Please check the examination details below before entering your candidate information

Candidate surname	Other names

Centre Number | Candidate Number

Pearson Edexcel Level 3 GCE

Time 2 hours

Paper reference **9MA0/01**

Mathematics

Advanced
PAPER 1: Pure Mathematics 1

You must have:
Mathematical Formulae and Statistical Tables (Green), calculator

Total Marks

Candidates may use any calculator allowed by Pearson regulations. Calculators must not have the facility for symbolic algebra manipulation, differentiation and integration, or have retrievable mathematical formulae stored in them.

Instructions

- Use **black** ink or ball-point pen.
- If pencil is used for diagrams/sketches/graphs it must be dark (HB or B).
- **Fill in the boxes** at the top of this page with your name, centre number and candidate number.
- Answer **all** questions and ensure that your answers to parts of questions are clearly labelled.
- Answer the questions in the spaces provided – *there may be more space than you need.*
- You should show sufficient working to make your methods clear. Answers without working may not gain full credit.
- Inexact answers should be given to three significant figures unless otherwise stated.

Information

- A booklet 'Mathematical Formulae and Statistical Tables' is provided.
- There are 16 questions in this question paper. The total mark for this paper is 100.
- The marks for **each** question are shown in brackets – *use this as a guide as to how much time to spend on each question.*

Advice

- Read each question carefully before you start to answer it.
- Try to answer every question.
- Check your answers if you have time at the end.

Turn over

P 6 9 6 0 1 A 0 1 4 8

Pearson

1. The point $P(-2, -5)$ lies on the curve with equation $y = \mathrm{f}(x)$, $x \in \mathbb{R}$

Find the point to which P is mapped, when the curve with equation $y = \mathrm{f}(x)$ is transformed to the curve with equation

(a) $y = \mathrm{f}(x) + 2$

(1)

(b) $y = |\mathrm{f}(x)|$

(1)

(c) $y = 3\mathrm{f}(x - 2) + 2$

(2)

a) +2 to y-value
$(-2, -3)$

b) negative y value turns positive
$(-2, 5)$

c) $f(x-2)$ adds 2 to x-coordinate
$(0, -5)$
$3f(x-2)$ multiplies y-coordinate by 3
$(0, -15)$
+2 adds 2 to y-coordinate
$(0, -13)$

2. $$f(x) = (x - 4)(x^2 - 3x + k) - 42 \quad \text{where } k \text{ is a constant}$$

Given that $(x + 2)$ is a factor of $f(x)$, find the value of k.

(3)

$(x+2)$ is a factor $\Rightarrow f(-2) = 0$

$f(-2) = (-2-4)((-2)^2 - 3(-2) + k) - 42$

$(-2-4)((-2)^2 - 3(-2) + k) - 42 = 0$

$-6(4 + 6 + k) - 42 = 0$

$-6(10 + k) - 42 = 0$

$-60 - 6k - 42 = 0$

$6k = -102$

$k = -17$

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3. A circle has equation

$$x^2 + y^2 - 10x + 16y = 80$$

(a) Find

(i) the coordinates of the centre of the circle,

(ii) the radius of the circle.

(3)

Given that P is the point on the circle that is furthest away from the origin O,

(b) find the exact length OP

(2)

a) i) $x^2 + y^2 - 10x + 16y = 80$

$(x-5)^2 - 25 + (y+8)^2 - 64 = 80$

$(x-5)^2 + (y+8)^2 - 89 = 80$

$(x-5)^2 + (y+8)^2 = 169$

Centre $= (5, -8)$

ii) Radius $= \sqrt{169}$

Radius $= 13$

b) Distance from O to centre is

$\sqrt{5^2+8^2} = \sqrt{25+64} = \sqrt{89}$

Distance from centre to furthest point $= 13$

Total distance $= 13 + \sqrt{89}$

4. (a) Express $\lim_{\delta x \to 0} \sum_{x=2.1}^{6.3} \frac{2}{x} \delta x$ as an integral.

(1)

(b) Hence show that

$$\lim_{\delta x \to 0} \sum_{x=2.1}^{6.3} \frac{2}{x} \delta x = \ln k$$

where k is a constant to be found.

