A-level

## MATHEMATICS

7357/2

## Paper 2

Mark scheme
June 2022
Version: 1.0 Final Mark Scheme

Mark schemes are prepared by the Lead Assessment Writer and considered, together with the relevant questions, by a panel of subject teachers. This mark scheme includes any amendments made at the standardisation events which all associates participate in and is the scheme which was used by them in this examination. The standardisation process ensures that the mark scheme covers the students' responses to questions and that every associate understands and applies it in the same correct way. As preparation for standardisation each associate analyses a number of students' scripts. Alternative answers not already covered by the mark scheme are discussed and legislated for. If, after the standardisation process, associates encounter unusual answers which have not been raised they are required to refer these to the Lead Examiner.

It must be stressed that a mark scheme is a working document, in many cases further developed and expanded on the basis of students' reactions to a particular paper. Assumptions about future mark schemes on the basis of one year's document should be avoided; whilst the guiding principles of assessment remain constant, details will change, depending on the content of a particular examination paper.

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## Mark scheme instructions to examiners

## General

The mark scheme for each question shows:

- the marks available for each part of the question
- the total marks available for the question
- marking instructions that indicate when marks should be awarded or withheld including the principle on which each mark is awarded. Information is included to help the examiner make his or her judgement and to delineate what is creditworthy from that not worthy of credit
- a typical solution. This response is one we expect to see frequently. However credit must be given on the basis of the marking instructions.

If a student uses a method which is not explicitly covered by the marking instructions the same principles of marking should be applied. Credit should be given to any valid methods. Examiners should seek advice from their senior examiner if in any doubt.

## Key to mark types

| $M$ | mark is for method |
| :--- | :--- |
| $R$ | mark is for reasoning |
| A | mark is dependent on M marks and is for accuracy |
| B | mark is independent of M marks and is for method and accuracy |
| E | mark is for explanation |
| F | follow through from previous incorrect result |

## Key to mark scheme abbreviations

| CAO | correct answer only |
| :--- | :--- |
| CSO | correct solution only |
| ft | follow through from previous incorrect result |
| 'their' | Indicates that credit can be given from previous incorrect result |
| AWFW | anything which falls within |
| AWRT | anything which rounds to |
| ACF | any correct form |
| AG | answer given |
| SC | special case |
| OE | or equivalent |
| NMS | no method shown |
| PI | possibly implied |
| sf | significant figure(s) |
| dp | decimal place(s) |

## AS/A-level Maths/Further Maths assessment objectives

| AO |  |  |
| :--- | :--- | :--- |
| AO1 | AO1.1a | Select routine procedures |
|  | AO1.1b | Correctly carry out routine procedures |
|  | AO1.2 | Accurately recall facts, terminology and definitions |
|  | AO2.1 | Construct rigorous mathematical arguments (including proofs) |
|  | AO2.2a | Make deductions |
|  | AO2.2b | Make inferences |
|  | AO2.4 | Explain their reasoning |
|  | AO2.5 | Use mathematical language and notation correctly |
|  | Translate problems in mathematical contexts into mathematical processes |  |
|  | AO3.1b | Translate problems in non-mathematical contexts into mathematical processes |
|  | AO3.2a | Interpret solutions to problems in their original context |
|  | AO3.2b | Where appropriate, evaluate the accuracy and limitations of solutions to problems |
|  | AO3.3 | Translate situations in context into mathematical models |
|  | AO3.4 | Use mathematical models |
|  | AO3.5a | Evaluate the outcomes of modelling in context |
|  | AO3.5b | Recognise the limitations of models |
|  | AO3.5c | Where appropriate, explain how to refine models |

Examiners should consistently apply the following general marking principles

## No Method Shown

Where the question specifically requires a particular method to be used, we must usually see evidence of use of this method for any marks to be awarded.

Where the answer can be reasonably obtained without showing working and it is very unlikely that the correct answer can be obtained by using an incorrect method, we must award full marks. However, the obvious penalty to students showing no working is that incorrect answers, however close, earn no marks.

Where a question asks the student to state or write down a result, no method need be shown for full marks.

Where the permitted calculator has functions which reasonably allow the solution of the question directly, the correct answer without working earns full marks, unless it is given to less than the degree of accuracy accepted in the mark scheme, when it gains no marks.

Otherwise we require evidence of a correct method for any marks to be awarded.

## Diagrams

Diagrams that have working on them should be treated like normal responses. If a diagram has been written on but the correct response is within the answer space, the work within the answer space should be marked. Working on diagrams that contradicts work within the answer space is not to be considered as choice but as working, and is not, therefore, penalised.

