Mark Scheme (Results)

Summer 2022

Pearson Edexcel GCE
AL Further Mathematics (9FM0)
Paper 4D Decision Mathematics 2

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June 2022
Publications Code 9FM0_4D_2206_MS

General General Marking Guidance

• All candidates must receive the same treatment. Examiners must mark the first candidate in exactly the same way as they mark the last.

• Mark schemes should be applied positively. Candidates must be rewarded for what they have shown they can do rather than penalised for omissions.

• Examiners should mark according to the mark scheme not according to their perception of where the grade boundaries may lie.

• There is no ceiling on achievement. All marks on the mark scheme should be used appropriately.

• All the marks on the mark scheme are designed to be awarded. Examiners should always award full marks if deserved, i.e. if the answer matches the mark scheme. Examiners should also be prepared to award zero marks if the candidate's response is not worthy of credit according to the mark scheme.

• Where some judgement is required, mark schemes will provide the principles by which marks will be awarded and exemplification may be limited.

• When examiners are in doubt regarding the application of the mark scheme to a candidate's response, the team leader must be consulted.

• Crossed out work should be marked UNLESS the candidate has replaced it with an alternative response.

PEARSON EDEXCEL GCE MATHEMATICS

General Instructions for Marking

1. The total number of marks for the paper is 75.

2. These mark schemes use the following types of marks:

   - **M** marks: Method marks are awarded for 'knowing a method and attempting to apply it', unless otherwise indicated.
   - **A** marks: Accuracy marks can only be awarded if the relevant method (M) marks have been earned.
   - **B** marks are unconditional accuracy marks (independent of M marks)

   Marks should not be subdivided.

3. Abbreviations

   These are some of the traditional marking abbreviations that will appear in the mark schemes.

   - **bod** – benefit of doubt
   - **ft** – follow through
   - the symbol $\surd$ will be used for correct ft
   - **cao** – correct answer only
   - **cso** - correct solution only. There must be no errors in this part of the question to obtain this mark
   - **isw** – ignore subsequent working
   - **awrt** – answers which round to
   - **SC**: special case
   - **o.e.** – or equivalent (and appropriate)
   - **d** or **dep** – dependent
   - **indep** – independent
   - **dp** decimal places
   - **sf** significant figures
   - $*$ The answer is printed on the paper or ag- answer given

4. All M marks are follow through.

   A marks are 'correct answer only' (cao.), unless shown, for example, as A1 ft to indicate that previous wrong working is to be followed through. After a misread however, the subsequent A marks affected are treated as A ft, but answers that don't logically make sense e.g. if an answer given for a probability is >1 or <0, should never be awarded A marks.
   be awarded A marks.

5. For misreading which does not alter the character of a question or materially simplify it, deduct

two from any A or B marks gained, in that part of the question affected.

6. Where a candidate has made multiple responses and indicates which response they wish to submit, examiners should mark this response. If there are several attempts at a question which have not been crossed out, examiners should mark the final answer which is the answer that is

the most complete.

7. Ignore wrong working or incorrect statements following a correct answer.

8. Mark schemes will firstly show the solution judged to be the most common response expected

from candidates. Where appropriate, alternatives answers are provided in the notes. If examiners are not sure if an answer is acceptable, they will check the mark scheme to see if an alternative answer is given for the method used. If no such alternative answer is provided but the response is deemed to be valid, examiners must escalate the response for a senior examiner to review.

Question	Scheme	Marks	AOs
1(a)	Reduce rows $\begin{bmatrix} 0 & 13 & 2 & 16 \\ 0 & 2 & 13 & 9 \\ 6 & 4 & 0 & 2 \\ 0 & 2 & 5 & 9 \end{bmatrix}$ and then columns $\begin{bmatrix} 0 & 11 & 2 & 14 \\ 0 & 0 & 13 & 7 \\ 6 & 2 & 0 & 0 \\ 0 & 0 & 5 & 7 \end{bmatrix}$	M1 A1	2.1 1.1b
	Followed by $\begin{bmatrix} 0 & 11 & 0 & 12 \\ 0 & 0 & 11 & 5 \\ 8 & 4 & 0 & 0 \\ 0 & 0 & 3 & 5 \end{bmatrix}$	M1 A1ft	2.1 1.1b
	A – 3, B – 1, C – 4, D – 2 or A – 3, B – 2, C – 4, D – 1	A1ft	2.2a
		(5)	
(b)	158	B1	1.1b
		(1)	
			(6 marks)

Notes:

(a) M1: simplifying the initial matrix by reducing rows and then columns. (Allow up to 2 independent slips).

