A-level

FURTHER MATHEMATICS

7367/2

Paper 2

Mark scheme

June 2022

Version: 1.0 Final Mark Scheme

226A7367/2/MS

Mark schemes are prepared by the Lead Assessment Writer and considered, together with the relevant questions, by a panel of subject teachers. This mark scheme includes any amendments made at the standardisation events which all associates participate in and is the scheme which was used by them in this examination. The standardisation process ensures that the mark scheme covers the students' responses to questions and that every associate understands and applies it in the same correct way. As preparation for standardisation each associate analyses a number of students' scripts. Alternative answers not already covered by the mark scheme are discussed and legislated for. If, after the standardisation process, associates encounter unusual answers which have not been raised they are required to refer these to the Lead Examiner.

It must be stressed that a mark scheme is a working document, in many cases further developed and expanded on the basis of students' reactions to a particular paper. Assumptions about future mark schemes on the basis of one year's document should be avoided; whilst the guiding principles of assessment remain constant, details will change, depending on the content of a particular examination paper.

Further copies of this mark scheme are available from aqa.org.uk

Mark scheme instructions to examiners

General

The mark scheme for each question shows:

- the marks available for each part of the question
- the total marks available for the question
- marking instructions that indicate when marks should be awarded or withheld including the principle on which each mark is awarded. Information is included to help the examiner make his or her judgement and to delineate what is creditworthy from that not worthy of credit
- a typical solution. This response is one we expect to see frequently. However credit must be given on the basis of the marking instructions.

If a student uses a method which is not explicitly covered by the marking instructions the same principles of marking should be applied. Credit should be given to any valid methods. Examiners should seek advice from their senior examiner if in any doubt.

Key to mark types

M	mark is for method
R	mark is for reasoning
A	mark is dependent on M marks and is for accuracy
B	mark is independent of M marks and is for method and accuracy
E	mark is for explanation
F	follow through from previous incorrect result

Key to mark scheme abbreviations

CAO	correct answer only
CSO	correct solution only
ft	follow through from previous incorrect result
'their'	indicates that credit can be given from previous incorrect result
AWFW	anything which falls within
AWRT	anything which rounds to
ACF	any correct form
AG	answer given
SC	special case
OE	or equivalent
NMS	no method shown
PI	possibly implied
sf	significant figure(s)
dp	decimal place(s)

Examiners should consistently apply the following general marking principles:

No Method Shown

Where the question specifically requires a particular method to be used, we must usually see evidence of use of this method for any marks to be awarded.

Where the answer can be reasonably obtained without showing working and it is very unlikely that the correct answer can be obtained by using an incorrect method, we must award **full marks**. However, the obvious penalty to candidates showing no working is that incorrect answers, however close, earn **no marks**.

Where a question asks the candidate to state or write down a result, no method need be shown for full marks.

Where the permitted calculator has functions which reasonably allow the solution of the question directly, the correct answer without working earns **full marks**, unless it is given to less than the degree of accuracy accepted in the mark scheme, when it gains **no marks**.

Otherwise we require evidence of a correct method for any marks to be awarded.

Diagrams

Diagrams that have working on them should be treated like normal responses. If a diagram has been written on but the correct response is within the answer space, the work within the answer space should be marked. Working on diagrams that contradicts work within the answer space is not to be considered as choice but as working, and is not, therefore, penalised.

Work erased or crossed out

Erased or crossed out work that is still legible and has not been replaced should be marked. Erased or crossed out work that has been replaced can be ignored.

Choice

When a choice of answers and/or methods is given and the student has not clearly indicated which answer they want to be marked, mark positively, awarding marks for all of the student's best attempts. Withhold marks for final accuracy and conclusions if there are conflicting complete answers or when an incorrect solution (or part thereof) is referred to in the final answer.

