Mark Scheme (Results)

Summer 2022

Pearson Edexcel GCE
In Further Mathematics (8FM0)
Paper 21 Further Pure Mathematics 1

Edexcel and BTEC Qualifications

Edexcel and BTEC qualifications are awarded by Pearson, the UK's largest awarding body. We provide a wide range of qualifications including academic, vocational, occupational and specific programmes for employers. For further information visit our qualifications websites at www.edexcel.com or www.btec.co.uk. Alternatively, you can get in touch with us using the details on our contact us page at www.edexcel.com/contactus.

Pearson: helping people progress, everywhere

Pearson aspires to be the world's leading learning company. Our aim is to help everyone progress in their lives through education. We believe in every kind of learning, for all kinds of people, wherever they are in the world. We've been involved in education for over 150 years, and by working across 70 countries, in 100 languages, we have built an international reputation for our commitment to high standards and raising achievement through innovation in education. Find out more about how we can help you and your students at: www.pearson.com/uk

Summer 2022
Question Paper Log Number P68617A*
Publications Code 8FM0_21_2206_MS*

General Marking Guidance

- All candidates must receive the same treatment. Examiners must mark the first candidate in exactly the same way as they mark the last.
- Mark schemes should be applied positively. Candidates must be rewarded for what they have shown they can do rather than penalised for omissions.
- Examiners should mark according to the mark scheme not according to their perception of where the grade boundaries may lie.
- There is no ceiling on achievement. All marks on the mark scheme should be used appropriately.
- All the marks on the mark scheme are designed to be awarded. Examiners should always award full marks if deserved, i.e. if the answer matches the mark scheme. Examiners should also be prepared to award zero marks if the candidate's response is not worthy of credit according to the mark scheme.
- Where some judgement is required, mark schemes will provide the principles by which marks will be awarded and exemplification may be limited.
- When examiners are in doubt regarding the application of the mark scheme to a candidate's response, the team leader must be consulted.
- Crossed out work should be marked UNLESS the candidate has replaced it with an alternative response.

EDEXCEL GCE MATHEMATICS
General Instructions for Marking

1. The total number of marks for the paper is 40.

2. The Edexcel Mathematics mark schemes use the following types of marks:

- **M** marks: method marks are awarded for ‘knowing a method and attempting to apply it’, unless otherwise indicated.
- **A** marks: Accuracy marks can only be awarded if the relevant method (M) marks have been earned.
- **B** marks are unconditional accuracy marks (independent of M marks)
- Marks should not be subdivided.

3. Abbreviations

These are some of the traditional marking abbreviations that will appear in the mark schemes.

- bod – benefit of doubt
- ft – follow through
- the symbol $\sqrt{}$ will be used for correct ft
- cao – correct answer only
- cso - correct solution only. There must be no errors in this part of the question to obtain this mark
- isw – ignore subsequent working
- awrt – answers which round to
- SC: special case
- oe – or equivalent (and appropriate)
- dep – dependent
- indep – independent
- dp decimal places
- sf significant figures
- $\ast$ The answer is printed on the paper
- The second mark is dependent on gaining the first mark

4. For misreading which does not alter the character of a question or materially simplify it, deduct two from any A or B marks gained, in that part of the question affected.

5. Where a candidate has made multiple responses <u>and indicates which response they wish to submit</u>, examiners should mark this response.
If there are several attempts at a question <u>which have not been crossed out</u>, examiners should mark the final answer which is the answer that is the <u>most complete</u>.

6. Ignore wrong working or incorrect statements following a correct answer.

7. Mark schemes will firstly show the solution judged to be the most common response expected from candidates. Where appropriate, alternatives answers are provided in the notes. If examiners are not sure if an answer is acceptable, they will check the mark scheme to see if an alternative answer is given for the method used.

