AQA

A-level

FURTHER MATHEMATICS 7367/3D

Paper 3 Discrete

Mark scheme

June 2022

Version: 1.0 Final Mark Scheme

226A7367/3D/MS

Mark schemes are prepared by the Lead Assessment Writer and considered, together with the relevant questions, by a panel of subject teachers. This mark scheme includes any amendments made at the standardisation events which all associates participate in and is the scheme which was used by them in this examination. The standardisation process ensures that the mark scheme covers the students' responses to questions and that every associate understands and applies it in the same correct way. As preparation for standardisation each associate analyses a number of students' scripts. Alternative answers not already covered by the mark scheme are discussed and legislated for. If, after the standardisation process, associates encounter unusual answers which have not been raised they are required to refer these to the Lead Examiner.

It must be stressed that a mark scheme is a working document, in many cases further developed and expanded on the basis of students' reactions to a particular paper. Assumptions about future mark schemes on the basis of one year's document should be avoided; whilst the guiding principles of assessment remain constant, details will change, depending on the content of a particular examination paper.

Further copies of this mark scheme are available from aqa.org.uk

Mark scheme instructions to examiners

General

The mark scheme for each question shows:

- the marks available for each part of the question
- the total marks available for the question
- marking instructions that indicate when marks should be awarded or withheld including the principle on which each mark is awarded. Information is included to help the examiner make his or her judgement and to delineate what is creditworthy from that not worthy of credit
- a typical solution. This response is one we expect to see frequently. However credit must be given on the basis of the marking instructions.

If a student uses a method which is not explicitly covered by the marking instructions the same principles of marking should be applied. Credit should be given to any valid methods. Examiners should seek advice from their senior examiner if in any doubt.

Key to mark types

M	mark is for method
R	mark is for reasoning
A	mark is dependent on M marks and is for accuracy
B	mark is independent of M marks and is for method and accuracy
E	mark is for explanation
F	follow through from previous incorrect result

Key to mark scheme abbreviations

CAO	correct answer only
CSO	correct solution only
ft	follow through from previous incorrect result
'their'	indicates that credit can be given from previous incorrect result
AWFW	anything which falls within
AWRT	anything which rounds to
ACF	any correct form
AG	answer given
SC	special case
OE	or equivalent
NMS	no method shown
PI	possibly implied
sf	significant figure(s)
dp	decimal place(s)

Examiners should consistently apply the following general marking principles:

No Method Shown

Where the question specifically requires a particular method to be used, we must usually see evidence of use of this method for any marks to be awarded.

Where the answer can be reasonably obtained without showing working and it is very unlikely that the correct answer can be obtained by using an incorrect method, we must award **full marks**. However, the obvious penalty to candidates showing no working is that incorrect answers, however close, earn **no marks**.

Where a question asks the candidate to state or write down a result, no method need be shown for full marks.

Where the permitted calculator has functions which reasonably allow the solution of the question directly, the correct answer without working earns **full marks**, unless it is given to less than the degree of accuracy accepted in the mark scheme, when it gains **no marks**.

Otherwise we require evidence of a correct method for any marks to be awarded.

Diagrams

Diagrams that have working on them should be treated like normal responses. If a diagram has been written on but the correct response is within the answer space, the work within the answer space should be marked. Working on diagrams that contradicts work within the answer space is not to be considered as choice but as working, and is not, therefore, penalised.

Work erased or crossed out

Erased or crossed out work that is still legible and has not been replaced should be marked. Erased or crossed out work that has been replaced can be ignored.

