AS

FURTHER MATHEMATICS
7366/2D

Paper 2 Discrete

Mark scheme

June 2022

Version: 1.0 Final Mark Scheme

Mark schemes are prepared by the Lead Assessment Writer and considered, together with the relevant questions, by a panel of subject teachers. This mark scheme includes any amendments made at the standardisation events which all associates participate in and is the scheme which was used by them in this examination. The standardisation process ensures that the mark scheme covers the students' responses to questions and that every associate understands and applies it in the same correct way. As preparation for standardisation each associate analyses a number of students' scripts. Alternative answers not already covered by the mark scheme are discussed and legislated for. If, after the standardisation process, associates encounter unusual answers which have not been raised they are required to refer these to the Lead Examiner.

It must be stressed that a mark scheme is a working document, in many cases further developed and expanded on the basis of students' reactions to a particular paper. Assumptions about future mark schemes on the basis of one year's document should be avoided; whilst the guiding principles of assessment remain constant, details will change, depending on the content of a particular examination paper.

Further copies of this mark scheme are available from aqa.org.uk

Mark scheme instructions to examiners

General

The mark scheme for each question shows:

- the marks available for each part of the question
- the total marks available for the question
- marking instructions that indicate when marks should be awarded or withheld including the principle on which each mark is awarded. Information is included to help the examiner make his or her judgement and to delineate what is creditworthy from that not worthy of credit
- a typical solution. This response is one we expect to see frequently. However credit must be given on the basis of the marking instructions.

If a student uses a method which is not explicitly covered by the marking instructions the same principles of marking should be applied. Credit should be given to any valid methods. Examiners should seek advice from their senior examiner if in any doubt.

Key to mark types

M	mark is for method
R	mark is for reasoning
A	mark is dependent on M marks and is for accuracy
B	mark is independent of M marks and is for method and accuracy
E	mark is for explanation
F	follow through from previous incorrect result

Key to mark scheme abbreviations

CAO	correct answer only
CSO	correct solution only
ft	follow through from previous incorrect result
‘their’	indicates that credit can be given from previous incorrect result
AWFW	anything which falls within
AWRT	anything which rounds to
ACF	any correct form
AG	answer given
SC	special case
OE	or equivalent
NMS	no method shown
PI	possibly implied
sf	significant figure(s)
dp	decimal place(s)

Examiners should consistently apply the following general marking principles:

No Method Shown

Where the question specifically requires a particular method to be used, we must usually see evidence of use of this method for any marks to be awarded.

Where the answer can be reasonably obtained without showing working and it is very unlikely that the correct answer can be obtained by using an incorrect method, we must award **full marks**. However, the obvious penalty to candidates showing no working is that incorrect answers, however close, earn **no marks**.

Where a question asks the candidate to state or write down a result, no method need be shown for full marks.

Where the permitted calculator has functions which reasonably allow the solution of the question directly, the correct answer without working earns **full marks**, unless it is given to less than the degree of accuracy accepted in the mark scheme, when it gains **no marks**.

Otherwise we require evidence of a correct method for any marks to be awarded.

Diagrams

Diagrams that have working on them should be treated like normal responses. If a diagram has been written on but the correct response is within the answer space, the work within the answer space should be marked. Working on diagrams that contradicts work within the answer space is not to be considered as choice but as working, and is not, therefore, penalised.

Work erased or crossed out

Erased or crossed out work that is still legible and has not been replaced should be marked. Erased or crossed out work that has been replaced can be ignored.

Choice

When a choice of answers and/or methods is given and the student has not clearly indicated which answer they want to be marked, mark positively, awarding marks for all of the student's best attempts. Withhold marks for final accuracy and conclusions if there are conflicting complete answers or when an incorrect solution (or part thereof) is referred to in the final answer.