Question	Scheme	Marks	AOs
1	$x=\frac{2x+15}{2x+3}\Rightarrow 2x^2+3x=2x+15\Rightarrow 2x^2+x-15=0\Rightarrow x=...$ Alternative 1: $(2x+3)^2x\geqslant(2x+3)(2x+15)\Rightarrow(2x+3)(2x^2+3x-2x-15)\geqslant 0$ $(2x+3)(x+3)(2x-5)\geqslant 0$ Alternative 2; $x-\frac{2x+15}{2x+3}\geqslant 0\Rightarrow\frac{x(2x+3)-2x-15}{2x+3}\geqslant 0\Rightarrow\frac{(x+3)(2x-5)}{2x+3}\geqslant 0$	**M1**	1.1b
	$\Rightarrow(x+3)(2x-5)=0\Rightarrow$ CVs are $-3,\ \frac{5}{2}$	**A1**	1.1b
	Also $2x+3=0\Rightarrow x=-\frac{3}{2}$ a CV	**B1**	2.3
	Hence from graph (oe) the solution set is $\left\{x\in\mathbb{R}:-3\leqslant x<-\frac{3}{2}, x\geqslant\frac{5}{2}\right\}\left\{x:-3\leqslant x<-\frac{3}{2}, x\geqslant\frac{5}{2}\right\}$	**M1** **A1** **A1**	1.1b 2.2a 2.5
		(6)	

(6 marks)

Notes:

M1: For a complete method to find the critical values other than $-\frac{3}{2}$.

Alternative 1: Multiplies by $(2x+3)^2$, collects terms onto one side and factorises into three brackets.

Alternative 2: Collects terms onto one side and combines into single fraction using a common denominator and factorises the numerator

A1: Correct critical values -3 and $\frac{5}{2}$

B1: For the critical value $-\frac{3}{2}$

M1: Selects the correct regions for their three CV's. Should include the right hand side open ended and another bounded region. CV's of a < b < c then must be of the form $a\leqslant x\leqslant b,\ x\geqslant c$ or $a<x<b,\ x>c$ the direction of the inequalities must be correct with or without strict inequalities.

A1: At least one correct interval identified. Alternatively allow for both intervals with correct end points but incorrect strict or inclusive inequalities

A1: Fully correct solution as a set – accept alternative set notations e.g. $\left[-3,-\frac{3}{2}\right)\cup\left[\frac{5}{2},\infty\right)$, but not just inequalities. Minimum use of set notation $-3\leqslant x<-\frac{3}{2}\cup x\geqslant\frac{5}{2}$

Note: Correct answer with no working scores M0 A0 but can score B1 M1 A1 A1

No working shown to factorise a cubic equation e.g.

$4x^3+8x^2-27x-45=(x+3)(2x+3)(2x-5)$ is M0 A0 but can still score B1 M1 A1 A1

A0 for $-3 \leqslant x < -\frac{3}{2} \cap x \geqslant \frac{5}{2}$ or $-3 \leqslant x < -\frac{3}{2}$ and $x \geqslant \frac{5}{2}$

Special case: If they have a repeated root final 3 marks M1 A1 A0 is possible e.g.

Question	Scheme	Marks	AOs
2	$t_0=\frac{1}{2}$ and steps are 2 months, so $h=\frac{1}{6}$ $\left(t_1=\frac{2}{3},t_2=\frac{5}{6}\right)$	B1	3.3
	$\left(\frac{\mathrm{d}P}{\mathrm{d}t}\right)_0=\frac{540}{5000}\left(1000-\frac{540\times\left(\frac{1}{2}+1\right)}{6\times\frac{1}{2}+5}\right)=...\left(97.065=\frac{19413}{200}\right)$	M1	3.4
	So when $t=\frac{2}{3}$, $P_1=540+\frac{'1'}{6}\times'97.065'=...$ Or starts with $'97.065'=\frac{y_1-540}{1/6}$ and rearranges to find $P_1=...$	M1	1.1b
	$=\frac{222471}{400}=556.1775$	A1	1.1b
	$\left(\frac{\mathrm{d}P}{\mathrm{d}t}\right)_1=\frac{'556.1775'}{5000}\left(1000-\frac{'556.1775'\times\left(\frac{2}{3}+1\right)}{6\times\frac{2}{3}+5}\right)=...(99.778...)$	M1	3.4
	So when $t=\frac{5}{6}$, $P_2='556.1775'+\frac{1}{6}\times'99.778...'=...(572.807\ldots)$	M1	1.1b
	So there are estimated to be 572 or 573 deer after 10 months.	A1	3.2a
		(7)	
			(7 marks)

Notes:

B1: Uses the given information to set up correct parameters for the model, $t_0=\frac{1}{2}$, $\left(t_1=\frac{2}{3},t_2=\frac{5}{6}\right)$ and $h=\frac{1}{6}$ seen or implied.