AS/A-level Maths/Further Maths assessment objectives

AO		Description
AO1	AO1.1a	Select routine procedures
	AO1.1b	Correctly carry out routine procedures
	AO1.2	Accurately recall facts, terminology and definitions
AO2	AO2.1	Construct rigorous mathematical arguments (including proofs)
	AO2.2a	Make deductions
	AO2.2b	Make inferences
	AO2.3	Assess the validity of mathematical arguments
	AO2.4	Explain their reasoning
	AO2.5	Use mathematical language and notation correctly
AO3	AO3.1a	Translate problems in mathematical contexts into mathematical processes
	AO3.1b	Translate problems in non-mathematical contexts into mathematical processes
	AO3.2a	Interpret solutions to problems in their original context
	AO3.2b	Where appropriate, evaluate the accuracy and limitations of solutions to problems
	AO3.3	Translate situations in context into mathematical models
	AO3.4	Use mathematical models
	AO3.5a	Evaluate the outcomes of modelling in context
	AO3.5b	Recognise the limitations of models
	AO3.5c	Where appropriate, explain how to refine models

Q	Marking instructions	AO	Marks	Typical solution
1	Circles correct answer	1.1b	B1	3
	Total		1	

Q	Marking instructions	AO	Marks	Typical solution
2	Circles correct answer	1.1b	B1	$2a^4$
	Total		1	

Q	Marking instructions	AO	Marks	Typical solution
3	Circles correct answer	1.1b	B1	p^2+12
	Total		1	

Q	Marking instructions	AO	Marks	Typical solution
4	Circles correct answer	2.2a	B1	$y = \text{sech}\, x$
	Total		1	

Q	Marking instructions	AO	Marks	Typical solution
5	Demonstrates the result for $n=1$ **and** states that it is true for $n=1$	2.2a	B1	Let $n=1$ Then $\left(\frac{1}{2}n(n+1)\right)^2 = 1^2 = 1$ and $\sum_{r=1}^{n} r^3 = 1$ So the result is true for $n=1$ Assume the result is true for $n=k$ Then $$\sum_{r=1}^{k} r^3 = \left(\frac{1}{2}k(k+1)\right)^2$$ and $$\sum_{r=1}^{k+1} r^3 = \left(\frac{1}{2}k(k+1)\right)^2 + (k+1)^3$$ $$= \frac{1}{4}(k+1)^2\left(k^2+4(k+1)\right)$$ $$= \frac{1}{4}(k+1)^2(k^2+4k+4)$$ $$= \frac{1}{4}(k+1)^2(k+2)^2$$ $$= \left(\frac{1}{2}(k+1)(k+2)\right)^2$$ The result is true for n = 1; if true for n = k, then it's also true for n = k + 1 and hence by induction $\sum_{r=1}^{n} r^3 = \left\{\frac{1}{2}n(n+1)\right\}^2$ for all integers $n \geq 1$
	Assumes the result true for $n=k$ (PI by algebraic working) **and** adds $(k+1)^3$ to $=\left(\frac{1}{2}k(k+1)\right)^2$	2.1	M1	
	Obtains $=\frac{1}{4}(k+1)^2(k+2)^2$ from $$=\left(\frac{1}{2}k(k+1)\right)^2 + (k+1)^3$$	1.1b	A1	
	Concludes a reasoned argument by stating that $$\sum_{r=1}^{n} r^3 = \left\{\frac{1}{2}n(n+1)\right\}^2$$ is true for n = 1, and that $$\sum_{r=1}^{k} r^3 = \left(\frac{1}{2}k(k+1)\right)^2$$ implies $$\sum_{r=1}^{k+1} r^3 = \left(\frac{1}{2}(k+1)(k+2)\right)^2$$ And hence, by induction, that $$\sum_{r=1}^{n} r^3 = \left\{\frac{1}{2}n(n+1)\right\}^2$$ is true for all integers $n \geq 1$	2.1	R1	
	Total		**4**	

Q	Marking instructions	AO	Marks	Typical solution
6	States that the area of the trapezium is greater than the integral Condone "the area of the trapezium is greater than the curve".	2.2a	B1	The area of the trapezium is greater than the integral. Area of trapezium $=(b-a)y_{\frac{1}{2}}$ The area of the trapezium is the same as the area of the rectangle from use of the mid-ordinate rule. This is equal to Sharon's estimate. The trapezium includes the area represented by the integral, so Sharon's estimate is an over-estimate.
	Explains that the area of the trapezium equals Sharon's estimate/ result of using mid-ordinate rule	2.4	E1	
	States that Sharon's estimate is an over-estimate and completes a reasoned argument to explain the required result	2.1	R1	
	Total		**3**	