A1:CAO

M1: develop an improved solution – need to see one double covered +e; one uncovered –e; and one single covered unchanged. 3 lines needed to 4 lines needed (lines may be implied). If lines are drawn they must be correct.

A1ft: CAO following on from row and column reduction final table. (f/t from previous table with no further slips).

A1ft: correct allocation ft their optimal table (both previous M marks must have been awarded in (a)) (Must be fully written. Do not accept just indicated on zeros on final matrix).

(b)B1:CAO – solution of original problem

Question	Scheme	Marks	AOs
2(a)	$(m+3)^2=0$	M1	3.1a
	$k_1=6$ and $k_2=9$	A1	1.1b
		(2)	
(b)	$u_0=1\Rightarrow A(-3)^0=1$ $u_1=1\Rightarrow (A+B)(-3)=1$	M1	1.1b
	$A=1$ and $B=-\frac{4}{3}$	A1	1.1b
		(2)	
			(4 marks)

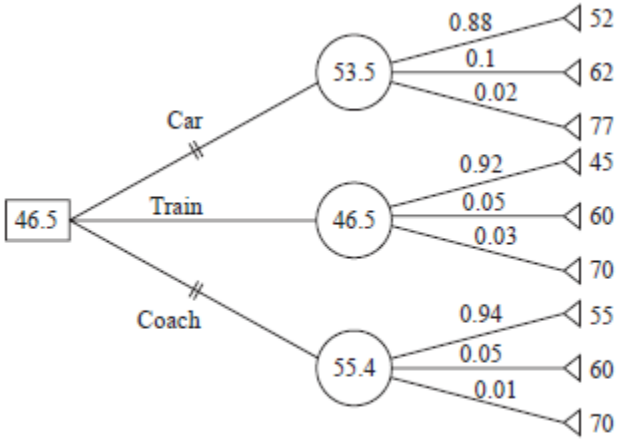
Notes:

(a) M1: Correct auxiliary equation (may be implied by 6, 9 correct) from $u_n=(A+Bn)(-3)^n$

A1: CAO – allow values stated implicitly i.e., $u_{n+2}+6u_{n+1}+9u_n=0$

(b) M1: Uses $u_0=u_1=1$ and attempts to find A and B

A1: CAO – allow values stated implicitly e.g. $u_n=\left(1-\frac{4}{3}n\right)(-3)^n$

Question	Scheme	Marks	AOs
3(a)		M1 A1 M1 M1 A1	3.3 1.1b 3.4 3.4 1.1b
		(5)	
(b)	Minimum expected travel time is 46.5 minutes Transport option is Train	B1ft B1	3.4 2.2a
		(2)	
			(7 marks)

Notes:

(a)M1: tree diagram with at least nine end pay-offs, one decision node and at least three chance nodes used correctly

A1: correct structure of tree diagram with each arc labelled correctly (including probabilities)

M1:at least three end-pay offs consistent with their stated probabilities (eg time 52 with probability 0.88); all nine attempted

M1: chance nodes attempted with their probabilities. Must be filled in on their diagram.

A1: cao for chance and decision nodes completed correctly

(b) B1ft: correct travel time from their completed tree diagram (dependent on all method marks earned in **(a)**)

B1: deduction of correct transport option (dependent on all method marks earned in **(a)**) including double line through inferior options in **(a)** (condone cross or single line here)