M1: Uses $P_0=540$ and their value for t_0 in the given equation to find a value for $\left(\frac{\mathrm{d}P}{\mathrm{d}t}\right)_0$

M1: Applies the approximation formula with 540, their h and their $\left(\frac{\mathrm{d}P}{\mathrm{d}t}\right)_0$ to find a value for P_1

A1: Correct approximation P at $t=\frac{2}{3}$. Accept awrt 556.2

M1: Uses $t_1=t_0+h$ and their P_1 in the given equation to find a value for $\left(\frac{\mathrm{d}P}{\mathrm{d}t}\right)_1$.

M1: Uses the approximation a second time with their h, their P_1 and their $\left(\frac{\mathrm{d}P}{\mathrm{d}t}\right)_1$. to find a value for P_2

A1: Correct answer. Accept either 572 or 573.

Useful table of values for reference

n	P_n	t	$\frac{\mathrm{d}P}{\mathrm{d}t}$	$h\frac{\mathrm{d}P}{\mathrm{d}t}$
0	540	$\frac{1}{2}$	97.065	16.1775
1	556.1775	$\frac{2}{3}$	99.77870698	16.6297845
2	572.807	$\frac{5}{6}$		

Note use of $t_0 = 6$ and $h = 2$ leads to $\left(\frac{\mathrm{d}P}{\mathrm{d}t}\right)_0 = \frac{100494}{1025} = 98.04...$ $P_1 = 736.08...$ $\left(\frac{\mathrm{d}P}{\mathrm{d}t}\right)_1 = 128.8...$
$P_2 = 993.7...$ which scores a maximum of B0 M1 M1 A0 M1 M1 A0

Question	Scheme	Marks	AOs
3(a)	Sight of $\sec\theta = \dfrac{1+t^2}{1-t^2}$ and $\tan\theta = \dfrac{2t}{1-t^2}$ at least once each.	B1	1.2
	$\dfrac{29-21\sec\theta}{20-21\tan\theta} = \dfrac{29-21\left(\dfrac{1+t^2}{1-t^2}\right)}{20-21\left(\dfrac{2t}{1-t^2}\right)}$	M1	1.1b
	$= \dfrac{29\left(1-t^2\right)-21\left(1+t^2\right)}{20\left(1-t^2\right)-21(2t)} = \dfrac{8-50t^2}{-20t^2-42t+20}$ or $= \dfrac{\dfrac{29\left(1-t^2\right)-21\left(1+t^2\right)}{\left(1-t^2\right)}}{\dfrac{20\left(1-t^2\right)-21(2t)}{\left(1-t^2\right)}} = \dfrac{8-50t^2}{-20t^2-42t+20}$	M1	2.1
	$= \dfrac{-2(5t-2)(5t+2)}{-2(5t-2)(2t+5)} = \dfrac{5t+2}{2t+5}$ * cso $= \dfrac{2(2+5t)(2-5t)}{2(2-5t)(5+2t)} = \dfrac{5t+2}{2t+5}$ * cso	A1*	1.1b
		(4)	
(b)	$\dfrac{20+21\tan\theta}{29+21\sec\theta} = \dfrac{20+21\left(\dfrac{2t}{1-t^2}\right)}{29+21\left(\dfrac{1+t^2}{1-t^2}\right)}$ $= \dfrac{20\left(1-t^2\right)+21(2t)}{29\left(1-t^2\right)+21\left(1+t^2\right)}$	M1	2.1
	$\dfrac{20+42t-20t^2}{50-8t^2} = \dfrac{2(5-2t)(5t+2)}{2(5-2t)(5+2t)} = \ldots$ $\dfrac{20+42t-20t^2}{50-8t^2} = \dfrac{-2(2t-5)(5t+2)}{-2(2t+5)(2t-5)} = \ldots$	M1	1.1b
	Achieves form correct working $= \dfrac{5t+2}{2t+5}$ * Then concludes hence the result is true or = RHS or $= \dfrac{5t+2}{2t+5} = \dfrac{29-21\sec\theta}{20-21\tan\theta}$ or	A1*	2.2a

	$\dfrac{20+21\tan\theta}{29+21\sec\theta}=\dfrac{29-21\sec\theta}{20-21\tan\theta}$ LHS = RHS		
		(3)	
			(7 marks)

Notes:

(a)

B1: Uses the correct identities at least once each. May be seen in (a) or (b)

M1: Substitutes their identities into the equation, they need not be correct

M1: Multiplies numerator and denominator by $1-t^2$ and simplifies to a quadratic with all terms collected