## Work erased or crossed out

Erased or crossed out work that is still legible and has not been replaced should be marked. Erased or crossed out work that has been replaced can be ignored.

## Choice

When a choice of answers and/or methods is given and the student has not clearly indicated which answer they want to be marked, mark positively, awarding marks for all of the student's best attempts. Withhold marks for final accuracy and conclusions if there are conflicting complete answers or when an incorrect solution (or part thereof) is referred to in the final answer.

| $\mathbf{Q}$ | Marking instructions | AO | Marks | Typical solution |
| :---: | :--- | :---: | :---: | :--- |
| $\mathbf{1}$ | Ticks correct box | 1.1 b | B 1 | $(x-4)^{2}+(y+5)^{2}=36$ |
|  | Question 1 Total |  | $\mathbf{1}$ |  |


| $\mathbf{Q}$ | Marking instructions | AO | Marks | Typical solution |
| :---: | :--- | :---: | :---: | :--- |
| $\mathbf{2}$ | Circles correct answer | 2.2 a | R 1 | -1 |
|  |  | Question 2 Total |  | $\mathbf{1}$ |


| $\mathbf{Q}$ | Marking instructions | $\mathbf{A O}$ | Marks | Typical solution |
| :---: | :--- | :---: | :---: | :--- |
| 3 | Ticks correct box | 1.2 | B 1 |  |
|  |  |  |  |  |


| Q | Marking instructions | AO | Marks | Typical solution |
| :---: | :---: | :---: | :---: | :---: |
| 4 | Uses the sine rule Or Substitutes correctly into the cosine rule | 1.1a | M1 | $\begin{aligned} & \frac{\sin \theta}{8.7}=\frac{\sin 38}{6.1} \\ & \theta=61.4 \\ & A=180-61.4 \\ & =118.58 \ldots \end{aligned}$ |
|  | Obtains a value of 61 or $61.410964 \ldots$ rounded or truncated Condone answer (in radians) of 1.0718... or 0.4364... <br> PI by correct obtuse angle or 81 <br> Or <br> Obtains correct length <br> $\mathrm{AB}=3.9367 \ldots$ <br> Or $\mathrm{AB}^{2}=15.4998 \ldots$ | 1.1b | A1 |  |
|  | Deduces the largest angle is $119$ <br> AWRT <br> CAO | 2.2a | A1 |  |
|  | Question 4 Total |  | 3 |  |


| Q | Marking instructions | AO | Marks | Typical solution |
| :---: | :--- | :---: | :---: | :--- |
| $\mathbf{5 ( a )}$ | Obtains 16 <br> Not incorrectly labelled | 1.1 b | B1 | $(2+5 x)^{4}=16+160 x+600 x^{2}$ <br> $+1000 x^{3}+625 x^{4}$ |
|  | Obtains 600 <br> Not incorrectly labelled | 1.1b | B1 | $A=16$ <br> $B=600$ |
|  | Subtotal |  | $\mathbf{2}$ |  |


| Q | Marking instructions | AO | Marks | Typical solution |
| :---: | :---: | :---: | :---: | :---: |
| 5(b) | Obtains the expansion of $\begin{aligned} & (2-5 x)^{4}= \\ & A-160 x+B x^{2}-1000 x^{3}+625 x^{4} \end{aligned}$ <br> Accept $A$ and $B$ unsubstituted or their $A$ and $B$ <br> Or <br> Uses a valid method and obtains one of $C=320$ or $D=2000$ | 1.1a | M1 | $\begin{aligned} & (2+5 x)^{4}-(2-5 x)^{4} \\ & =16+160 x+600 x^{2}+1000 x^{3}+625 x^{4} \\ & -\left(16-160 x+600 x^{2}-1000 x^{3}+625 x^{4}\right) \\ & =320 x+2000 x^{3} \end{aligned}$ |
|  | Completes reasoned argument to show $(2+5 x)^{4}-(2-5 x)^{4}=320 x+2000 x^{3}$ <br> Accept $A$ and $B$ unsubstituted or their $A$ and $B$ <br> Must finish with $320 x+2000 x^{3}$ don't accept just $\mathrm{C}=320$ and $D=2000$ | 2.1 | R1F |  |
|  | Subtotal |  | 2 |  |