Question	Scheme	Marks	AOs
4(a)	AE, BE, BF, BG, DB, DT, SB	**B1**	1.1b
		(1)	
(b)	95	**B1**	1.1b
		(1)	
(c)	The maximum feasible flow into F is 22 (from BF and DF) but the maximum feasible flow out of F is 24 so therefore FT cannot be full to capacity	**B1**	2.4
		(1)	
(d)	$C_1\left(=33+41+30\right)=104$ $C_2\left(=53+30+14+0+17\right)=114$	**B1** **B1**	1.1b 1.1b
		(2)	
(e)	SABDFT	**B1**	1.1b
		(1)	
(f)	Use of max-flow min-cut theorem Identification of cut through AE, BE, BG, BF, DF and DT, Value of cut = 98, Value of flow = 98 Therefore it follows that flow is maximal	**M1** **A1** **A1**	2.1 3.1a 2.2a
		(3)	
			(9 marks)

Notes:

(a) B1: CAO

(b)B1: CAO

(c) B1: Correct reasoning – argument must be numerical in nature (e.g. as a minimum comparison of 22 with 24)

(d) B1: CAO for C_1

B1:CAO for C_2

(e) B1: CAO

(f)M1: Construct argument based on max-flow min-cut theorem (e.g. attempt to find a cut through saturated arcs – must contain source on one side and sink on the other). Allow a cut drawn on the diagram (need not be the correct one)

A1: Use appropriate process of finding a minimum cut – AE, BE, BG, BF, DF and DT plus value correct and value of flow through the network stated correctly (98)

A1: Correct deduction that the flow is maximal – must use all four words 'maximum', flow', 'minimum' and 'cut' (allow abbreviations for maximum and minimum) dependent on previous A1**.**

<table>
<tr><th>Question</th><th>Scheme</th><th>Marks</th><th>AOs</th></tr>
<tr><td>5(a)</td><td>$k = 39$</td><td>B1</td><td>2.2a</td></tr>
<tr><td></td><td></td><td>(1)</td><td></td></tr>
<tr><td>(b)</td><td>To ensure that the total amount transported to destination R from the four supply points cannot be less than the demand of 44</td><td>B2, 1, 0</td><td>2.4
2.4</td></tr>
<tr><td></td><td></td><td>(2)</td><td></td></tr>
<tr><td>(c)</td><td><table><tr><td></td><td>R</td><td>S</td><td>T</td></tr><tr><td>A</td><td>34</td><td></td><td></td></tr><tr><td>B</td><td>10</td><td>17</td><td></td></tr><tr><td>C</td><td></td><td>20</td><td>21</td></tr><tr><td>D</td><td></td><td></td><td>18</td></tr></table></td><td>B1</td><td>1.1b</td></tr>
<tr><td></td><td>2942</td><td>B1</td><td>2.2a</td></tr>
<tr><td></td><td></td><td>(2)</td><td></td></tr>
<tr><td>(d)</td><td><table><tr><td></td><td></td><td>23</td><td>37</td><td>39</td></tr><tr><td></td><td></td><td>R</td><td>S</td><td>T</td></tr><tr><td>0</td><td>A</td><td>X</td><td>−20</td><td>−15</td></tr><tr><td>−8</td><td>B</td><td>X</td><td>X</td><td>1</td></tr><tr><td>−12</td><td>C</td><td>14</td><td>X</td><td>X</td></tr><tr><td>−14</td><td>D</td><td>10</td><td>−3</td><td>X</td></tr></table><table><tr><td></td><td>R</td><td>S</td><td>T</td></tr><tr><td>A</td><td>$34-\theta$</td><td>θ</td><td></td></tr><tr><td>B</td><td>$10+\theta$</td><td>$17-\theta$</td><td></td></tr><tr><td>C</td><td></td><td></td><td></td></tr><tr><td>D</td><td></td><td></td><td></td></tr></table><table><tr><td></td><td>R</td><td>S</td><td>T</td></tr><tr><td>A</td><td>17</td><td>17</td><td></td></tr><tr><td>B</td><td>27</td><td></td><td></td></tr><tr><td>C</td><td></td><td>20</td><td>21</td></tr><tr><td>D</td><td></td><td></td><td>18</td></tr></table>Entering cell is AS and exiting cell is BS</td><td>M1
A1