Special case: If they make an error with the identities award M1 if they use the correct method to write the numerator and denominator as a single fraction and then divides to achieve an expression of the form $\dfrac{a}{b}$

A1*: Cancels factors to achieve the given expression, with no errors seen

Condone $\dfrac{50t^2-8}{20t^2+42t-20}=\dfrac{(5t+2)(5t-2)}{(5t-2)(2t+5)}=\dfrac{5t+2}{2t+5}$ and $\dfrac{-8+50t^2}{-20t^2-42t+20}=\dfrac{(5t+2)(5t-2)}{(5t-2)(2t+5)}=\dfrac{5t+2}{2t+5}$

(b)

M1: Substitutes into LHS of equation and multiplies numerator and denominator by $1-t^2$ to work towards the other side.

M1: Simplifies to a quadratic with all terms collected and factorises then cancels terms in the LHS in an attempt to try and match expressions.

A1*: Achieves correct expressions for both sides and gives a conclusion deducing that the result is true.

M1A0 for $\dfrac{20t^2-42t-20}{8t^2-50}=\dfrac{(5t+2)(2t-5)}{(2t-5)(2t+5)}=\dfrac{5t+2}{2t+5}$ and

$\dfrac{-20t^2-42t+20}{-8+50t^2}=\dfrac{(5t+2)(2t-5)}{(2t-5)(2t+5)}=\dfrac{5t+2}{2t+5}$

Question	Scheme	Marks	AOs
4(a)	$\left(\frac{5}{2},0\right)$ o.e.	B1	2.2a
		(1)	
(b)	$\frac{\mathrm{d}y}{\mathrm{d}x}=\frac{5}{q}$	B1	1.1b
	At P, $x=\frac{q^2}{10}$ so tangent has equation $y-q=\text{their}\frac{5}{q}\left(x-\frac{q^2}{10}\right)$ or $q=\left(\text{their}\frac{5}{q}\right)\left(\frac{q^2}{10}\right)+c\Rightarrow c=...$ to reach an equation for y	M1	1.1b
	$\Rightarrow qy-q^2=5x-\frac{q^2}{2}\Rightarrow 10x-2qy+q^2=0$ * cso or $\Rightarrow y=\frac{5}{q}x+\frac{q}{2}\Rightarrow 10x-2qy+q^2=0$ * cso	A1*	2.1
		(3)	
(c)	B is $\left(-\frac{5}{2},q\right)$ o.e.	B1	2.2a
	So diagonal BF has equation $\frac{y-0}{q-0}=\frac{x-\frac{5}{2}}{-\frac{5}{2}-\frac{5}{2}}$ or $y=-\frac{q}{5}\left(x-\frac{5}{2}\right)$	M1	1.1b
	(AP is a tangent so) diagonals meet when $10x-2q\left(-\frac{q}{5}\left(x-\frac{5}{2}\right)\right)+q^2=0$ or $x=\frac{2qy-q^2}{10}$ therefore $y=-\frac{q}{5}\left(\frac{2qy-q^2}{10}-\frac{5}{2}\right)$ leading to $y=\ldots$ $\left\{y=\frac{25q+q^3}{50+2q^2}\right\}$	dM1	3.1a
	$\Rightarrow 10x+\frac{2q^2}{5}x-q^2+q^2=0\Rightarrow x\left(10+\frac{2q^2}{5}\right)=0$ or $x=\frac{1}{10}\left(2q\left(\frac{25q+q^3}{50+2q^2}\right)-q^2\right)$	M1	1.1b
	But $10+\frac{2q^2}{5}>0$ so not zero, hence $x=0$, so the intersection lies on the y-axis.	A1	2.4

	Or achieves $x = 0$ (with no errors), so the intersection lies on the y axis.		
		(5)	
	Alternative for the last three marks		
	When $x = 0$ for BF $y = -\frac{q}{5}\left(-\frac{5}{2}\right) = ...$ **or** for AP $2qy = q^2 \Rightarrow y = ...$	**M1**	1.1b
	For BF y intercept is $\frac{q}{2}$ **and** for AP y intercept is $\frac{q}{2}$	**M1**	3.1a
	Since both diagonals always cross the y-axis at the same place, their intersection must always be on the y axis.	**A1**	2.4
			(9 marks)

Notes:

(a)
B1: Deduces correct coordinates.