(2)

a) $\lim_{\delta x \to 0} \sum_{x=2.1}^{6.3} \frac{2}{x} \delta x = \int_{2.1}^{6.3} \frac{2}{x} dx$

b) $\lim_{\delta x \to 0} \sum_{x=2.1}^{6.3} \frac{2}{x} \delta x = \int_{2.1}^{6.3} \frac{2}{x} dx$

$= \left[2\ln(x) \right]_{2.1}^{6.3}$

$= 2\ln(6.3) - 2\ln(2.1)$

$= 2(\ln(6.3) - \ln(2.1))$

$= 2\ln(6.3 \div 2.1)$

$= 2\ln(3)$

$= \ln(3^2)$

$= \ln(9)$

5. The height, h metres, of a tree, t years after being planted, is modelled by the equation

$$h^2 = at + b \qquad 0 \leqslant t < 25$$

where a and b are constants.

Given that

- the height of the tree was 2.60 m, exactly 2 years after being planted
- the height of the tree was 5.10 m, exactly 10 years after being planted

(a) find a complete equation for the model, giving the values of a and b to 3 significant figures.

(4)

Given that the height of the tree was 7 m, exactly 20 years after being planted

(b) evaluate the model, giving reasons for your answer.

(2)

5)a) $h^2 = at + b$

$2.6^2 = 2a + b$

$2a + b = 6.76$ ①

$h^2 = at + b$

$5.1^2 = 10a + b$

$26.01 = 10a + b$ ②

② - ① $10a + b - (2a + b) = 26.01 - 6.76$

$10a + b - 2a - b = 19.25$

$8a = 19.25$

$a = 2.40625$

$a = 2.41$ 3sf

Sub into ① $2(2.40625) + b = 6.76$

$4.8125 + b = 6.76$

$b = 1.9475$

$b = 1.95$ 3sf

b) $h^2 = 2.40625t + 1.9475$

Substitute $h = 7$

$7^2 = 2.40625t + 1.9475$

$49 = 2.40625t + 1.9475$

$47.0525 = 2.40625t$

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Question 5 continued

$t = 19.5542\ldots$ years.
This is close to 20 years, so the model is accurate.

(Total for Question 5 is 6 marks)

6.

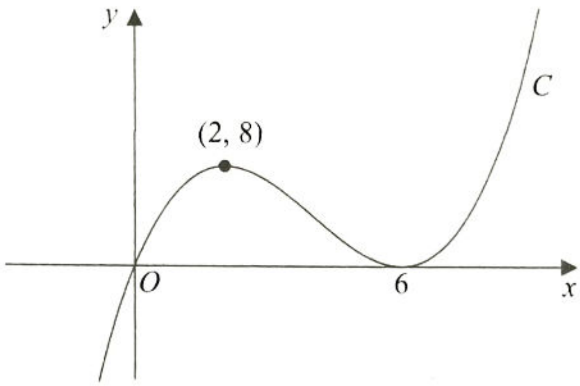

Figure 1

Figure 1 shows a sketch of a curve C with equation $y = f(x)$ where $f(x)$ is a cubic expression in x.

The curve

- passes through the origin
- has a maximum turning point at $(2, 8)$
- has a minimum turning point at $(6, 0)$

(a) Write down the set of values of x for which

$$f'(x) < 0$$

(1)

The line with equation $y = k$, where k is a constant, intersects C at only one point.

(b) Find the set of values of k, giving your answer in set notation.

(2)

(c) Find the equation of C. You may leave your answer in factorised form.

(3)

a) This is where the curve is decreasing:
$2 < x < 6$

b) Must be above the highest turning point ($y = 8$)
or below the lowest turning point ($y = 0$)
$k < 0$, $k > 8$ $\{k : k < 0\} \cup \{k : k > 8\}$

c) $f(x)$ has a root at $x = 0$ and a double root at $x = 6$
$f(x)$ has a factor of x and a double factor of $(x - 6)$

Question 6 continued

$g(x) = ax(x-6)^2$.

Substitute $(2, 8)$ to find a:

$g(2) = a \times 2 \times (2-6)^2$

$a \times 2 \times (2-6)^2 = 8$

$2a \times (-4)^2 = 8$

$2a \times 16 = 8$

$32a = 8$

$a = \frac{1}{4}$.

$g(x) = \frac{1}{4}x(x-6)^2$.

(Total for Question 6 is 6 marks)

7. (i) Given that p and q are integers such that

$$pq \text{ is even}$$

use algebra to prove by contradiction that at least one of p or q is even.

(3)

(ii) Given that x and y are integers such that

- $x < 0$
- $(x + y)^2 < 9x^2 + y^2$

show that $y > 4x$

(2)

i) Suppose both p and q are odd

$p = 2m - 1$ for integer m

$q = 2n - 1$ for integer n

$pq = (2m-1)(2n-1)$

$pq = 4mn - 2m - 2n + 1$

$pq = 2(2mn - m - n) + 1$

$2mn - m - n$ is an integer because m and n are integers.