| Q | Marking instructions | AO | Marks | Typical solution |
| :---: | :--- | :---: | :---: | :--- |
| 5(c) | Integrates one term correctly <br> Accept $C$ and $D$ unsubstituted or <br> their $C$ and $D$ <br> Or <br> Uses reverse of chain rule to <br> obtain at least one term of the <br> form <br> $P(2 \pm 5 x)^{5}, P= \pm \frac{1}{5}$ or $\pm \frac{1}{25}$ | 1.1a | M1 |  |
|  | Obtains $\frac{320}{2} x^{2}+\frac{2000}{4} x^{4}+c$ <br> FT $C$ and $D$ unsubstituted or <br> their $C$ and $D$ <br> Or <br> $\frac{(2+5 x)^{5}}{5 \times 5}+\frac{(2-5 x)^{5}}{5 \times 5}+c$ <br> Condone missing $+c$ | 1.1 b | A1F | $=\int\left(320 x+2000 x^{3}\right) \mathrm{d} x$ |


| Q | Marking instructions | AO | Marks | Typical solution |
| :---: | :--- | :---: | :---: | :--- |
| $\mathbf{6 ( a )}$ | Squares a number with two or <br> more digits and adds its digits. <br> Must be explicit | 1.1 a | M1 |  |
|  | Completes argument to show <br> that Asif's method is incorrect <br> Must compare sum of digits with <br> last digit of square number | 2.3 | R 1 | $1+2=3$ <br> $3 \neq 4$ |
|  | Subtotal |  | $\mathbf{2}$ |  |


| $\mathbf{Q}$ | Marking instructions | AO | Marks | Typical solution |
| :---: | :--- | :---: | :---: | :--- |
| $\mathbf{6 ( b )}$ | Obtains 1 | 1.1b | B1 | 1 |
|  | Subtotal |  | $\mathbf{1}$ |  |


| Q | Marking instructions | AO | Marks | Typical solution |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| 6(c) | Lists at least four single digits and their squares Or Explains why odd digits do not need to be considered | 1.1a | M1 | $\begin{aligned} & 0^{2}=0 \\ & 1^{2}=1 \\ & 2^{2}=4 \\ & 3^{2}=9 \end{aligned}$ | $\begin{aligned} 4^{2} & =16 \\ 5^{2} & =25 \\ 6^{2} & =36 \\ 7^{2} & =49 \end{aligned}$ | $\begin{aligned} & 8^{2}=64 \\ & 9^{2}=81 \end{aligned}$ |
|  | Completes rigorous argument to prove that no square number has a last digit of 8 <br> OE <br> CSO | 2.1 | R1 | Therefore, there can be no square number which has a last digit of 8 |  |  |
| Subtotal |  |  | 2 |  |  |  |
| Question 6 Total |  |  | 5 |  |  |  |


| Q | Marking instructions | AO | Marks | Typical solution |
| :---: | :--- | :---: | :---: | :---: |
| 7(a) | Identifies the height of the <br> triangle or rectangle as $15-q^{2}$ <br> PI by $\left(q, 15-q^{2}\right), h=15-q^{2}$ or <br> $y=15-q^{2}$ may be indicated on <br> diagram | 3.1 a | M1 |  |
|  | Completes rigorous argument to <br> show the given result. It must be <br> clear how they have defined the <br> base and height with use of <br> $\frac{1}{2} \times 2 q\left(15-q^{2}\right)$ <br> for whole triangle <br> Or <br> $\left[\begin{array}{l}\left.\frac{1}{2} q\left(15-q^{2}\right)\right] \times 2 \text { for two half } \\ \text { triangles } \\ \text { Or } \\ \text { Explains why the area of the } \\ \text { triangle is given by } \\ q\left(15-q^{2}\right) \text { with reference to the } \\ \text { rectangle on either side of y-axis }\end{array}\right.$ | 2.1 | R 1 |  |



|  | Question 7 Total |  | 9 |  |
| :--- | :--- | :--- | :--- | :--- |


| Q | Marking instructions | AO | Marks | Typical solution |
| :---: | :--- | :---: | :---: | :--- |
| 8(a) | Sketches a branch of the curve <br> with correct shape in 1st or 2nd <br> quadrant with correct <br> asymptotes and not touching the <br> axes | 1.2 | M1 |  |
|  | Sketches a fully correct curve <br> with correct asymptotes and not <br> touching the axes | 1.1 b | A1 |  |
|  | Subtotal |  | $\mathbf{2}$ |  |