Q	Marking instructions	AO	Marks	Typical solution
7(a)	Deduces correct value of b	2.2a	B1	$2 \times \frac{9}{2} + b = 0$ $b = -9$ $\frac{a}{2} = 3$ $a = 6$
	Deduces correct value of a	2.2a	B1	
	Total		**2**	

Q	Marking instructions	AO	Marks	Typical solution
7(b)	Selects a suitable method to solve the inequality, for example Multiplies by square of denominator or sketches graphs of their $y = \mathrm{f}(x)$ and $y = x + 2$	3.1a	M1	$x + 2 \geq \frac{6x - 5}{2x - 9}$ $(x + 2)(2x - 9)^2 \geq (6x - 5)(2x - 9)$ $(2x - 9)\{(x + 2)(2x - 9) - (6x - 5)\} \geq 0$ $(2x - 9)\{2x^2 - 5x - 18 - 6x + 5\} \geq 0$ $(2x - 9)\{2x^2 - 11x - 13\} \geq 0$ $(2x - 9)(x + 1)(2x - 13) \geq 0$ Change of sign occurs at $x = -1, x = \frac{9}{2}, x = \frac{13}{2}$ Exclude $x = \frac{9}{2}$ (f not defined) $-1 \leq x < \frac{9}{2}$ or $x \geq \frac{13}{2}$
	Simplifies their inequality or equation or Indicates points of intersection of the two graphs	1.1a	M1	
	Obtains at least two critical values of their inequality or equation	1.1b	A1F	
	Excludes $x = \frac{9}{2}$ (PI by final answer)	2.2a	A1	
	Deduces correct solution set for their inequality or graph. Condone inclusion of $x = \frac{9}{2}$ Follow through their answers to part a)	2.2a	A1F	
	Obtains completely correct solution OE with each step clearly shown	2.1	R1	
	Total		**6**	

	Question total		**8**	

Q	Marking instructions	AO	Marks	Typical solution
8(a)(i)	Differentiates twice correctly	1.1b	B1	$f'(x) = \sec x \tan x$ $f''(x) = \sec x(\sec^2 x) + \tan x(\sec x \tan x)$ $= \sec x(\sec^2 x + \tan^2 x)$ $f^{(3)}(x) = \sec x \tan x(\sec^2 x + \tan^2 x) + \sec x(2\sec x(\sec x \tan x) + 2\tan x \sec^2 x)$ $= 5\sec^3 x \tan x + \sec x \tan^3 x$ $f^{(4)}(x) = \tan x\left(15\sec^2 x(\sec x \tan x)\right) + 5\sec^5 x + \sec x(3\tan^2 x \sec^2 x) + \tan^3 x(\sec x \tan x)$ $= 18\sec^3 x \tan^2 x + 5\sec^5 x + \tan^4 x \sec x$ $f^{(4)}(0) = 0 + 5 + 0 = 5$
	Differentiates three times	1.1a	M1	
	Obtains a correct expression for the fourth derivative of $f(x)$	1.1b	A1	
	Obtains the required result from correct working	1.1b	A1	
	Total		**4**	

Q	Marking instructions	AO	Marks	Typical solution
8(a)(ii)	Substitutes zero and uses in Maclaurin expansion. Condone no division by factorial	1.1a	M1	$f(0) = 1$ $f'(0) = 0$ $f''(0) = 1$ $f^{(3)}(0) = 0$ $f(x) = 1 + \frac{x^2}{2} + \frac{5x^4}{24} + \cdots$
	Obtains correct result (allow factorial notation)	1.1b	A1	
	Total		**2**	