M1

A1</td><td>2.1
1.1b

1.1b

2.2a</td></tr>
<tr><td></td><td></td><td>(4)</td><td></td></tr>
<tr><td colspan="4">(9 marks)</td></tr>
<tr><td colspan="4">Notes:</td></tr>
<tr><td colspan="4">(a) B1:CAO
(b) B1: Partial correct reasoning – must include at least two of ‘destination R’, ‘supply points’, ‘cannot be less’/’must be at least’, ‘demand of 44’ oe (do not accept ‘greater than or equal to’)
B1: Fully correct reasoning – all points covered as stated above. No incorrect statement.
(c) B1: CAO for north-west corner method (six correct figures in correct cells only, no zeros)
B1: CAO for initial solution (2942)

(d) M1: Finding 7 shadow costs and 6 improvement indices
A1: CAO</td></tr>
</table>

M1: A valid route shown, their most negative II chosen, only one empty square used, θ's balance
A1:cao – (no zeros) including deducing entering and exiting cells

For reference:

	R	S	T	Supply
A	23	17	24	34
B	15	29	32	27
C	25	25	27	41
D	19	20	25	18
Demand	44	37	*k*	

Qu	Scheme	Marks	AOs

6

Stage	State	Action	Dest	Value
May	2	0	0	160 = 160*
(2)	1	1	0	80 + 35 = 115*
	0	2	0	70 = 70*
April	3	2	0	240 + 70 + 70 = 380*
(5)		3	1	240 + 105 + 250 + 115 = 710
		4	2	240 + 140 + 250 + 160= 790
	2	3	0	160 + 105 + 250 + 70 = 585*
		4	1	160 + 140 + 250 + 115 = 665
	1	4	0	80 + 140 + 250 + 70= 540*
March	3	1	1	240 + 35 + 540= 815*
(3)		2	2	240 + 70 + 585 = 895
		3	3	240 + 105 + 250 + 380= 975
	2	2	1	160 + 70 + 540= 770*
		3	2	160 + 105 + 250 + 585= 1100
		4	3	160 + 140 + 250 + 380 = 930
	1	3	1	80 + 105 + 250 + 540 = 975*
		4	2	80+ 140 + 250 + 585= 1055
	0	4	1	140 + 250 + 540 = 930*
Feb	3	0	0	240 + 930 = 1170
(3)		1	1	240 + 35 + 975= 1250
		2	2	240 + 70 + 770 = 1080*
		3	3	240 + 105 + 250 + 815 = 1410
	2	1	0	160 + 35 + 930= 1125*
		2	1	160 + 70+ 975= 1205
		3	2	160 + 105 + 250 + 770 = 1285
		4	3	160 + 140 + 250 + 815 = 1365
	1	2	0	80+ 70+ 930= 1080*
		3	1	80 + 105 + 250 + 975 = 1410
		4	2	80 + 140 + 250 + 770 = 1240
	0	3	0	105 + 250 + 930 = 1285*
		4	1	140 + 250 + 975 = 1365
Jan	0	1	0	35 + 1285 = 1320
(1)		2	1	70 + 1080= 1150*
		3	2	105 + 250 + 1125= 1480
		4	3	140 + 250 + 1080 = 1470

Month	January	February	March	April	May
Number made	2	2	4	4	2

Minimum Cost: (£) 1150

Marks	AOs
M1	3.1a
A1	1.1b
A1	1.1b
M1	3.1a
A1ft	1.1b
A1	1.1b
M1	1.1b
A1ft	1.1b
A1ft	1.1b
A1	1.1b
M1	1.1b
A1	1.1b
B1	2.2a
B1	1.1b
(14 marks)	

Notes:

All M marks – must bring earlier optimal results into calculations. Ignore extra rows. Must have right 'ingredients' (storage costs, overhead costs and additional workers) at least once per stage. Ingredients may be summed into a single figure. Correct total figure implies correct calculation. Penalise lack of * only once per question.