(b)
B1: Using or deriving $\frac{dy}{dx} = \frac{5}{q}$

M1: Finds the equation of the tangent using the equation of a line formula with $y_1 = q$, $x = \frac{q^2}{10}$ (or clear attempt at it) and $m = \frac{2 \times \text{their}'a'}{q}$.

If uses $y = mx + c$ must find a value for c and substitute back to find an equation for the tangent
A1*: Completes correctly to the given equation, no errors seen.

(c)
B1: B is $\left(-\frac{5}{2}, q\right)$ seen or used.

M1: A correct method to find the equation of the diagonal BF using their coordinates of F and B
dM1: Uses the printed answer in (b) and their equation of the diagonal BF to form an equation just involving x or solves the two diagonals simultaneously to find an expression for y
M1: Correctly factors out the x to achieve $x(...) = 0$ or uses their expression for y to find an expression for x

A1: Conclusion given including reference to $10 + \frac{2q^2}{5} \neq 0$

Alternative for last three marks
M1: Attempts to find the y intercept for at least one of the two diagonals.
M1: Finds y intercept for both diagonals in order to compare
A1: Both intercepts correct and suitable conclusion giving reference to both diagonals always crossing y-axis at same point.

Question	Scheme	Marks	AOs
5(a)	A correct method to find one coordinates of M, N or P For example $\overrightarrow{AB}=\begin{pmatrix}6\\-6\\6\end{pmatrix}$ so $\overrightarrow{OM}=\begin{pmatrix}3\\2\\-4\end{pmatrix}+\frac{4}{6}\begin{pmatrix}6\\-6\\6\end{pmatrix}=\ldots$ $\overrightarrow{AC}=\begin{pmatrix}-9\\-12\\12\end{pmatrix}$ so $\overrightarrow{ON}=\begin{pmatrix}3\\2\\-4\end{pmatrix}+\frac{4}{12}\begin{pmatrix}-9\\-12\\12\end{pmatrix}=\ldots$ $\overrightarrow{AD}=\begin{pmatrix}-7\\-7\\14\end{pmatrix}$ so $\overrightarrow{OP}=\begin{pmatrix}3\\2\\-4\end{pmatrix}+\frac{4}{14}\begin{pmatrix}-7\\-7\\14\end{pmatrix}=\ldots$	M1	3.1a
	One of $(M=)(7,-2,0)$, $(N=)(0,-2,0)$ or $(P=)(1,0,0)$	A1	1.1b
	All of $(M=)(7,-2,0)$, $(N=)(0,-2,0)$ and $(P=)(1,0,0)$	A1	1.1b
		(3)	
(b)	Correct method, e.g. realises MN is parallel to x axis, so base is 7 and height 2, hence area of intersection is $\frac{1}{2}\times 7\times 2=\ldots$ **Alternatively** using $\frac{1}{2}\lvert a\times b\rvert$ $\overrightarrow{PM}=\pm\begin{pmatrix}6\\-2\\0\end{pmatrix}$ $\overrightarrow{PN}=\pm\begin{pmatrix}-1\\-2\\0\end{pmatrix}$ $\overrightarrow{NM}=\pm\begin{pmatrix}7\\0\\0\end{pmatrix}$ For example $\frac{1}{2}\left\lvert\overrightarrow{MP}\times\overrightarrow{PN}\right\rvert=\frac{1}{2}\begin{vmatrix}\mathbf{i}&\mathbf{j}&\mathbf{k}\\6&-2&0\\-1&-2&0\end{vmatrix}=\frac{1}{2}\left\lvert\begin{pmatrix}0\\0\\-14\end{pmatrix}\right\rvert=\ldots$	M1	1.1b
	$=7$ cso	A1	1.1b
		(2)	
(c)	Vol $NMPA=\frac{1}{3}A_bh=\frac{1}{3}\times 7\times 4=\frac{28}{3}$ Or using triple scalar product $NMPA=\frac{1}{6}\left\lvert\overrightarrow{AM}\cdot\left(\overrightarrow{AN}\times\overrightarrow{AP}\right)\right\rvert=\frac{1}{6}\left\lvert\begin{pmatrix}4\\-4\\4\end{pmatrix}\cdot\left(\begin{pmatrix}-3\\-4\\4\end{pmatrix}\times\begin{pmatrix}-2\\-2\\4\end{pmatrix}\right)\right\rvert$ $=\frac{1}{6}\left\lvert\begin{pmatrix}4\\-4\\4\end{pmatrix}\cdot\begin{pmatrix}-8\\4\\-2\end{pmatrix}\right\rvert=\frac{28}{3}$	M1 A1	3.1a 1.1b