| Q | Marking instructions | AO | Marks | Typical solution |
| :---: | :---: | :---: | :---: | :---: |
| 9 | Uses a log (or index) law correctly on an algebraic term $\log A \pm \log B$ <br> $n \log A$ | 1.1b | B1 | $\begin{aligned} & \log _{2} x^{3}-\log _{2} y^{2}=9 \\ & \log _{2} \frac{x^{3}}{y^{2}}=9 \\ & \frac{x^{3}}{y^{2}}=2^{9} \\ & x^{3}=2^{9} y^{2} \\ & x=8 y^{\frac{2}{3}} \end{aligned}$ |
|  | Raises 2 to the power of both sides (removal of $\log _{2}$ ) <br> Or <br> writes 9 as $9 \log _{2} 2$ or $\log _{2} 512$ OE | 1.1a | M1 |  |
|  | Obtains correct equation without logs Or obtains $\log _{2}(x)=\log _{2}\left(8 y^{2 / 3}\right)$ | 1.1b | A1 |  |
|  | Completes a reasoned argument to obtain $x=8 y^{\frac{2}{3}}$ | 2.1 | R1 |  |
|  | Question 9 Total |  | 4 |  |


| Q | Marking instructions | AO | Marks | Typical solution |
| :---: | :--- | :---: | :---: | :--- |
| $\mathbf{1 0 ( a ) ( i )}$ | Forms correct model <br> Or <br> applies repeated percentage <br> increase 4 times <br> Pl by AWRT 75.9 | 3.3 | B1 |  |
|  | Substitutes $t=5$ into their model <br> Or <br> Applies repeated percentage <br> increase 5 times | 3.4 | M1 | $t=5 \Rightarrow x=25 \times 1.32^{5}=100.18 \ldots$ |
|  | Obtains 101 <br> Condone 100 <br> CAO | 3.2 a | A1 | $x=25 \times 1.32^{t}$ |
|  | Subtotal |  | $\mathbf{3}$ |  |


| Q | Marking instructions | AO | Marks | Typical solution |
| :---: | :--- | :---: | :---: | :--- |
| $\mathbf{1 0 ( a ) ( i i ) ~}$ | Explains that the model grows <br> exponentially | 3.5 b | E1 | The value predicted by the <br> exponential model will grow without <br> limit. |
| Must refer to model and <br> exponential | E1 | This can't be true as there are only <br> 900 tomato plants in the <br> greenhouse. |  |  |
|  | Refers to 900 plants. <br> eg this can't be true as there are <br> only 900 tomato plants <br> Condone reference to <br> "tomato(es)" or "plants" in place <br> of "tomato plants" | 3.5 a | Subtotal |  |
|  | $\mathbf{2}$ |  |  |  |


| Q | Marking instructions | AO | Marks | Typical solution |
| :---: | :---: | :---: | :---: | :---: |
| 10(b)(i) | Rearranges to obtain one of the following: $\begin{aligned} & \frac{P}{x(900-x)} \frac{\mathrm{d} x}{\mathrm{~d} t}=\frac{1}{Q} \\ & \frac{P}{x(900-x)} \mathrm{d} x=\frac{1}{Q} \mathrm{~d} t \\ & \frac{P}{x(900-x)}=\frac{1}{Q} \frac{\mathrm{dt}}{\mathrm{~d} x} \end{aligned}$ <br> where $P \times Q=2700$ <br> If their $\mathrm{P}=2700$ no need to see explicit $\frac{1}{Q}$ with $d t$, or $\frac{\mathrm{d} t}{\mathrm{~d} x}$ <br> May include integral signs | 3.1a | B1 | $\begin{aligned} & \frac{\mathrm{d} x}{\mathrm{~d} t}=\frac{x(900-x)}{2700} \\ & \frac{2700}{x(900-x)} \frac{\mathrm{d} x}{\mathrm{~d} t}=1 \\ & \int\left(\frac{A}{x}+\frac{B}{900-x}\right) d x=\int d t \\ & \frac{2700}{x(900-x)} \equiv \frac{A}{x}+\frac{B}{900-x} \\ & 2700 \equiv A(900-x)+B x \\ & x=0 \Rightarrow A=\frac{2700}{900}=3 \\ & x=900 \Rightarrow B=3 \\ & \therefore \end{aligned}$ |
|  | Forms partial fraction equation with correct denominators and uses an appropriate method to find their numerators PI by correct A and B without incorrect working | 3.1a | M1 | $\left(\frac{3}{x}+\frac{300-x}{900} d x\right.$ |
|  | Obtains correct $A$ and $B$ and concludes with $\int\left(\frac{3}{x}+\frac{3}{900-x}\right) d x=\int d t$ <br> Accept $\int 1 d t$ <br> Condone missing brackets | 2.1 | R1 |  |
|  | Subtotal |  | 3 |  |


