi}{4}$.

$$x = 2\tan\left(-\frac{\pi}{4}\right) + 1 = -1$$

$$y = 2\sec^2\left(-\frac{\pi}{4}\right) + 3 = 7$$

Gradient $= -\frac{1}{2}$

$$y - y_1 = m(x - x_1)$$

$$y - 7 = -\frac{1}{2}(x + 1)$$

$$y - 7 = -\frac{1}{2}x - \frac{1}{2}$$

$$y = -\frac{1}{2}x + \frac{13}{2}$$

Upper bound for k

Hence: $\boxed{\frac{43}{8} \leq k \leq \frac{13}{2}}$