	Vol $ABCD = \frac{1}{6}\left\|\overrightarrow{AB}\cdot\left(\overrightarrow{AC}\times\overrightarrow{AD}\right)\right\| = \ldots$ $= \frac{1}{6}\left\|\begin{pmatrix}6\\-6\\6\end{pmatrix}\cdot\left(\begin{pmatrix}-9\\-12\\12\end{pmatrix}\times\begin{pmatrix}-7\\-7\\14\end{pmatrix}\right)\right\| = \frac{1}{6}\left\|\begin{pmatrix}6\\-6\\6\end{pmatrix}\cdot\begin{pmatrix}-84\\42\\-21\end{pmatrix}\right\| = \ldots$	**M1**	1.1b
	$= 147$	**A1**	1.1b
	So volume required is $'147' - \frac{'28'}{3} = \ldots$	**M1**	3.1a
	$= \frac{413}{3}$	**A1**	1.1b
		(6)	
			(11 marks)

Notes:

(a)
M1: Correct method for finding at least one of the three points. Allow one slip in coordinates but should have correct fraction to make the value of z to be 0.
A1: Any one of the three points correct, ignoring the labelling.
A1: All three points correct, ignoring the labelling

(b)
M1: Correct method for finding the area of the triangle, e.g realises that MN is parallel to the x-axis so uses $\frac{1}{2}bh$ with $b = MN$ and h is distance of MN from axis.

Alternative using $\frac{1}{2}|a\times b|$ with vectors $\overrightarrow{PM} = \pm\begin{pmatrix}6\\-2\\0\end{pmatrix}$ $\overrightarrow{PN} = \pm\begin{pmatrix}-1\\-2\\0\end{pmatrix}$ $\overrightarrow{NM} = \pm\begin{pmatrix}7\\0\\0\end{pmatrix}$ follow through on their answers in part (a). Condone sign slips except they must be using $-\mathbf{j}$ in the cross product

For example $\frac{1}{2}\left|\overrightarrow{MP}\times\overrightarrow{PN}\right| = \frac{1}{2}\begin{vmatrix}\mathbf{i} & \mathbf{j} & \mathbf{k}\\6 & -2 & 0\\-1 & -2 & 0\end{vmatrix} = \frac{1}{2}\left|\begin{pmatrix}0\\0\\-14\end{pmatrix}\right| = \ldots$

$$\frac{1}{2}\left|\overrightarrow{PN}\times\overrightarrow{NM}\right| = \frac{1}{2}\begin{vmatrix}\mathbf{i} & \mathbf{j} & \mathbf{k}\\-1 & -2 & 0\\7 & 0 & 0\end{vmatrix} = \frac{1}{2}\left|\begin{pmatrix}0\\0\\14\end{pmatrix}\right| = \ldots$$

$$\frac{1}{2}\left|\overrightarrow{PM}\times\overrightarrow{NM}\right| = \frac{1}{2}\begin{vmatrix}\mathbf{i} & \mathbf{j} & \mathbf{k}\\6 & -2 & 0\\7 & 0 & 0\end{vmatrix} = \frac{1}{2}\left|\begin{pmatrix}0\\0\\14\end{pmatrix}\right| = \ldots$$

or attempting to find an angle using dot product or cosine rule followed by $\frac{1}{2}ab\sin C$.

A1: Correct area of 7 from correct vectors

(c) On ePen this is M1 A1 M1 M1 M1 A1

M1: Formulates a correct method to find the volume of *NMPA*. May use method shown, or e.g. $\frac{1}{6}\left|\overrightarrow{AM}\cdot\left(\overrightarrow{AN}\times\overrightarrow{AP}\right)\right|$ or equivalent method.

A1: For $\frac{28}{3}$.