Therefore, $2(2mn - m - n)$ is even.

Hence, $2(2mn - m - n) + 1$ must be odd.

Hence, pq is odd.

Therefore, by contradiction, if pq is even, then at least one of p and q must be even.

ii) $(x+y)^2 < 9x^2 + y^2$

$x^2 + 2xy + y^2 < 9x^2 + y^2$

$x^2 + 2xy < 9x^2$

$2xy < 8x^2$

$xy < 4x^2$

Since $x < 0$ we flip the sign when dividing by x:

$y > 4x$

8.

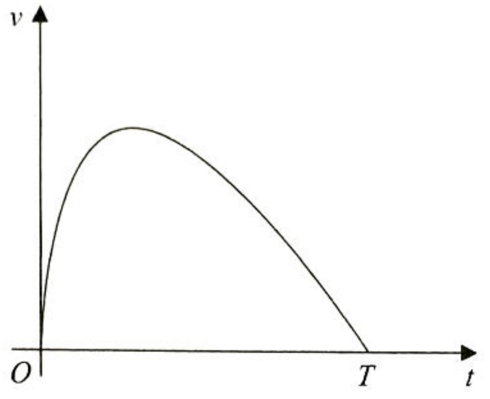

Figure 2

A car stops at two sets of traffic lights.

Figure 2 shows a graph of the speed of the car, $v\,\mathrm{ms}^{-1}$, as it travels between the two sets of traffic lights.

The car takes T seconds to travel between the two sets of traffic lights.

The speed of the car is modelled by the equation

$$v = (10 - 0.4t)\ln(t + 1) \qquad 0 \leqslant t \leqslant T$$

where t seconds is the time after the car leaves the first set of traffic lights.

According to the model,

(a) find the value of T

(1)

(b) show that the maximum speed of the car occurs when

$$t = \frac{26}{1 + \ln(t + 1)} - 1$$

(4)

Using the iteration formula

$$t_{n+1} = \frac{26}{1 + \ln(t_n + 1)} - 1$$

with $t_1 = 7$

(c) (i) find the value of t_3 to 3 decimal places,

(ii) find, by repeated iteration, the time taken for the car to reach maximum speed.

(3)

a) $v = (10 - 0.4t)\ln(t+1)$

At T, $v = 0$

$(10 - 0.4T)\ln(T+1) = 0$

Question 8 continued

$10-0.4T=0$ or $\ln(T+1)=0$

$0.4T=10$ $\quad T+1=1$

$T=25s$ $\quad T=0$

Discount this solution

$T=25$ only

b) $v=(10-0.4t)\ln(t+1)$

$$\frac{dv}{dt}=\frac{10-0.4t}{t+1}-0.4\ln(t+1)$$

$\frac{dv}{dt}=0$ at maximum.

$$\frac{10-0.4t}{t+1}-0.4\ln(t+1)=0$$

$$10-0.4t-0.4(t+1)\ln(t+1)=0$$

$$10-0.4t-0.4t\ln(t+1)-0.4\ln(t+1)=0$$

$$10-0.4\ln(t+1)=0.4t+0.4t\ln(t+1)$$

$$25-\ln(t+1)=t+t\ln(t+1)$$

$$25-\ln(t+1)=t(1+\ln(t+1))$$

$$t=\frac{25-\ln(t+1)}{1+\ln(t+1)}$$

$$t=\frac{26-1-\ln(t+1)}{1+\ln(t+1)}$$

$$t=\frac{26}{1+\ln(t+1)}-\frac{1+\ln(t+1)}{1+\ln(t+1)}$$

$$t=\frac{26}{1+\ln(t+1)}-1$$

c) i) $t_{n+1}=\dfrac{26}{1+\ln(t_n+1)}-1$

$t_1=7$

$t_2=7.443\ldots$

$t_3=7.298$ (3dp)