| Q | Marking instructions | AO | Marks | Typical solution |
| :---: | :---: | :---: | :---: | :---: |
| 10(b)(ii) | Integrates to obtain $\ln x$ or $\pm \ln (900-x)$ <br> Condone missing brackets for this mark | 3.1a | M1 | $\begin{aligned} & 3(\ln x-\ln (900-x))+c=t \\ & 3(\ln 25-\ln (900-25))+c=0 \\ & c=10.67 \\ & t=3(\ln x-\ln (900-x))+10.67 \end{aligned}$ |
|  | Integrates to obtain $\ln x$ and $\pm \ln (900-x)$ <br> Condone missing brackets for this mark | 1.1a | M1 |  |
|  | Integrates to obtain $3(\ln x-\ln (900-x))+c=t$ <br> OE <br> Condone missing $+c$ | 1.1b | A1 |  |
|  | Uses $t=0, x=25$, to obtain a value for $c$ | 3.4 | M1 |  |
|  | Obtains correct equation for $t$ in terms of $x$ <br> ACF <br> If c given as a decimal accept <br> AWRT 11 <br> eg $\begin{aligned} & t=3(\ln x-\ln (900-x))+3 \ln 35 \\ & t=3 \ln \left(\frac{35 x}{900-x}\right) \end{aligned}$ | 1.1b | A1 |  |
|  | Subtotal |  | 5 |  |


| Q | Marking instructions | AO | Marks | Typical solution |
| :---: | :--- | :---: | :---: | :---: |
| 10(b)(iii) | Substitutes $x=450$ into their <br> model from part (b)(ii) | 3.4 | M1 | $3(\ln 450-\ln (450))+10.67$ <br> $=10.67 \ldots$ |
|  | Obtains 11 <br> CAO | 3.2 a | A1 | It takes 11 days from when the <br> damage is first noticed until half of <br> the plants are damaged by insects |
|  | Subtotal |  | $\mathbf{2}$ |  |
|  | Question 10 Total |  |  | $\mathbf{1 5}$ |


| $\mathbf{Q}$ | Marking instructions | AO | Marks | Typical solution |
| :---: | :--- | :---: | :---: | :---: |
| $\mathbf{1 1}$ | Circles correct answer | 1.1 b | B1 | $1.63 \mathrm{~m} \mathrm{~s}^{-2}$ |
|  | Question 11 Total |  | $\mathbf{1}$ |  |


| $\mathbf{Q}$ | Marking instructions | AO | Marks | Typical solution |
| :---: | :--- | :---: | :---: | :---: |
| $\mathbf{1 2}$ | Circles correct answer | 1.1 b | B1 | $a=0.6 u$ |
|  | Question 12 Total |  | 1 |  |


| Q | Marking instructions | AO | Marks | Typical solution |
| :---: | :---: | :---: | :---: | :---: |
| 13(a) | Obtains $7 \sin \theta$ for vertical component of initial velocity | 1.1b | B1 |  |
|  | Uses $v^{2}=u^{2}+2 a s$ with $v=0$ Or uses appropriate constant acceleration equations that form a complete method to obtain $h$ eg finds $t$ then subs into $s=u t+\frac{1}{2} a t^{2}$ | 3.3 | M1 | $u=7 \sin \theta$ $v^{2}=u^{2}+2 a s$ $0=49 \sin ^{2} \theta-19.6 h$ |
|  | Completes argument substituting $s=h, v=0$, $u=7 \sin \theta$ and $a=-9.8$ to show the given result. Accept negative values used consistently AG | 2.1 | R1 | $\stackrel{\text { so }}{h=2.5 \sin ^{2} \theta}$ |
|  | Subtotal |  | 3 |  |


| Q | Marking instructions | AO | Marks | Typical solution |
| :---: | :---: | :---: | :---: | :---: |
| 13(b) | Substitutes any value of $\theta$ in the range $0<\theta \leq 60$ to obtain a height greater than zero | 1.1a | M1 | $\begin{gathered} \theta=60^{\circ} \\ h=2.5 \sin ^{2} 60 \\ h=1.9 \end{gathered}$ |
|  | $\begin{aligned} & \text { Deduces } h=1.9 \\ & \text { AWRT } 1.9 \text { or } \frac{15}{8} \end{aligned}$ | 2.2a | A1 |  |
|  | Subtotal |  | 2 |  |


| Q Marking instructions AO Marks Typical solution <br> 13(c) States that Nisha is incorrect <br> and <br> refers to air resistance or ball <br> modelled as a particle 3.5 a E1 Nisha is incorrect <br> The model ignores air resistance     <br> which gets greater as ball gets     <br> larger     |
| :--- |
|  |