Note there are many ways to find the required volume of *AMNP* applying the triple scalar product to a combination of the following vectors

$$\overrightarrow{AM}=\begin{pmatrix}4\\-4\\4\end{pmatrix}\ \overrightarrow{AN}=\begin{pmatrix}-3\\-4\\4\end{pmatrix}\ \overrightarrow{AP}=\begin{pmatrix}-2\\-2\\4\end{pmatrix}\ \overrightarrow{NA}=\begin{pmatrix}3\\4\\-4\end{pmatrix}\ \overrightarrow{NM}=\begin{pmatrix}7\\0\\0\end{pmatrix}\ \overrightarrow{NP}=\begin{pmatrix}1\\2\\0\end{pmatrix}$$

$$\overrightarrow{MA}=\begin{pmatrix}-4\\4\\-4\end{pmatrix}\ \overrightarrow{MN}=\begin{pmatrix}-7\\0\\0\end{pmatrix}\ \overrightarrow{MP}=\begin{pmatrix}-6\\2\\0\end{pmatrix}\ \overrightarrow{PA}=\begin{pmatrix}2\\2\\-4\end{pmatrix}\ \overrightarrow{PM}=\begin{pmatrix}6\\-2\\0\end{pmatrix}\ \overrightarrow{PN}=\begin{pmatrix}-1\\-2\\0\end{pmatrix}$$

For example

$$\frac{1}{6}\left|\overrightarrow{AM}\cdot\left(\overrightarrow{AN}\times\overrightarrow{AP}\right)\right|=\frac{1}{6}\left|\begin{pmatrix}4\\-4\\4\end{pmatrix}\cdot\left(\begin{pmatrix}-3\\-4\\4\end{pmatrix}\times\begin{pmatrix}-2\\-2\\4\end{pmatrix}\right)\right|=\frac{1}{6}\left|\begin{pmatrix}4\\-4\\4\end{pmatrix}\cdot\begin{pmatrix}-8\\4\\-2\end{pmatrix}\right|=\frac{1}{6}\times 56$$

$$\frac{1}{6}\left|\overrightarrow{NA}\cdot\left(\overrightarrow{NM}\times\overrightarrow{NP}\right)\right|=\frac{1}{6}\left|\begin{pmatrix}3\\4\\-4\end{pmatrix}\cdot\left(\begin{pmatrix}7\\0\\0\end{pmatrix}\times\begin{pmatrix}1\\2\\0\end{pmatrix}\right)\right|=\frac{1}{6}\left|\begin{pmatrix}3\\4\\-4\end{pmatrix}\cdot\begin{pmatrix}0\\0\\14\end{pmatrix}\right|=\frac{1}{6}\times 56$$

$$\frac{1}{6}\left|\overrightarrow{MA}\cdot\left(\overrightarrow{MN}\times\overrightarrow{MP}\right)\right|=\frac{1}{6}\left|\begin{pmatrix}-4\\4\\-4\end{pmatrix}\cdot\left(\begin{pmatrix}-7\\0\\0\end{pmatrix}\times\begin{pmatrix}-6\\2\\0\end{pmatrix}\right)\right|=\frac{1}{6}\left|\begin{pmatrix}-4\\4\\-4\end{pmatrix}\cdot\begin{pmatrix}0\\0\\14\end{pmatrix}\right|=\frac{1}{6}\times 56$$

$$\frac{1}{6}\left|\overrightarrow{PA}\cdot\left(\overrightarrow{PM}\times\overrightarrow{PN}\right)\right|=\frac{1}{6}\left|\begin{pmatrix}2\\2\\-4\end{pmatrix}\cdot\left(\begin{pmatrix}6\\2\\0\end{pmatrix}\times\begin{pmatrix}-1\\-2\\0\end{pmatrix}\right)\right|=\frac{1}{6}\left|\begin{pmatrix}2\\2\\-4\end{pmatrix}\cdot\begin{pmatrix}0\\0\\-14\end{pmatrix}\right|=\frac{1}{6}\times 56$$

Note candidates may write as $\frac{1}{6}\begin{vmatrix}4&-4&4\\-3&-4&4\\-2&-2&4\end{vmatrix}=\frac{1}{6}\left|4(-16+8)+4(-12+8)+4(6-8)\right|=\frac{1}{6}|-56|=\frac{28}{3}$

M1: A complete attempt at the volume of *ABCD*, with correct method for cross product (oe in other methods). Condone sign slips except they must be using – **j** in the cross product

A1 (M1 on ePen): 147

M1: Finds difference of the two volumes must have used a correct method to find the volumes.