AS/A-level Maths/Further Maths assessment objectives

AO		Description
AO1	AO1.1a	Select routine procedures
	AO1.1b	Correctly carry out routine procedures
	AO1.2	Accurately recall facts, terminology and definitions
AO2	AO2.1	Construct rigorous mathematical arguments (including proofs)
	AO2.2a	Make deductions
	AO2.2b	Make inferences
	AO2.3	Assess the validity of mathematical arguments
	AO2.4	Explain their reasoning
	AO2.5	Use mathematical language and notation correctly
AO3	AO3.1a	Translate problems in mathematical contexts into mathematical processes
	AO3.1b	Translate problems in non-mathematical contexts into mathematical processes
	AO3.2a	Interpret solutions to problems in their original context
	AO3.2b	Where appropriate, evaluate the accuracy and limitations of solutions to problems
	AO3.3	Translate situations in context into mathematical models
	AO3.4	Use mathematical models
	AO3.5a	Evaluate the outcomes of modelling in context
	AO3.5b	Recognise the limitations of models
	AO3.5c	Where appropriate, explain how to refine models

Q	Marking instructions	AO	Marks	Typical solution
1(a)	Circles correct answer	1.2	B1	3
	Subtotal		**1**	

Q	Marking instructions	AO	Marks	Typical solution
1(b)	Circles correct answer	1.1b	B1	5
	Subtotal		**1**	

	Question total		**2**	

Q	Marking instructions	AO	Marks	Typical solution
2(a)	Determines the correct value of the cut Condone missing units	1.1b	B1	$110 + 120 + 45 + 55 + 70$ $= 400$ m^3 s^{-1}
	Subtotal		**1**	

Q	Marking instructions	AO	Marks	Typical solution
2(b)	Writes down the correct cut	1.1b	B1	$\{A, B, C, D, E, G, H, I\}$ $\{F\}$
	Subtotal		**1**	

Q	Marking instructions	AO	Marks	Typical solution
2(c)	Deduces that the maximum flow cannot exceed the minimum of their answer to **(a)** and 300 Condone strict inequality but not equality	2.2a	B1F	As $300 < 400$, the maximum flow through the network is less than or equal to 300 m^3 s^{-1} by the maximum flow–minimum cut theorem
	Explains their answer with reference to the maximum flow–minimum cut theorem Must be weak inequality	2.4	E1F	
	Subtotal		**2**	

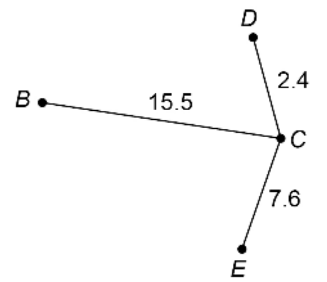
	Question total		**4**	

Q	Marking instructions	AO	Marks	Typical solution
3(a)	States the correct critical path and no others	1.1b	B1	*ADEHK*
	Subtotal		**1**	

Q	Marking instructions	AO	Marks	Typical solution
3(b)	Uses the model to assess the effect of the change of activity *G* on the earliest start times of activities *I* and *J* or states that *G* remains non-critical (PI by A1)	3.5c	M1	The earliest start times of activity *I* and activity *J* increase to 14 Activities *I* and *J* remain non-critical, so the earliest start time and latest finish time of activity *K* is unchanged. Therefore, the latest finish times for activities *I* and *J* are unchanged.
	States explicitly that the earliest start time and latest finish time of activity *K* remain unchanged	1.1b	B1	
	Deduces that the earliest start times of *I* and *J* increase to 14 and that the latest finish times of *I* and *J* are unchanged	2.2a	A1	
	Subtotal		**3**	

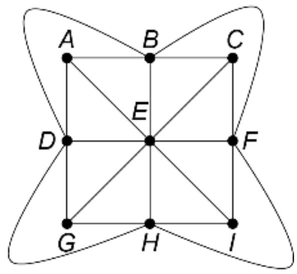
	Question total		**4**	

Q	Marking instructions	AO	Marks	Typical solution
4(a)	Uses the nearest neighbour algorithm from Deganwy by finding the first two arcs of the Hamiltonian cycle PI by 2.4 + 7.6	1.1a	M1	Deganwy – Conwy – E'bach – Aber – Bangor – Deganwy 2.4 + 7.6 + 17.1 + 9.1 + 17.8 = 54.0
	Determines correctly an upper bound for the TSP problem Accept 54	1.1b	A1	
	Subtotal		**2**	