Q	Marking instructions	AO	Marks	Typical solution
8(b)	Deduces correct expansion of cosh x or justifies the use of l'Hôpital's rule (must see 0/0 at least once)	2.2a	B1	$\cosh x = \frac{1}{2}\left(1 + x + \frac{x^2}{2!} + \cdots\right) + \frac{1}{2}\left(1 - x + \frac{x^2}{2!} + \cdots\right)$ $= 1 + \frac{x^2}{2!} + \frac{x^4}{4!} + \cdots$
	Substitutes expansions into expression PI or applies l'Hôpital's rule four times	1.1a	M1	$\lim_{x\to 0}\left(\frac{\sec x - \cosh x}{x^4}\right)$ $= \lim_{x\to 0}\frac{\left(1 + \frac{x^2}{2!} + \frac{5x^4}{4!} + \cdots\right) - \left(1 + \frac{x^2}{2!} + \frac{x^4}{4!} + \cdots\right)}{x^4}$
	Simplifies correctly	1.1b	A1	$= \lim_{x\to 0}\frac{\frac{x^4}{6} + \text{higher powers}}{x^4}$ $= \lim_{x\to 0}\left(\frac{1}{6} + \text{higher powers}\right) = \frac{1}{6}$
	Completes a rigorous argument to prove the required result, including a clear demonstration of the limiting process. Must show evidence of higher powers until the limit is taken or must justify l'Hôpital's rule at each stage	2.1	R1	$\therefore \lim_{x\to 0}\left(\frac{\sec x - \cosh x}{x^4}\right) = \frac{1}{6}$
	Total		**4**	

	Question total		**10**	

Q	Marking instructions	AO	Marks	Typical solution
9(a)	Uses Euler method once (condone one slip)	1.1a	M1	$x_0 = 5$ $y_0 = 12.3$ $h = 0.1$
	Obtains correct value of y_1 Allow AWRT 13.5	1.1b	A1	$y_1 = 12.3 + 0.1\left(4 + \frac{2 \times 5 \times 12.3}{16}\right)$
	Uses midpoint formula once (condone one slip)	3.1a	M1	$= \frac{431}{32}$ $x_1 = 5.1$
	Obtains the correct value to 6 sig. fig.	1.1b	A1	$y_2 = 12.3 + 0.2\left(\frac{9\sqrt{21}}{10} + \frac{2 \times 5.1 \times \frac{431}{32}}{17.01}\right)$ $= 14.7402$
	Total		**4**	

Q	Marking instructions	AO	Marks	Typical solution
9(b)(i)	Applies the Integrating Factor Method	3.1a	M1	$\frac{\mathrm{d}y}{\mathrm{d}x} - \left(\frac{2x}{x^2-9}\right)y = (x^2-9)^{\frac{1}{2}}$
	Finds correct integrating factor	1.1b	A1	$\int P\mathrm{d}x = -\int \frac{2x}{x^2-9}\mathrm{d}x$ $= -\ln(x^2-9)$
	Multiplies equation by their integrating factor	1.1a	M1	Integrating factor = $\mathrm{e}^{\int P\mathrm{d}x} = \frac{1}{x^2-9}$
	Integrates LHS to obtain $\frac{y}{x^2-9}$	1.1b	A1	$\left(\frac{1}{x^2-9}\right)\frac{\mathrm{d}y}{\mathrm{d}x} - \left(\frac{2x}{(x^2-9)^2}\right)y = (x^2-9)^{-\frac{1}{2}}$ $\frac{\mathrm{d}}{\mathrm{d}x}\left(\frac{y}{x^2-9}\right) = (x^2-9)^{-\frac{1}{2}}$
	Uses inverse cosh or logarithmic equivalent to integrate RHS	1.1a	M1	$\frac{y}{x^2-9} = \int (x^2-9)^{-\frac{1}{2}}\,\mathrm{d}x$ $y = (x^2-9)\left(\cosh^{-1}\frac{x}{3} + c\right)$
	Finds correct solution including constant of integration. ACF Accept $\frac{y}{x^2-9} = \cosh^{-1}\frac{x}{3} + c$	1.1b	A1	
	Total		**6**	