ii) $t_4=7.3440\ldots$

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Question 8 continued

$10 - 0.4T = 0$ or $\ln(T+1) = 0$

$0.4T = 10$ $T + 1 = 1$

$T = 25\text{s}$ $T = 0$

Discount this solution

$T = 25$ only

b) $v = (10 - 0.4t)\ln(t+1)$

$$\frac{dv}{dt} = \frac{10 \cdot 0.4t}{t+1} - 0.4\ln(t+1)$$

$\frac{dv}{dt} = 0$ at maximum

$$\frac{10 - 0.4t}{t+1} - 0.4\ln(t+1) = 0$$

$$10 - 0.4t - 0.4(t+1)\ln(t+1) = 0$$

$$10 - 0.4t - 0.4t\ln(t+1) - 0.4\ln(t+1) = 0$$

$$10 - 0.4\ln(t+1) = 0.4t + 0.4t\ln(t+1)$$

$$25 - \ln(t+1) = t + t\ln(t+1)$$

$$25 - \ln(t+1) = t(1 + \ln(t+1))$$

$$t = \frac{25 - \ln(t+1)}{1 + \ln(t+1)}$$

$$t = \frac{26 - 1 - \ln(t+1)}{1 + \ln(t+1)}$$

$$t = \frac{26}{1 + \ln(t+1)} - \frac{1 + \ln(t+1)}{1 + \ln(t+1)}$$

$$t = \frac{26}{1 + \ln(t+1)} - 1$$

c) i) $t_{n+1} = \frac{26}{1 + \ln(t_n + 1)} - 1$

$t_1 = 7$

$t_2 = 7.443\ldots$

$t_3 = 7.298$ (3dp)

ii) $t_4 = 7.3440\ldots$

Question 8 continued

$t_5 = 7.3292\ldots$
$t_6 = 7.3339\ldots$
$t_7 = 7.3324\ldots$
$t_8 = 7.3329\ldots$
$t_9 = 7.3327\ldots$

t_8 and t_9 are both 7.333 to 3dp.
Hence, $t = 7.333$ s.

9.

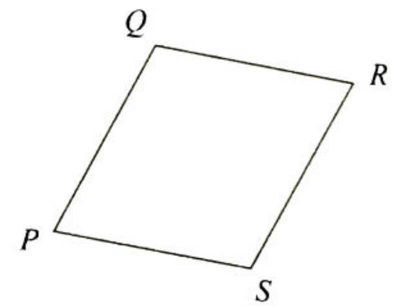

Figure 3

Figure 3 shows a sketch of a parallelogram $PQRS$.

Given that

- $\overrightarrow{PQ} = 2\mathbf{i} + 3\mathbf{j} - 4\mathbf{k}$
- $\overrightarrow{QR} = 5\mathbf{i} - 2\mathbf{k}$

(a) show that parallelogram $PQRS$ is a rhombus.

(2)

(b) Find the exact area of the rhombus $PQRS$.

(4)

a) Rhombus has equal length sides i.e. $|\overrightarrow{PQ}| = |\overrightarrow{QR}|$

$|\overrightarrow{PQ}| = \sqrt{2^2+3^2+(-4)^2}$
$= \sqrt{4+9+16}$
$= \sqrt{29}$
$|\overrightarrow{QR}| = \sqrt{5^2+(-2)^2}$
$= \sqrt{25+4}$
$= \sqrt{29}$

$|\overrightarrow{PQ}| = |\overrightarrow{QR}|$ so this is a rhombus

b) Area of rhombus $= \frac{1}{2} \times$ product of diagonals.
In this case $\frac{1}{2}|\overrightarrow{PR}||\overrightarrow{QS}|$

$|\overrightarrow{PQ}| = |2i+3j-4k|$
$|\overrightarrow{QR}| = |5i-2k|$
$|\overrightarrow{RS}| = |-2i-3j+4k|$ parallel to $|\overrightarrow{PQ}|$
$|\overrightarrow{SP}| = |-5i+2k|$ parallel to $|\overrightarrow{QR}|$

Question 9 continued

$$|\overrightarrow{PR}| = |2i + 3j - 4k + 5i - 2k|$$
$$= |7i + 3j - 6k|$$
$$= \sqrt{7^2 + 3^2 + (-6)^2}$$
$$= \sqrt{49 + 9 + 36}$$
$$= \sqrt{94}$$

$$|\overrightarrow{QS}| = |5i - 2k - 2i - 3j + 4k|$$
$$= |3i - 3j + 2k|$$
$$= \sqrt{3^2 + (-3)^2 + 2^2}$$
$$= \sqrt{9 + 9 + 4}$$
$$= \sqrt{22}$$

$$\text{Area} = \frac{1}{2}|\overrightarrow{PR}||\overrightarrow{QS}|$$
$$= \frac{1}{2}\sqrt{94}\sqrt{22}$$
$$= \sqrt{\frac{1}{4} \times 94 \times 22}$$
$$= \sqrt{47 \times 11}$$
$$= \sqrt{517}$$

10. A scientist is studying the number of bees and the number of wasps on an island.

The number of bees, measured in thousands, N_b, is modelled by the equation

$$N_b = 45 + 220\,e^{0.05t}$$

where t is the number of years from the start of the study.