| Q | Marking instructions | AO | Marks | Typical solution |
| :---: | :---: | :---: | :---: | :---: |
| 14(a) | Forms a dimensionally correct non-zero moment. <br> For example $\begin{aligned} & 0.012 g \times 0.066 \\ & 0.012 g \times 66 \\ & 12 g \times 66 \\ & 160 \mathrm{mg} \end{aligned}$ | 3.3 | B1 | $66 \times 12 \mathrm{~g}=80 \mathrm{mg}$ |
|  | Obtains a correct single equation where the only unknown is the mass. This might come from correct moments about a point other than the centre with forces correctly resolved in the vertical direction, but these equations must be combined to eliminate the force at the centre. PI by 9.9 | 1.1a | M1 | $m=9.9$ |
|  | Obtains 9.9 <br> Condone $m=0.0099 \mathrm{~kg}$ OE provided units are included if $m$ is not in grams | 1.1b | A1 |  |
|  | Subtotal |  | 3 |  |


| Q Marking instructions AO Marks Typical solution <br> $\mathbf{1 4 ( b )}$ States a valid assumption about <br> the £2 coin with no <br> contradictions or incorrect <br> statements <br> For example <br> Models the coin as uniform, <br> or weight acts through the <br> centre <br> or centre of mass is at the <br> centre <br> Do not accept 'mass acts' or <br> contre of mass acts' 'at the <br> centre or 'centre of weight' or <br> 'weight is at' 3.5 a E1 The coin is uniform <br>  Subtotal  1  |
| :--- |


| Q | Marking instructions | AO | Marks | Typical solution |
| :---: | :--- | :---: | :---: | :---: |
| $\mathbf{1 5}$ | Obtains a correct expression for <br> the area of a triangle above the <br> time axis in terms of a variable <br> for time | 1.1 b | B1 | Let $t$ be the next time when $v=0$ <br> Area above $=2 t$ |
|  | Obtains a correct expression for <br> the area of the triangle or <br> trapezium below the time axis in <br> terms of a variable for time <br> accept negative area for <br> displacement | 1.1 b | B1 |  |
|  | Forms equation with a single <br> variable using their expressions <br> for area consistent with <br> area above - area below $= \pm k$ <br> Or <br> Forms equation with a single <br> variable using their expressions <br> for displacement consistent with <br> disp above + disp below $=k$ | 3.1 b | M1 |  |
| Where $k$ is one of $3,7,13$ or 17 | Area below $=2(10-t)+20$ |  |  |  |


| Q | Marking instructions | AO | Marks | Typical solution |
| :---: | :---: | :---: | :---: | :---: |
| 16(a) | States or uses the direction of motion is $\left[\begin{array}{c}3 \\ -4\end{array}\right]$ or $\left[\begin{array}{c}9 \\ c+1\end{array}\right]$ <br> Or <br> States or uses the gradient of the direction of motion is $-\frac{4}{3}$ or $\frac{c+1}{9}$ | 3.1a | M1 |  |
|  | Obtains a correct vector equation eg $\left[\begin{array}{c} 10 \\ c \end{array}\right]=\left[\begin{array}{c} 1 \\ -1 \end{array}\right]+k\left[\begin{array}{c} 3 \\ -4 \end{array}\right]$ <br> OE <br> Or <br> Obtains both gradients or both direction vectors $\left[\begin{array}{c}3 \\ -4\end{array}\right],\left[\begin{array}{c}9 \\ c+1\end{array}\right]$ or <br> Obtains a correct cartesian equation for Q . <br> eg $y+1=-\frac{4}{3}(x-1)$ | 1.1b | A1 | $\begin{gathered} {\left[\begin{array}{c} 10 \\ c \end{array}\right]-\left[\begin{array}{c} 1 \\ -1 \end{array}\right]=\left[\begin{array}{c} 9 \\ c+1 \end{array}\right]} \\ {\left[\begin{array}{c} 9 \\ c+1 \end{array}\right]=k\left[\begin{array}{c} 3 \\ -4 \end{array}\right]} \\ k=3 \\ c+1=-12 \end{gathered}$ |
|  | Obtains or eliminates parameter in their vector equation Or Equates gradients or the reciprocals $\frac{c+1}{9}=-\frac{4}{3}$ <br> Or substitutes $x=10$ into their cartesian equation | 1.1a | M1 |  |
|  | Shows that $c=-13$ <br> AG <br> A correct verification method using the given $c=-13$ scores a maximum of M1A1M0A0 | 1.1b | A1 |  |
|  | Subtotal |  | 4 |  |