M1: Second stage completed. 6 rows, something in each cell.
A1: Any two states correct
A1: CAO for second stage

M1: Third stage completed. 9 rows with correct state, action and destination. Something in each cell. Condone at least 8 rows, with correct state, action and destination.
A1ft: Any two states correct – ft their optimal values
A1: CAO for third stage

M1: Fourth stage completed. 13 rows, with correct state, action and destination. Something in each cell. Condone at least 11 rows, with correct state, action and destination.
A1ft: Any two states correct – ft their optimal values
A1ft: Any three states correct – ft their optimal values
A1: CAO for fourth stage

M1: Fifth stage completed. 4 rows, something in each cell.
A1: CAO for fifth stage

B1: Correct allocation (dependent on all previous M marks)
B1: Correct minimum cost (dependent on all previous M marks)

<table>
<tr><th>Qu</th><th>Scheme</th><th>Marks</th><th>AOs</th></tr>
<tr><td>7(a)</td><td>Row minima are –2, min(k, –4) and –3</td><td>M1</td><td>1.1b</td></tr>
<tr><td></td><td>e.g., If the row minimum for option R is –4 then the play safe is Q (as –2 is greater than –4 and –3)
If the row minimum for option R is k then –4 >k and the play safe is still Q</td><td>A1</td><td>2.3</td></tr>
<tr><td></td><td></td><td>(2)</td><td></td></tr>
<tr><td>(b)</td><td>Column maxima are 4, 6, 3 and max(k, –2)</td><td>B1</td><td>1.1b</td></tr>
<tr><td></td><td>Since the play-safe is option Z, $k < 3$ or $k \leq 2$</td><td>B1</td><td>2.2a</td></tr>
<tr><td></td><td></td><td>(2)</td><td></td></tr>
<tr><td>(c)</td><td>e.g. Option Y dominates option X</td><td>B1</td><td>1.2</td></tr>
<tr><td></td><td>Because e.g. –1 < 3, –4 < 5 and 3 < 6</td><td>B1</td><td>2.4</td></tr>
<tr><td></td><td></td><td>(2)</td><td></td></tr>
<tr><td>(d)
(i)</td><td>$\begin{pmatrix} 4 & -1 & -2 \\ -3 & -4 & k \\ -1 & 3 & -3 \end{pmatrix} \rightarrow \begin{pmatrix} 8 & 3 & 2 \\ 1 & 0 & k+4 \\ 3 & 7 & 1 \end{pmatrix}$</td><td>B1</td><td>1.1b</td></tr>
<tr><td></td><td>$V - 8p_1 - p_2 - 3p_3 + r = 0$
$V - 3p_1 - 7p_3 + s = 0$
$V - 2p_1 - (k+4)p_2 - p_3 + t = 0$</td><td>M1
A1</td><td>3.3
2.5</td></tr>
<tr><td></td><td>$p_1 + p_2 + p_3 + u = 1$</td><td>B1</td><td>3.3</td></tr>
<tr><td>(ii)</td><td>e.g.
<table>
<tr><th>b.v.</th><th>V</th><th>p_1</th><th>p_2</th><th>p_3</th><th>r</th><th>s</th><th>t</th><th>u</th><th>Value</th></tr>
<tr><td>r</td><td>1</td><td>–8</td><td>–1</td><td>–3</td><td>1</td><td>0</td><td>0</td><td>0</td><td>0</td></tr>
<tr><td>s</td><td>1</td><td>–3</td><td>0</td><td>–7</td><td>0</td><td>1</td><td>0</td><td>0</td><td>0</td></tr>
<tr><td>t</td><td>1</td><td>–2</td><td>$-(k+4)$</td><td>–1</td><td>0</td><td>0</td><td>1</td><td>0</td><td>0</td></tr>
<tr><td>u</td><td>0</td><td>1</td><td>1</td><td>1</td><td>0</td><td>0</td><td>0</td><td>1</td><td>1</td></tr>
<tr><td>P</td><td>–1</td><td>0</td><td>0</td><td>0</td><td>0</td><td>0</td><td>0</td><td>0</td><td>0</td></tr>
</table></td><td>B1
M1
A1</td><td>1.2
3.3
1.1b</td></tr>
<tr><td></td><td></td><td>(7)</td><td></td></tr>
<tr><td>(e)</td><td>$p_3 = \frac{13}{37}$</td><td>B1</td><td>1.1b</td></tr>
<tr><td></td><td>$V - 8\left(\frac{7}{37}\right) - \left(\frac{17}{37}\right) - 3\left(\frac{13}{37}\right) + 0 = 0$ or $V - 3\left(\frac{7}{37}\right) - 7\left(\frac{13}{37}\right) + 0 = 0$</td><td>M1</td><td>3.1a</td></tr>
</table>