According to the model,

(a) find the number of bees at the start of the study, **(1)**

(b) show that, exactly 10 years after the start of the study, the number of bees was increasing at a **rate** of approximately 18 thousand per year. **(3)**

The number of wasps, measured in thousands, N_w, is modelled by the equation

$$N_w = 10 + 800\,e^{-0.05t}$$

where t is the number of years from the start of the study.

When $t = T$, according to the models, there are an equal number of bees and wasps.

(c) Find the value of T to 2 decimal places. **(4)**

a) $N_b = 45 + 220e^{0.05t}$

At start of study, $t=0$

$N_b = 45+220e^{0.05\times 0}$

$N_b = 45+220e^{0}$

$N_b = 45+220\times 1$

$N_b = 45+220$

$N_b = 265$

265 000 bees

b) $\frac{dN_b}{dt} = 0.05e^{0.05t} \times 220$

$\frac{dN_b}{dt} = 11e^{0.05t}$

At $t=10$:

$\frac{dN_b}{dt} = 11e^{0.05\times 10}$

$\frac{dN_b}{dt} = 11e^{0.5}$

Question 10 continued

$\frac{dN_b}{dt} = 18.1$ (3sf)

18.1 thousand or 18100

c) $N_b = 45 + 220e^{0.05t}$

$N_w = 10 + 800e^{-0.05t}$

At $t = T$, $N_b = N_w$.

$45 + 220e^{0.05T} = 10 + 800e^{-0.05T}$

$220e^{0.05T} + 35 - 800e^{-0.05T} = 0$

$220(e^{0.05T})^2 + 35e^{0.05T} - 800 = 0$

$$e^{0.05T} = \frac{-35 \pm \sqrt{35^2 - 4 \times 220 \times (-800)}}{2 \times 220}$$

$e^{0.05T} > 0$ always so discount negative option

$$e^{0.05T} = \frac{-35 + \sqrt{35^2 - 4 \times 220 \times (-800)}}{2 \times 220}$$

$$e^{0.05T} = \frac{-35 + \sqrt{1225 + 704000}}{440}$$

$$e^{0.05T} = -\frac{7}{88} + \frac{\sqrt{705225}}{440}$$

$$e^{0.05T} = \frac{-7 + \sqrt{28209}}{88}$$

$$0.05T = \ln\left(\frac{-7 + \sqrt{28209}}{88}\right)$$

$$T = 20\ln\left(\frac{-7 + \sqrt{28209}}{88}\right)$$

$T = 12.08$ yr (2dp)

11.

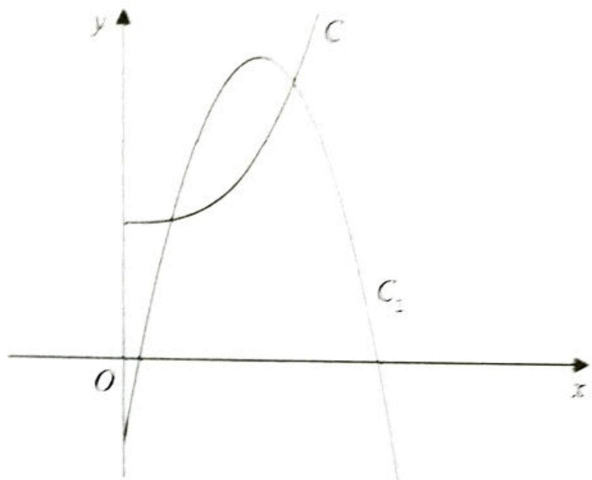

Figure 4

Figure 4 shows a sketch of part of the curve C_1 with equation

$$y = 2x^3 + 10 \qquad x > 0$$

and part of the curve C_2 with equation

$$y = 42x - 15x^2 - 7 \qquad x > 0$$

(a) Verify that the curves intersect at $x = \frac{1}{2}$

(2)

The curves intersect again at the point P

(b) Using algebra and showing all stages of working, find the exact x coordinate of P

(5)

a) C_1 : $y = 2x^3 + 10$

At $x = \frac{1}{2}$:

$y = 2(\frac{1}{2})^3 + 10$

$y = 2(\frac{1}{8}) + 10$

$y = \frac{1}{4} + 10$

$y = \frac{41}{4}$

C_2 : $y = 42x - 15x^2 - 7$

At $x = \frac{1}{2}$:

$y = 42(\frac{1}{2}) - 15(\frac{1}{2})^2 - 7$

$y = 21 - 15(\frac{1}{4}) - 7$

$y = 14 - \frac{15}{4}$

$y = \frac{41}{4}$

Question 11 continued

b) $2x^3 + 10 = 42x - 15x^2 - 7$

$2x^3 + 15x^2 - 42x + 17 = 0$

Since the curves intersect at $x = \frac{1}{2}$, $(2x-1)$ must be a factor.