| $\mathbf{Q}$ | Marking instructions | AO | Marks | Typical solution |
| :---: | :--- | :---: | :---: | :--- |
| $\mathbf{1 6 ( b ) ( i ) ~}$ | Obtains <br> $\binom{-4}{5}+\frac{1}{2}\binom{3}{-4} t^{2}$ metres <br> OE <br> Condone missing units | 2.2 a | B 1 |  |
|  | Subtotal |  | $\mathbf{r}=\left[\begin{array}{c}-4 \\ 5\end{array}\right]+\frac{1}{2}\left[\begin{array}{c}3 \\ -4\end{array}\right] t^{2}$ |  |



| Q | Marking instructions | AO | Marks | Typical solution |
| :---: | :---: | :---: | :---: | :---: |
| 17 | Differentiates with evidence of correct use of product rule. Condone sign errors | 3.4 | M1 | $\begin{aligned} & \mathbf{v}=\frac{\mathrm{d} \mathbf{r}}{\mathrm{~d} t}= \\ & \left(\mathrm{e}^{t} \cos t-\mathrm{e}^{t} \sin t\right) \mathbf{i}+\left(\mathrm{e}^{t} \sin t+\mathrm{e}^{t} \cos t\right) \mathbf{j} \\ & \mathbf{a}=\frac{\mathrm{d} \mathbf{v}}{\mathrm{~d} t}= \\ & \left(\mathrm{e}^{t} \cos t-\mathrm{e}^{t} \sin t-\mathrm{e}^{t} \sin t-\mathrm{e}^{t} \cos t\right) \mathbf{i}+ \\ & \left(\mathrm{e}^{t} \sin t+\mathrm{e}^{t} \cos t+\mathrm{e}^{t} \cos t-\mathrm{e}^{t} \sin t\right) \mathbf{j} \\ & =-2 \mathrm{e}^{t} \sin t \mathbf{i}+2 \mathrm{e}^{t} \cos t \mathbf{j} \\ & \quad=\sqrt{4 \mathrm{e}^{2 t}\left(\sin ^{2} t+\cos ^{2} t\right)} \\ & \therefore\|\mathbf{a}\|=2 \mathrm{e}^{t} \end{aligned}$ |
|  | Finds expression for $\mathbf{v}$ or $\frac{\mathrm{d} \mathbf{r}}{\mathrm{d} t}$ with either $\mathbf{i}$ or $\mathbf{j}$ component fully correct | 1.1a | M1 |  |
|  | Finds fully correct expression for $\mathbf{v}$ or $\frac{\mathrm{d} \mathbf{r}}{\mathrm{d} t}$ $\begin{gathered} \left(\mathrm{e}^{t} \cos t-\mathrm{e}^{t} \sin t\right) \mathbf{i}+ \\ \left(\mathrm{e}^{t} \sin t+\mathrm{e}^{t} \cos t\right) \mathbf{j} \end{gathered}$ <br> Condone poor use of brackets provided fully correct acceleration seen | 1.1b | A1 |  |
|  | Differentiates their $\mathbf{v}$ or $\frac{\mathrm{d} \mathbf{r}}{\mathrm{d} t}$ with evidence of correct use of product rule to find an expression for a with at least one component correct. Condone sign errors | 3.4 | M1 |  |
|  | Finds correct expression for a May be unsimplified | 1.1b | A1 |  |
|  | Obtains an expression for the magnitude of their a provided their a has non-zero $\mathbf{i}$ and $\mathbf{j}$ components | 1.1a | M1 |  |
|  | Completes reasoned argument from a correct a to show given result. Must see a factor of $\begin{aligned} & \left(\sin ^{2} t+\cos ^{2} t\right) \\ & \text { eg } \sqrt{4 \mathrm{e}^{2 t}\left(\sin ^{2} t+\cos ^{2} t\right)} \\ & \mathrm{AG} \end{aligned}$ | 2.1 | R1 |  |
|  | Question 17 Total |  | 7 |  |


| Q | Marking instructions | AO | Marks | Typical solution |
| :---: | :--- | :---: | :---: | :--- |
| 18(a) | Obtains both <br> $\sin ^{-1}\left(\frac{0.6}{0.8}\right)$ and $\sin ^{-1}\left(\frac{0.6}{1.2}\right)$ <br> OE <br> Accept complementary angles <br> or exact values $\frac{\sqrt{3}}{2}$ and $\frac{\sqrt{7}}{4}$ | 1.1 b | B 1 | Angle for $O A=\sin ^{-1}\left(\frac{0.6}{0.8}\right)=48.59^{\circ}$ |
|  | Angle for $O B=\sin ^{-1}\left(\frac{0.6}{1.2}\right)=30^{\circ}$ |  |  |  |
| Resolves forces horizontally to <br> form equilibrium equation, one <br> component correct <br> Or <br> Uses a triangle of forces and <br> applies the sine rule | 3.3 | M1 | $T_{O A} \cos A=T_{O B} \cos B$ |  |
| Obtains correct equation with <br> angles substituted | 1.1 b | A 1 |  |  |