Choice

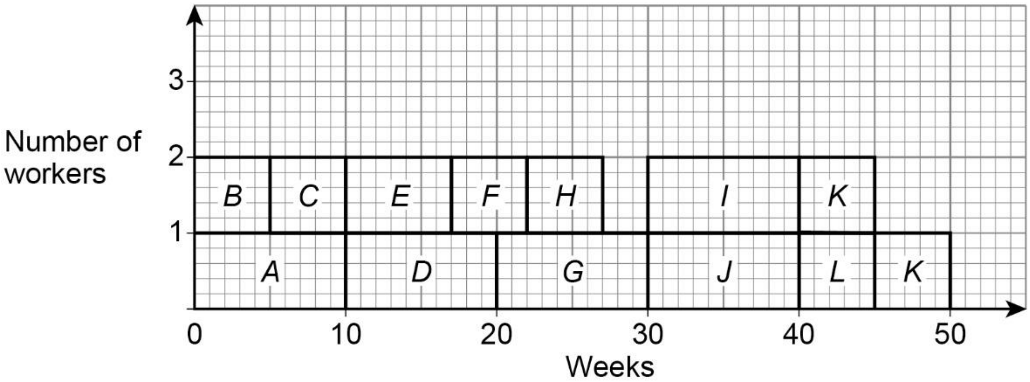
When a choice of answers and/or methods is given and the student has not clearly indicated which answer they want to be marked, mark positively, awarding marks for all of the student's best attempts. Withhold marks for final accuracy and conclusions if there are conflicting complete answers or when an incorrect solution (or part thereof) is referred to in the final answer.

AS/A-level Maths/Further Maths assessment objectives

AO		Description
AO1	AO1.1a	Select routine procedures
	AO1.1b	Correctly carry out routine procedures
	AO1.2	Accurately recall facts, terminology and definitions
AO2	AO2.1	Construct rigorous mathematical arguments (including proofs)
	AO2.2a	Make deductions
	AO2.2b	Make inferences
	AO2.3	Assess the validity of mathematical arguments
	AO2.4	Explain their reasoning
	AO2.5	Use mathematical language and notation correctly
AO3	AO3.1a	Translate problems in mathematical contexts into mathematical processes
	AO3.1b	Translate problems in non-mathematical contexts into mathematical processes
	AO3.2a	Interpret solutions to problems in their original context
	AO3.2b	Where appropriate, evaluate the accuracy and limitations of solutions to problems
	AO3.3	Translate situations in context into mathematical models
	AO3.4	Use mathematical models
	AO3.5a	Evaluate the outcomes of modelling in context
	AO3.5b	Recognise the limitations of models
	AO3.5c	Where appropriate, explain how to refine models

Q	Marking instructions	AO	Marks	Typical solution
1	Ticks correct box	1.2	B1	*G* is not planar
	Total		**1**	

Q	Marking instructions	AO	Marks	Typical solution
2	Circles correct answer	1.1b	B1	8
	Total		**1**	

Q	Marking instructions	AO	Marks	Typical solution
3	Ticks correct box	1.1b	B1	2^{nd} Option (see below)
				
	Total		**1**	

Q	Marking instructions	AO	Marks	Typical solution
4(a)	Explains correctly that strategy **D** is dominated by strategy **C**, so strategy **D** should not be played	3.5c	B1	As $6 \leq 7$, $-2 \leq -1$ and $1 \leq 1$, strategy **C** dominates strategy **D** Ben should never play strategy **D**
	Total		**1**	

Q	Marking instructions	AO	Marks	Typical solution
4(b)	Translates the problem of finding a stable solution into identifying all correct row minima or all correct column maxima May be seen around table Ignore inclusion of strategy **D**	3.1a	M1	row minima: –3, –4, –1 column maxima: 7, 2, 3 max(row minima) = –1 min(column maxima) = 2 max(row minima) = $-1 \neq 2$ = min(col maxima), therefore, a stable solution does not exist
	Identifies the correct max(row minima) and the correct min(col maxima)	1.1b	A1	
	Clearly shows that the correct value max(row minima) and correct value min(col maxima) are not equal and concludes that a stable solution does not exist using correct terminology	3.2a	R1	
	Total		**3**	