Q	Marking instructions	AO	Marks	Typical solution
4(b)	Translates the problem to a mathematical process by finding at least 2 arcs of the MST	3.1b	M1	
	Finds the minimum spanning tree for *B*, *C*, *D* & *E*	1.1b	A1	
	Finds a lower bound for the distance by using their minimum spanning tree (deleting Aber) and the two shortest arcs from Aber	1.1b	B1F	MST excluding Aber = 15.5 + 2.4 + 7.6 = 25.5 The two shortest arcs from Aber are 9.1 and 10.0 Lower bound = 25.5 + 19.1 = 44.6
	Subtotal		**3**	

	Question total		**5**	

Q	Marking instructions	AO	Marks	Typical solution
5(a)	Uses Euler's formula for connected planar graphs or Obtains $f = 13$ without using Euler's formula for connected planar graphs	1.1a	M1	$v - e + f = 2$ For graph J: $9 - 20 + f = 2$ $f = 13$
	Obtains the correct value for f CSO	1.1b	A1	
	Subtotal		**2**	

Q	Marking instructions	AO	Marks	Typical solution
5(b)	Draws J correctly in planar form	1.1b	B1	
	Subtotal		**1**	

	Question total		**3**	

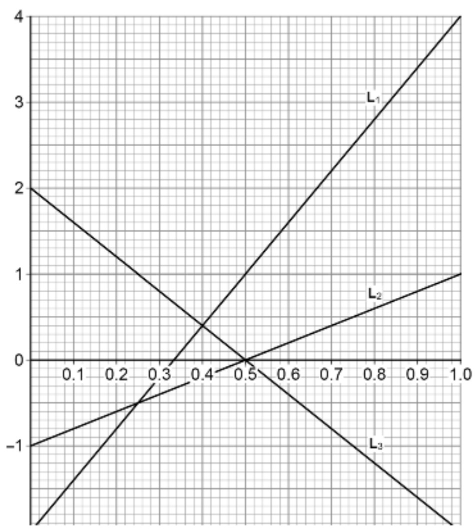
<table>
<tr><th>Q</th><th>Marking instructions</th><th>AO</th><th>Marks</th><th>Typical solution</th></tr>
<tr><td rowspan="2">6(a)</td><td>Completes correctly at least 2 rows or at least 2 columns</td><td>1.1a</td><td>M1</td><td rowspan="2">
<table>
<tr><th></th><th>A</th><th>B</th><th>C</th><th>D</th></tr>
<tr><td>A</td><td>A</td><td>A</td><td>D</td><td>D</td></tr>
<tr><td>B</td><td>A</td><td>B</td><td>C</td><td>D</td></tr>
<tr><td>C</td><td>D</td><td>C</td><td>C</td><td>D</td></tr>
<tr><td>D</td><td>D</td><td>D</td><td>D</td><td>D</td></tr>
</table>
</td></tr>
<tr><td>Completes Cayley table correctly</td><td>1.1b</td><td>A1</td></tr>
<tr><td></td><td>Subtotal</td><td></td><td>2</td><td></td></tr>
</table>

Q	Marking instructions	AO	Marks	Typical solution
6(b)	Explains that left and right multiplication with **B** leaves the matrix unchanged	2.4	E1	The row and column for **B** are the same as the row and column heading, therefore **B** is the identity element of *S*
	Subtotal		**1**	

Q	Marking instructions	AO	Marks	Typical solution
6(c)	Explains that matrix multiplication is commutative for *S* or Explains that matrix multiplication, in general, is not commutative	2.4	M1	As the Cayley table is symmetrical about the leading diagonal so matrix multiplication is commutative for *S* However, matrix multiplication is not a commutative operation in general. Therefore, Sam’s statement is not valid.
	States that matrix multiplication is commutative for *S* but, in general, is not commutative and concludes that Sam’s statement is not valid.	2.3	A1	
	Subtotal		**2**	