$(2x-1)(x^2 + 8x - 17) = 0$

Want positive root of $x^2 + 8x - 17 = 0$

$$x = \frac{-8 \pm \sqrt{8^2 - 4 \times 1 \times (-17)}}{2 \times 1}$$

$$x = \frac{-8 \pm \sqrt{64 + 68}}{2}$$

$$x = -4 \pm \frac{\sqrt{132}}{2}$$

$$x = -4 \pm \sqrt{33}$$

$x = -4 + \sqrt{33}$ only

12. **In this question you must show all stages of your working.**

Solutions relying on calculator technology are not acceptable.

Show that

$$\int_1^{e^2} x^3 \ln x \, dx = ae^8 + b$$

where a and b are rational constants to be found.

(5)

$I = \int_1^{e^2} x^3 \ln x \, dx$

$u = \ln x \qquad v = \frac{1}{4}x^4$

$\frac{du}{dx} = \frac{1}{x} \qquad \frac{dv}{dx} = x^3$

$I = uv - \int v \frac{du}{dx}$

$I = \left[\frac{1}{4}x^4 \ln(x)\right]_1^{e^2} - \int_1^{e^2} \frac{1}{4}x^4 \times \frac{1}{x} \, dx$

$I = \frac{1}{4}(e^2)^4 \ln(e^2) - \frac{1}{4}(1)^4 \ln(1) - \int_1^{e^2} \frac{1}{4}x^3 \, dx$

$I = \frac{1}{4}e^8 \times 2 - \frac{1}{4} \times 1 \times 0 - \left[\frac{1}{16}x^4\right]_1^{e^2}$

$I = \frac{1}{2}e^8 - \frac{1}{16}(e^2)^4 + \frac{1}{16}(1)^4$

$I = \frac{1}{2}e^8 - \frac{1}{16}e^8 + \frac{1}{16}$

$I = \frac{7}{16}e^8 + \frac{1}{16}$

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13. (i) In an arithmetic series, the first term is a and the common difference is d.

Show that

$$S_n = \frac{n}{2}[2a + (n-1)d]$$

(3)

(ii) James saves money over a number of weeks to buy a printer that costs £64

He saves £10 in week 1, £9.20 in week 2, £8.40 in week 3 and so on, so that the weekly amounts he saves form an arithmetic sequence.

Given that James takes n weeks to save exactly £64

(a) show that

$$n^2 - 26n + 160 = 0$$

(2)

(b) Solve the equation

$$n^2 - 26n + 160 = 0$$

(1)

(c) Hence state the number of weeks James takes to save enough money to buy the printer, giving a brief reason for your answer.

(1)

i) $S = a + a+d + a+2d \ldots + l-2d + l-d + l$

$S = l + l-d + l-2d \ldots + a+2d + a+d + a$

where l is the last term $l = a+(n-1)d$

Add these together:

$2S = a+l + a+l + \ldots + a+l$

and there are n of these terms.

$2S = n(a+l)$

$S = \frac{n}{2}(a+l)$

$S = \frac{n}{2}(a + a + (n-1)d)$

$S = \frac{n}{2}(2a + (n-1)d)$

ii) a) This is an arithmetic series with

$a = 10 \quad d = -0.8$

$S = \frac{n}{2}(2 \times 10 + (n-1) \times (-0.8))$

$S = 64$

$\frac{n}{2}(2 \times 10 + (n-1) \times (-0.8)) = 64$

$n(20 - 0.8(n-1)) = 128$

Question 13 continued

$20n - 0.8n(n-1) = 128$

$25n - n(n-1) = 160$

$25n - n^2 + n = 160$

$26n - n^2 = 160$

$n^2 - 26n + 160 = 0.$

b) $n^2 - 26n + 160 = 0$

$(n-16)(n-10) = 0$

$n = 10, \quad n = 16$

c) 10 weeks because this is the first time he hits the goal so it makes sense to buy the printer at this point.

14. **In this question you must show all stages of your working.**

Solutions relying entirely on calculator technology are not acceptable.