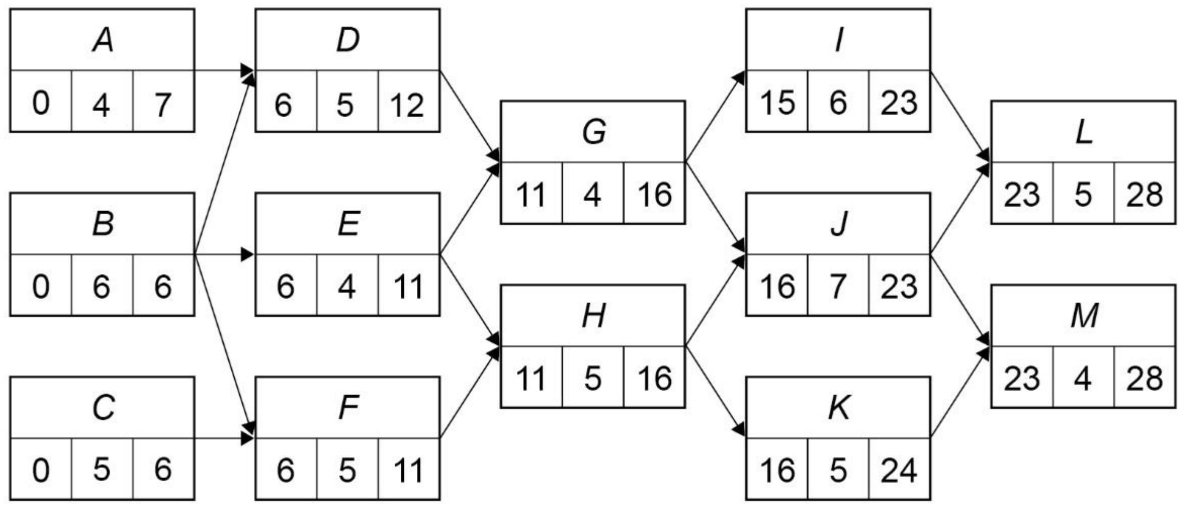
Q	Marking instructions	AO	Marks	Typical solution
4(c)	Explains that the play-safe strategy for Jadzia is **Y**	2.4	E1	Play-safe strategy for Jadzia = **Y** Therefore, Ben should play strategy **A**
	States that Ben should play strategy **A**	1.1b	B1	
	Total		**2**	

	Question total		**6**	

Q	Marking instructions	AO	Marks	Typical solution
5(a)	Sets up a model by identifying the problem as a route inspection problem by noting that *B*, *D*, *H* and *J* are odd-degree nodes (PI)	3.3	M1	Odd nodes: *B*, *D*, *H*, *J* Shortest Distances *B*–*D*: 225 *H*–*J*: 275 *B*–*H*: 425 *D*–*J*: 75 *B*–*J*: 250 *D*–*H*: 200 Pairings (*B*–*D*)(*H*–*J*) = 500 (*B*–*H*)(*D*–*J*) = 500 (*B*–*J*)(*D*–*H*) = 450* Minimum distance the council worker can walk in order to count all street lights is 2250 + 450 = 2700 m Hence x = 2700
	Uses the model to find at least five correct shortest distances between the odd nodes	3.4	M1	
	Uses the model to find the correct shortest distance for the correct pairing	1.1b	A1	
	Determines correctly the value of x using the shortest distance of their pairing	3.2a	A1F	
	Total		**4**	

Q	Marking instructions	AO	Marks	Typical solution
5(b)	Evaluates the council regulation by considering the total distance of all the streets in the village OR Evaluates the council regulation by finding the minimum number of street lights that would be required	3.5a	M1	There are 91 street lights spread over a total distance of 2250 m, so there is, on average, a street light every 24.7 m. Hence, as 24.7 < 25, the village will meet the new council regulation.
	Correctly compares two comparable values and concludes that the village will meet the new council regulation	3.2a	R1	
	Total		**2**	

	Question total		**6**	

Q	Marking instructions	AO	Marks	Typical solution
6(a)(i)	Finds correctly the earliest start time for activities *E*, *G* and *H*	1.1a	M1	
	Finds correctly the earliest start time for activities each activity on the network	1.1b	A1	
	Finds correctly the latest finish time for each activity on the network Condone inclusion of ‘END’ activity	1.1b	B1	
				
	Total		**3**	

Q	Marking instructions	AO	Marks	Typical solution
6(a)(ii)	Identifies correctly all critical activities and no others	1.1b	B1	*B, F, H, J, L*
	Total		**1**	

Q	Marking instructions	AO	Marks	Typical solution
6(b)	Identifies that activity J should have its duration reduced	3.2a	M1	Activity J should be selected, as a 2 week reduction in the duration of J will result in a 2 week reduction in the minimum completion time for the project.
	Explains that reducing activity J's duration by 2 weeks reduces the minimum completion time for the project by 2 weeks	2.4	R1	
	Total		**2**	