	Question total		**5**	

Q	Marking instructions	AO	Marks	Typical solution
7(a)	States that strategy $\mathbf{K}_3$ dominates strategy $\mathbf{K}_2$	1.1b	B1	$-4 < -2,\ -2 < -1,\ 0 < 2$ hence strategy $\mathbf{K}_3$ dominates strategy $\mathbf{K}_2$ Therefore, Kez should never play strategy $\mathbf{K}_2$
	Subtotal		**1**	

Q	Marking instructions	AO	Marks	Typical solution
7(b)	Introduces and defines a probability variable (PI)	3.3	B1	Let Kez choose strategy $\mathbf{K}_1$ with probability p and strategy $\mathbf{K}_3$ with probability $1 - p$ If Lui plays:
	Uses the model to find one expected gain for Kez in terms of the probability variable	3.4	M1	$\mathbf{L}_1$: expected gain for Kez $= 4p - 2(1 - p) = 6p - 2$ $\mathbf{L}_2$: expected gain for Kez $= p - (1 - p) = 2p - 1$ $\mathbf{L}_3$: expected gain for Kez $= -2p + 2(1 - p) = -4p + 2$
	Finds correctly all three expected gains for Kez in terms of the probability variable	1.1b	A1	
	Uses a graph with straight lines and at least one vertical axis and sketches one of 'their' expected gains correctly (PI)	1.1a	M1	
	Identifies correctly the optimal point of intersection from the graph and finds the value of the probability variable	1.1b	A1	$2p - 1 = -4p + 2$ $p = 0.5$
	Determines the number of times that Kez plays strategy $\mathbf{K}_3$	3.2a	B1F	$0.5 \times 20 = 10$ Kez is expected to play strategy $\mathbf{K}_3$ 10 times out of 20
	Subtotal		**6**	

	Question 7 total		**7**	

Q	Marking instructions	AO	Marks	Typical solution
8(a)	Explains correctly how each term in the expression relates to the total area that Alli plants	1.1b	B1	$\frac{1}{16}x =$ area required for the garlic cloves $\frac{1}{36}y =$ area required for the leek seedlings
	Subtotal		**1**	

Q	Marking instructions	AO	Marks	Typical solution
8(b)(i)	Obtains at least one correct non-trivial constraint for x or y Condone strict inequality	3.3	M1	Maximise $\frac{1}{16}x + \frac{1}{36}y$ subject to: $15x + 10y \leq 1500$ $y \geq 50$ $y \leq x$ x, y are integer
	Obtains three correct constraints in x and/or y Condone strict inequality	1.1b	A1	
	Formulates the linear programming problem correctly with all constraints correct and use of 'maximise' Condone inclusion of $x \geq 0$	2.5	A1	
	Subtotal		**3**	

Q	Marking instructions	AO	Marks	Typical solution
8(b)(ii)	Recognises a limitation of the model in the context of the problem with reference to area	3.5b	B1	The linear programming problem does not take into account the area of Alli's garden.
	Subtotal		**1**	

Q	Marking instructions	AO	Marks	Typical solution
8(c)(i)	Evaluates the new model and identifies that the constraint modelling the money has changed	3.5a	E1	$15x + 10y \leq 1500$ has changed. This means that the number of cloves & seedlings that Alli can buy has increased as the amount of money available has increased.
	Infers a change in the financial context of the problem, such as the total money available increased or cost of cloves/seedlings has decreased or Infers the implications of the change, such as Alli can now plant more cloves/seedlings	2.2b	B1	
	Subtotal		**2**	

Q	Marking instructions	AO	Marks	Typical solution
8(c)(ii)	Using the model, identifies a vertex of the feasible region (PI)	3.4	M1	Optimal point at (100, 50) $\frac{1}{16} \times 100 + \frac{1}{36} \times 50 = 7.64$ m^2
	Obtains correct coordinates of optimal vertex	1.1b	A1	
	Calculates the correct maximum total area AWRT 7.6 from correct working Condone missing units	1.1b	A1	
	Subtotal		**3**	

	Question total		**10**	

	Question Paper total		**40**	