| Q | Marking instructions | AO | Marks | Typical solution |
| :---: | :---: | :---: | :---: | :---: |
| 18(b) | Obtains $T_{O A}=2 g \times$ their ratio | 1.1b | B1 | $\begin{gathered} m \mathrm{~g}=T_{O A} \sin A+T_{O B} \sin B \\ m \mathrm{~g}=2.6 g \sin 48.59+2 g \sin 30 \\ m=3.0 \end{gathered}$ |
|  | Resolves forces vertically to form a three-term equilibrium equation, with at least two terms correct <br> Or <br> Uses a triangle of forces and applies the sine rule | 3.3 | M1 |  |
|  | Forms fully correct equation of forces in equilibrium <br> This mark can be awarded for $m g=T_{O A} \sin A+T_{O B} \sin B$ | 1.1b | A1 |  |
|  | Substitutes their $T_{O A}$ and $T_{O B}$ and correct values for angles into the correct equation and obtains AWRT m = 3 <br> Might come from 2.96.. <br> FT their ratio from part (a) provided their $\mathrm{m}=$ AWRT 3 | 3.4 | A1F |  |
|  | Subtotal |  | 4 |  |
|  | Question 18 Total |  | 8 |  |


| Q | Marking instructions | AO | Marks | Typical solution |
| :---: | :---: | :---: | :---: | :---: |
| 19(a) | States $F=\mu R$ seen anywhere PI by use of $\mu R$ in their 4-term equation of motion or on diagram | 3.3 | B1 |  |
|  | Resolves the weight parallel to the slope to obtain $m g \sin 25$ or better | 1.1b | B1 |  |
|  | Resolves perpendicular to the slope to obtain $R=m g \cos 25$ or better | 1.1b | B1 |  |
|  | Uses F = ma to form a four-term equation with consistent signs. eg $T-\text { weight }- \text { Friction }=m a$ <br> Condone omission of g in weight and friction component | 3.3 | M1 | $\begin{gathered} T-\text { weight }- \text { Friction }=m a \\ T-20 g \sin 25-F=m \times 1.2 \\ T-20 g \sin 25-20 g \cos 25 \mu=m \times 1.2 \end{gathered}$ |
|  | Substitutes $T=230$ and $F=\mu m g \cos 25$ into their four term $F=m a$ equation with consistent signs. Condone ' $m g a$ ' in $F=m a$ for this mark | 1.1a | M1 | $\mu=0.69$ |
|  | Obtains single correct equation with all numerical values substituted. <br> eg $230-196 \sin 25-196 \cos 25 \mu=24$ <br> Scores B1B1B1M1M1A1 | 1.1b | A1 |  |
|  | Obtains $\mu=0.69$ CAO | 3.2a | A1 |  |
|  | Subtotal |  | 7 |  |


| Q | Marking instructions | AO | Marks | Typical solution |
| :---: | :---: | :---: | :---: | :---: |
| 19(b)(i) | $\begin{aligned} & \text { Substitutes } u=0, a=1.2 \text { and } \\ & t=3.8 \text { into } s=u t+\frac{1}{2} a t^{2} \end{aligned}$ <br> Or uses appropriate constant acceleration equations that forms a complete method to obtain $s$ | 1.1a | M1 | $\begin{gathered} s=u t+\frac{1}{2} a t^{2} \\ s=\frac{1}{2} \times 1.2 \times 3.8^{2}=8.664 \\ \\ \begin{aligned} O A & =10-8.664 \\ & =1.3 \mathrm{~m} \end{aligned} \end{gathered}$ |
|  | Obtains AWRT 8.7 | 1.1b | A1 |  |
|  | Obtains 10 - their 8.7 <br> AWRT 1.3 <br> or <br> FT their 8.7 provided it is less than 10 <br> Condone missing units | 1.1b | A1F |  |
|  | Subtotal |  | 3 |  |


| Q | Marking instructions | AO | Marks | Typical solution |
| :---: | :--- | :---: | :---: | :--- |
| 19(b)(ii) | States the crate has been <br> modelled as a particle <br> OE | 3.5 b | E1 | The crate is a particle |
|  | Subtotal |  | $\mathbf{1}$ |  |


|  | Question 19 Total |  | 11 |  |
| :--- | :--- | :--- | :--- | :--- |


|  | Question Paper Total |  | 100 |  |
| :--- | :--- | :--- | :--- | :--- |