	Question total		**6**	

Q	Marking instructions	AO	Marks	Typical solution
7(a)	Deduces correctly that *G* has two (distinct) subgroups PI by reference to subgroups of order 1 and p	2.2a	M1	As p is prime, it has exactly two factors: 1 and p. Hence by Lagrange's theorem, *G* has two subgroups.
	Completes rigorous mathematical argument with reference to 1 and p as the factors of the prime number p or 1 and p as the orders of the two subgroups and by naming or explaining Lagrange's theorem	2.1	R1	
	Total		**2**	

Q	Marking instructions	AO	Marks	Typical solution
7(b)(i)	Uses correct mathematical language of 'generator' to name the element g	2.5	B1	Generator
	Total		**1**	

Q	Marking instructions	AO	Marks	Typical solution
7(b)(ii)	States correctly that *G* is isomorphic to the cyclic group of order p (or a specific example of a cyclic group of order p)	1.1b	B1	Cyclic group of order p
	Total		**1**	

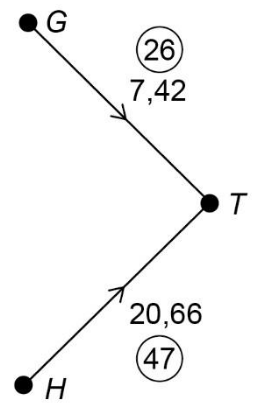
Q	Marking instructions	AO	Marks	Typical solution
7(c)	Uses the definition of an inverse element to set up a relationship of the form $g^r * x = e$ or $x * g^r = e$ PI	1.1a	M1	Let g^k be the inverse of g^r $g^r * g^k = e = g^p$ $g^{r+k} = g^p \quad \Rightarrow \quad k = p - r$ The inverse of g^r is g^{p-r}
	Finds correctly g^{p-r}, the inverse of g^r	1.1b	A1	
	Total		**2**	

Q	Marking instructions	AO	Marks	Typical solution
7(d)(i)	Explains correctly that addition modulo 5 between two elements of *G* will only ever result in 0, 1, 2, 3 or 4 which are all in *G*	2.4	E1	For any a and b in *G*, $a + b$ (mod 5) is either 0, 1, 2, 3 or 4, all of which are in *G*.
	Total		**1**	

Q	Marking instructions	AO	Marks	Typical solution
7(d)(ii)	Completes the Cayley table correctly	1.1b	B1	see table below
	Total		**1**	

$*$	0	1	2	3	4
0	0	1	2	3	4
1	1	2	3	4	0
2	2	3	4	0	1
3	3	4	0	1	2
4	4	0	1	2	3

	Question total		**8**	

Q	Marking instructions	AO	Marks	Typical solution
8(a)	Connects supersink T to nodes G and H with two directed arcs	1.1a	M1	
	Includes a correct upper capacity and a correct lower capacity on both directed arcs GT and GH	1.1b	A1	
	Total		**2**	

Q	Marking instructions	AO	Marks	Typical solution
8(b)	Finds at least one correct augmenting path and the flow (may be seen on diagram) Condone T not included in augmenting path	3.1a	M1	**Augmenting Path** / **Flow**: *SAEGT* – 2 *SBDEGT* – 2 *SCFHT* – 2 Maximum flow = 79 $m^3\ s^{-1}$
	Finds a second correct augmenting path and the flow Condone T not included in augmenting path	1.1b	A1	
	Finds a third correct augmenting path and the flow, and no incorrect paths Condone T not included in augmenting path	1.1b	A1	
	Clearly states the correct maximum flow with correct units	3.2a	B1	
	Total		**4**	

Q	Marking instructions	AO	Marks	Typical solution
8(c)	Translates the problem of proving the flow is maximum into explicitly identifying the correct minimum cut of the network Condone *T* not included in sink set	3.1a	M1	{*S, B, C*} {*A, D, E, F, G, H, T*} = 25 + 19 + 10 + 25 = 79 As the flow (79) is equal to the value of the minimum cut (79), the maximum flow through the network is 79 by the maximum flow-minimum cut theorem.
	Completes a reasoned argument by comparing the value of the correct minimum cut and the flow of 79 $m^3\ s^{-1}$ and states this flow is maximum by naming or explaining the maximum flow-minimum cut theorem	2.1	R1	
	Total		**2**	