A1: $\frac{413}{3}$

Note there are many ways to find the required volume of *ABCD* applying the triple scalar product to a combination of the following vectors

$\overrightarrow{AB}=\begin{pmatrix}6\\-6\\6\end{pmatrix}$ $\overrightarrow{AC}=\begin{pmatrix}-9\\-12\\12\end{pmatrix}$ $\overrightarrow{AD}=\begin{pmatrix}-7\\-7\\14\end{pmatrix}$ $\overrightarrow{BA}=\begin{pmatrix}-6\\6\\-6\end{pmatrix}$ $\overrightarrow{BC}=\begin{pmatrix}15\\-6\\6\end{pmatrix}$ $\overrightarrow{BD}=\begin{pmatrix}-13\\-1\\8\end{pmatrix}$

$\overrightarrow{CA}=\begin{pmatrix}9\\12\\-12\end{pmatrix}$ $\overrightarrow{CB}=\begin{pmatrix}-15\\6\\-6\end{pmatrix}$ $\overrightarrow{CD}=\begin{pmatrix}2\\5\\2\end{pmatrix}$ $\overrightarrow{DA}=\begin{pmatrix}7\\7\\-14\end{pmatrix}$ $\overrightarrow{DB}=\begin{pmatrix}13\\1\\-8\end{pmatrix}$ $\overrightarrow{DC}=\begin{pmatrix}-2\\-5\\2\end{pmatrix}$

For example

$$\frac{1}{6}\left|\overrightarrow{AB}\cdot\left(\overrightarrow{AC}\times\overrightarrow{AD}\right)\right|=\frac{1}{6}\left|\begin{pmatrix}6\\-6\\6\end{pmatrix}\cdot\left(\begin{pmatrix}-9\\-12\\12\end{pmatrix}\times\begin{pmatrix}-7\\-7\\14\end{pmatrix}\right)\right|=\frac{1}{6}\left|\begin{pmatrix}6\\-6\\6\end{pmatrix}\cdot\begin{pmatrix}-84\\42\\-21\end{pmatrix}\right|=\frac{1}{6}\times 882=147$$

$$\frac{1}{6}\left|\overrightarrow{BA}\cdot\left(\overrightarrow{BC}\times\overrightarrow{BD}\right)\right|=\frac{1}{6}\left|\begin{pmatrix}-6\\6\\-6\end{pmatrix}\cdot\left(\begin{pmatrix}15\\-6\\6\end{pmatrix}\times\begin{pmatrix}-13\\-1\\8\end{pmatrix}\right)\right|=\frac{1}{6}\left|\begin{pmatrix}-6\\6\\-6\end{pmatrix}\cdot\begin{pmatrix}-42\\42\\-63\end{pmatrix}\right|=\frac{1}{6}\times 882=147$$

$$\frac{1}{6}\left|\overrightarrow{CA}\cdot\left(\overrightarrow{CD}\times\overrightarrow{CB}\right)\right|=\frac{1}{6}\left|\begin{pmatrix}9\\12\\-12\end{pmatrix}\cdot\left(\begin{pmatrix}2\\5\\2\end{pmatrix}\times\begin{pmatrix}15\\6\\-6\end{pmatrix}\right)\right|=\frac{1}{6}\left|\begin{pmatrix}9\\12\\-12\end{pmatrix}\cdot\begin{pmatrix}-42\\42\\-63\end{pmatrix}\right|=\frac{1}{6}\times 882=147$$

$$\frac{1}{6}\left|\overrightarrow{DA}\cdot\left(\overrightarrow{DB}\times\overrightarrow{DC}\right)\right|=\frac{1}{6}\left|\begin{pmatrix}7\\7\\-14\end{pmatrix}\cdot\left(\begin{pmatrix}13\\1\\-8\end{pmatrix}\times\begin{pmatrix}-2\\-5\\2\end{pmatrix}\right)\right|=\frac{1}{6}\left|\begin{pmatrix}7\\7\\-14\end{pmatrix}\cdot\begin{pmatrix}42\\-42\\63\end{pmatrix}\right|=\frac{1}{6}\times 882=147$$

Note candidates may write as

$$\frac{1}{6}\begin{vmatrix}6 & -6 & 6\\-9 & -12 & 12\\-7 & -7 & 14\end{vmatrix}=\frac{1}{6}\left|6(-168+84)+6(-126+84)+6(63-84)\right|=\frac{1}{6}\left|-882\right|=147$$

Pearson Education Limited. Registered company number 872828
with its registered office at 80 Strand, London, WC2R 0RL, United Kingdom