	$V=\frac{112}{37}\Rightarrow\frac{112}{37}-2\left(\frac{7}{37}\right)-(k+4)\left(\frac{17}{37}\right)-\left(\frac{13}{37}\right)+0=0$	dM1	3.4
	$k=1$	A1	2.2a
		(4)	
			(17 marks)

Notes:

(a)M1: Attempt to calculate row minima. Condone 'k or -4' for min(k,-4).

A1: Correct argument/conditions for why the play-safe for player A is always their option Q

(b)B1: Attempt to calculate column maxima. Condone 'k or -2' for max(k,-2)

B1: CAO ignore lower limit figure, if given

(c) B1: Correct statement – must include the word 'dominate'. Also e.g. option Z dominates option X

B1: Correct inequalities – must be clear that all three inequalities must hold

(d)(i) B1: Correct augmentation – possibly implied by later working, X column may be included

M1: At least three equations in V, p_1, p_2, p_3 and at least one dummy variable seen

A1: CAO (ignore extra probability equation, if seen, see note below)

B1: Correct probability equation

(ii) B1: Correct row and column labels for Simplex tableau

M1: Any one (numerical in nature) row correct

A1: CAO

(e) B1: p_3 correctly stated

M1: Attempts to calculate V using either equation from (d) not involving k, or attempts to eliminate V from two equations, one involving k

dM1: Dependent on previous M mark – either uses equation in k and their V to calculate k, or eliminates V from two equations to calculate k

A1: CAO $k = 1$

Special case:

(d) If augmentation with +5 maximum marks possible B0 M1 A1 B1 B1 M1 A0

Note: (d)(i) **A1** The extra probability equation you may see is $V - 7p_1 - 9p_2 - 10p_3 + \text{slack} = 0$

Qu	Scheme	Marks	AOs
8(a)	$u_{n+1} = pu_n + k$	B1	3.3
	(aux equation $m - p = 0 \Rightarrow$) complementary function is $A(p)^n$	B1	1.1b
	Consider a trial solution of the form $u_n = \lambda$ so $\lambda - p\lambda = k$	M1	1.1b
	General solution is $u_n = A(p)^n + \dfrac{k}{1-p}$	A1	1.1b
	$u_1 = 5000 \Rightarrow 5000 = A(p) + \dfrac{k}{1-p}$ and solve for A	M1	3.4
	$u_n = \left(5000 - \dfrac{k}{1-p}\right)p^{n-1} + \dfrac{k}{1-p}$	A1	2.2a
		(6)	
(b)	Set k = 10 000, p = 0.95 and $u_m \ldots 135\,000$	B1	3.1b
	$\left(5000 - \dfrac{10000}{1-0.95}\right)(0.95)^{m-1} + \dfrac{10000}{1-0.95} \ldots 135000$		
	$(0.95)^{m-1}$ „ $\dfrac{1}{3} \Rightarrow (m-1)\log(0.95)$ „ $\log\left(\dfrac{1}{3}\right) \Rightarrow m \ldots..$	M1	1.1b
	$m \ldots 22.418...$ so 23 months after the company was first up	A1	3.2a
		(3)	
			(9 marks)

Notes:

(a) B1:CAO

B1:CAO

M1: substituting their trial solution into the recurrence relation in an attempt to find their λ (which if correct is $\dfrac{k}{1-p}$)

A1: CAO for the general solution

M1:using the conditions in the model to calculate A(which if correct is $p^{-1}\left(5000 - \dfrac{k}{1-p}\right)$)

A1: CAO for the particular solution (oe)

(b)B1: Applying $u_m \ldots 135000$ (or equality or strict inequality) to their general solution together with correct values for k and p (dependent on both M marks in **(a)**)

M1: dependent on previous B mark – solving their equation using logarithms

A1: CAO – must be rounded to 23

Special case: A common misread is 500 for 5000. Mark as a misread so final A marks in (a) and (b) deducted.