Q	Marking instructions	AO	Marks	Typical solution
8(d)	States or explains that *AG* is currently not limiting the maximum flow through the network	1.1b	M1	Increasing the upper capacity of *AG* would not increase the maximum flow through the network as flow into node *A* is already maximum. Hence, the trainee engineer's claim is incorrect.
	Explains why *AG* is not limiting the maximum flow through the network and then concludes that the trainee engineer's claim is incorrect	2.3	R1	
	Total		**2**	

	Question total		**10**	

Q	Marking instructions	AO	Marks	Typical solution
9(a)(i)	Deduces the operation is commutative by considering both $x \oplus y$ and $y \oplus x$ and shows that they are the same Condone $\left(\text{mod } k^2 - 16k + 74\right)$ missing	2.2a	B1	$x \oplus y = x + y + 8 \quad \left(\text{mod } k^2 - 16k + 74\right)$ $y \oplus x = y + x + 8 \quad \left(\text{mod } k^2 - 16k + 74\right)$ $= x + y + 8 \quad \left(\text{mod } k^2 - 16k + 74\right)$
	Total		**1**	

Q	Marking instructions	AO	Marks	Typical solution
9(a)(ii)	Sets up test for associativity by considering combinations of the form $(x \oplus y) \oplus z$ and $x \oplus (y \oplus z)$ and simplifies one combination to obtain $x + y + z + 16$ Condone $\left(\text{mod } k^2 - 16k + 74\right)$ missing	1.1a	M1	$(x \oplus y) \oplus z$ $= (x + y + 8) + z + 8 \quad \left(\text{mod } k^2 - 16k + 74\right)$ $\therefore (x \oplus y) \oplus z$ $= x + y + z + 16 \quad \left(\text{mod } k^2 - 16k + 74\right)$ $x \oplus (y \oplus z)$ $= x + (y + z + 8) + 8 \quad \left(\text{mod } k^2 - 16k + 74\right)$ $\therefore x \oplus (y \oplus z)$ $= x + y + z + 16 \quad \left(\text{mod } k^2 - 16k + 74\right)$ $\therefore x \oplus (y \oplus z) = (x \oplus y) \oplus z$ Hence, the binary operation is associative.
	Completes a reasoned argument and concludes that the binary operation is associative. Condone $\left(\text{mod } k^2 - 16k + 74\right)$ missing	2.1	R1	
	Total		**2**	

Q	Marking instructions	AO	Marks	Typical solution
9(b)	Translates problem by applying binary operation between 3 and another positive integer Condone $\left(\text{mod } k^2 - 16k + 74\right)$ missing	3.1a	M1	$3 \oplus x = 3 + x + 8 \;\left(\text{mod } k^2 - 16k + 74\right)$ $3 \oplus x = x + 11 \;\left(\text{mod } k^2 - 16k + 74\right)$ $3 \oplus x = x \;\left(\text{mod } k^2 - 16k + 74\right)$ $\Rightarrow \; k^2 - 16k + 74 = 11$ $k^2 - 16k + 63 = 0$ $k = 7, 9$
	Uses the condition for 3 to be an identity element to deduce that the addition must be modulo 11	2.2a	A1	
	Solves the quadratic equation to find the two correct values for k and no others	3.2a	A1	
	Total		**3**	

	Question total		**6**	

Q	Marking instructions	AO	Marks	Typical solution
10(a)(i)	Explains correctly why the condition is necessary	2.4	E1	The sum of all probabilities cannot be greater than 1 and this condition ensures this.
	Total		**1**	

Q	Marking instructions	AO	Marks	Typical solution
10(a)(ii)	Explains correctly why the condition is necessary	2.4	E1	Probabilities cannot be negative, and this condition ensures this.
	Total		**1**	

Q	Marking instructions	AO	Marks	Typical solution
10(b)	Uses Kira's model for the linear programming problem to find at least two correct elements	3.4	M1	See below
	Finds at least two correct columns or two correct rows in the pay-off matrix	1.1b	A1	
	Finds all the correct elements in the pay-off matrix	1.1b	A1	
	Total		**3**	

Julian

Kira

Strategy	**J_1**	**J_2**	**J_3**
K_1	7	3	9
K_2	1	7	2
K_3	8	2	4

	Question total		**5**	

	Paper total		**50**	