(a) Given that

$$2\sin(x - 60°) = \cos(x - 30°)$$

show that

$$\tan x = 3\sqrt{3}$$

(4)

(b) Hence or otherwise solve, for $0 \leqslant \theta < 180°$

$$2\sin 2\theta = \cos(2\theta + 30°)$$

giving your answers to one decimal place.

(4)

a) $2\sin(x-60) = \cos(x-30)$

$2(\sin(x)\cos(60) - \cos(x)\sin(60)) = \cos(x)\cos(30) + \sin(x)\sin(30)$

$2\sin(x)(\frac{1}{2}) - 2\cos(x)(\frac{\sqrt{3}}{2}) = \frac{\sqrt{3}}{2}\cos(x) + \frac{1}{2}\sin(x)$

$\sin(x) - \sqrt{3}\cos(x) = \frac{\sqrt{3}}{2}\cos(x) + \frac{1}{2}\sin(x)$

$2\sin(x) - 2\sqrt{3}\cos(x) = \sqrt{3}\cos(x) + \sin(x)$

$\sin(x) - 2\sqrt{3}\cos(x) = \sqrt{3}\cos(x)$

$\sin(x) = 3\sqrt{3}\cos(x)$

$\frac{\sin(x)}{\cos(x)} = 3\sqrt{3}$

$\tan(x) = 3\sqrt{3}$

b) $2\sin(2\theta) = \cos(2\theta + 30)$

$x - 60 = 2\theta$ $\qquad$ $x - 30 = 2\theta + 30$

$\Rightarrow x = 2\theta + 60$

$\tan(2\theta + 60) = 3\sqrt{3}$

$2\theta + 60 = 79.1°, 259.1°$

$2\theta = 19.1°, 199.1°$ $\qquad$ all to 2dp

$\theta = 9.6°, 99.6°$

15.

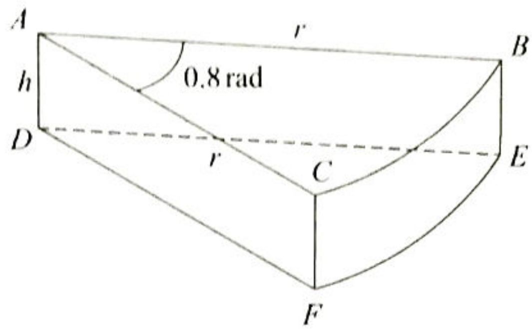

Figure 5

A company makes toys for children.

Figure 5 shows the design for a solid toy that looks like a piece of cheese.

The toy is modelled so that

- face ABC is a sector of a circle with radius r cm and centre A
- angle $BAC = 0.8$ radians
- faces ABC and DEF are congruent
- edges AD, CF and BE are perpendicular to faces ABC and DEF
- edges AD, CF and BE have length h cm

Given that the volume of the toy is $240\,\text{cm}^3$

(a) show that the surface area of the toy, $S\,\text{cm}^2$, is given by

$$S = 0.8r^2 + \frac{1680}{r}$$

making your method clear.

(4)

Using algebraic differentiation,

(b) find the value of r for which S has a stationary point.

(4)

(c) Prove, by further differentiation, that this value of r gives the minimum surface area of the toy.

(2)

a) Find surface area in terms of r and h.
Top face: $\frac{1}{2}r^2\theta = \frac{1}{2} \times r^2 \times 0.8 = 0.4r^2$
Bottom face = top face $= 0.4r^2$
Rectangle face $= rh$
Other rectangle face $= rh$
Curved face $= hr\theta = 0.8rh$

Question 15 continued

DO NOT WRITE IN THIS AREA

$\text{Total} = 0.4r^2 + 0.4r^2 + rh + rh + 0.8rh$

$\text{Total} = 0.8r^2 + 2.8rh$.

Use volume to find equation linking r and h.

$V = \frac{1}{2}r^2\theta h$

$\frac{1}{2}r^2\theta h = 240$

$\frac{1}{2} \times 0.8 \times r \times rh = 240$

$0.4r \times rh = 240$

$rh = \frac{240}{0.4r}$

$rh = \frac{600}{r}$

Substitute into area equation:

$S = 0.8r^2 + 2.8\left(\frac{600}{r}\right)$

$S = 0.8r^2 + \frac{1680}{r}$

b) $\frac{dS}{dr} = 1.6r - \frac{1680}{r^2}$

$1.6r - \frac{1680}{r^2} = 0$

$1.6r^3 - 1680 = 0$

$r^3 - 1050 = 0$

$r^3 = 1050$

$r = 10.2$ (3sf)

c) $\frac{d^2S}{dr^2} = 1.6 + \frac{3360}{r^3}$

$r^3 = 1050$ at stationary point

$\frac{d^2S}{dr^2} = 1.6 + \frac{3360}{1050}$

$\frac{d^2S}{dr^2} = 4.8 > 0$

Since this is positive, the point must be a minimum.

P 6 9 6 0 1 A 0 4 3 4 8

16.

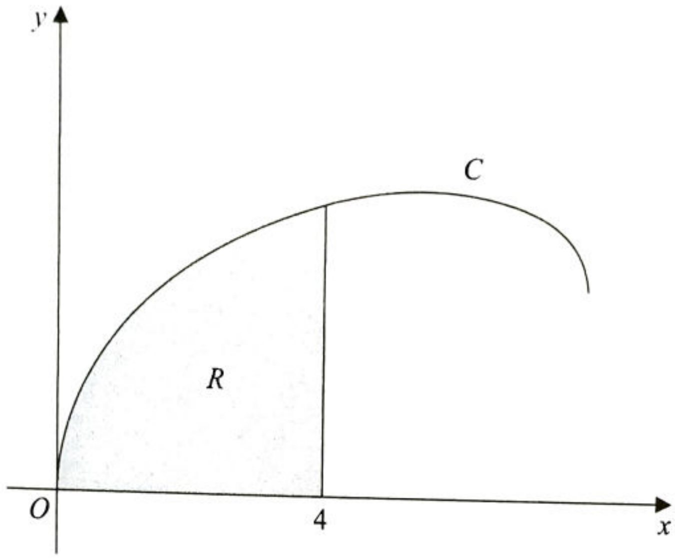

Figure 6

Figure 6 shows a sketch of the curve C with parametric equations

$$x = 8\sin^2 t \qquad y = 2\sin 2t + 3\sin t \qquad 0 \leqslant t \leqslant \frac{\pi}{2}$$

The region R, shown shaded in Figure 6, is bounded by C, the x-axis and the line with equation $x = 4$

(a) Show that the area of R is given by

$$\int_0^a (8 - 8\cos 4t + 48\sin^2 t\cos t)\,\mathrm{d}t$$

where a is a constant to be found.

(5)

(b) Hence, using algebraic integration, find the exact area of R.

(4)

a) $R = \int_{x=0}^{x=4} y\,dx$

At $x=0$, $t=0$

For $x=4$: $8\sin^2 t = 4$

$\sin^2 t = \frac{1}{2}$

$\sin t = \frac{\sqrt{2}}{2}$

$t = \frac{\pi}{4}$

$R = \int_{t=0}^{t=\pi/4} y\,dx$

$y = 2\sin 2t + 3\sin t$

$x = 8\sin^2 t$

$dx = 16\sin t\cos t\,dt$

Question 16 continued

$R = \int_0^{\pi/4} (2\sin 2t + 3\sin t)(16\sin t\cos t)\,dt$

$R = \int_0^{\pi/4} (4\sin t\cos t + 3\sin t)(16\sin t\cos t)\,dt$

$R = \int_0^{\pi/4} 64\sin^2 t\cos^2 t + 48\sin^2 t\cos t\,dt$

$R = \int_0^{\pi/4} 16(4\sin^2 t\cos^2 t) + 48\sin^2 t\cos t\,dt$

$R = \int_0^{\pi/4} 16(2\sin t\cos t)^2 + 48\sin^2 t\cos t\,dt$

$R = \int_0^{\pi/4} 16\sin^2 2t + 48\sin^2 t\cos t\,dt$

$R = \int_0^{\pi/4} 16\left(\tfrac{1}{2} - \tfrac{1}{2}\cos 4t\right) + 48\sin^2 t\cos t\,dt$

$R = \int_0^{\pi/4} 8 - 8\cos 4t + 48\sin^2 t\cos t\,dt$

b) First note that $\frac{d}{dt}(\sin^3 t) = 3\sin^2 t\cos t$

$\Rightarrow \quad R = \left[8t - 2\sin 4t + 16\sin^3 t\right]_0^{\pi/4}$

$R = 8\left(\frac{\pi}{4}\right) - 2\sin\left(4\left(\frac{\pi}{4}\right)\right) + 16\sin^3\left(\frac{\pi}{4}\right)$

$-8(0) + 2\sin(4(0)) - 16\sin^3(0)$

$R = 2\pi - 2\sin(\pi) + 16\left(\frac{\sqrt{2}}{2}\right)^3$

$R = 2\pi + \frac{16 \times 2 \times \sqrt{2}}{8}$

$R = 2\pi + 4\sqrt{2}$