Q	Marking instructions	AO	Marks	Typical solution
9(b)(ii)	Uses initial conditions to find constant of integration	3.1a	M1	$\frac{12.3}{16} = \cosh^{-1}\frac{5}{3} + c$ $c = \frac{12.3}{16} - \cosh^{-1}\frac{5}{3} = -0.32986\ldots$
	Substitutes $x = 5.2$ and their constant of integration into their solution of DE	1.1a	M1	When $x = 5.2$
	Obtains correct answer	1.1b	A1	$y = (5.2^2 - 9)\left(\cosh^{-1}\frac{5.2}{3} - 0.32986\ldots\right)$ $y = 14.7434$ (6 sig. fig.)
	Total		**3**	

Q	Marking instructions	AO	Marks	Typical solution
9(c)	Compares the two correct values to make a correct evaluation	3.2b	E1	The value from part (a) is equal to the answer to part (b)(ii) to four significant figures, meaning that the estimate in part (a) is very accurate.
	Total		**1**	

	Question total		**14**	

Q	Marking instructions	AO	Marks	Typical solution
10(a)	Deduces that a stretch, only along the y-axis, is required	2.2a	M1	$$\frac{x^2}{25} - \frac{y^2}{4} = 1$$ Stretch s.f. ½ along y-axis gives $$\frac{x^2}{25} - \frac{(2y)^2}{4} = 1$$ $$\frac{x^2}{25} - y^2 = 1$$
	Deduces that a translation with two non-zero components is needed (or a translation parallel to the x-axis **and** a translation parallel to the y-axis)	2.2a	M1	
	Obtains correct stretch and/or correct translation (accept $\begin{bmatrix} 3 \\ -8 \end{bmatrix}$)	1.1b	A1	Translation by vector $\begin{bmatrix} 3 \\ -4 \end{bmatrix}$ gives $$\frac{(x-3)^2}{25} - (y+4)^2 = 1$$
	States correct order of completely correct transformations (accept any correct sequence)	2.4	E1	which is equivalent to the equation for C_2 Stretch, scale factor ½ , along y-axis followed by translation by vector $\begin{bmatrix} 3 \\ -4 \end{bmatrix}$
	Total		**4**	

Q	Marking instructions	AO	Marks	Typical solution
10(b)	Obtains the correct asymptotes of C_1 (PI) or uses a correct method to obtain both asymptotes of C_2 directly PI by an equation of the form $5y \pm x + k = 0$	1.1a	M1	Asymptotes of C_1: $y = \pm\frac{2x}{5}$ $$y = \frac{2x}{5}$$ Stretch $\Rightarrow 2y = \frac{2x}{5} \Rightarrow 5y = x$ Translation $\Rightarrow 5(y+4) = x - 3$ $$5y - x + 23 = 0$$
	Correctly applies their sequence of at least two different types of transformations to one asymptote or obtains an equation of the form $5y \pm x + k = 0$ where k is a non-zero integer constant	3.1a	M1	$$y = -\frac{2x}{5}$$ Stretch $\Rightarrow 2y = -\frac{2x}{5} \Rightarrow 5y = -x$ Translation $\Rightarrow 5(y+4) = -x - 3$ $$5y + x + 17 = 0$$
	Obtains both correct results	1.1b	A1	
	Total		**3**	

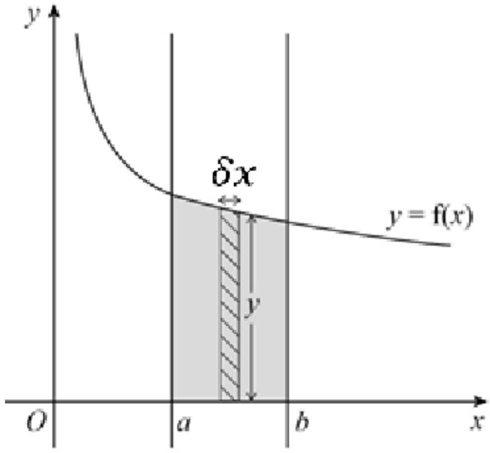
	Question total		**7**	

Q	Marking instructions	AO	Marks	Typical solution
11(a)	Forms correct characteristic equation and solves. PI by correct eigenvalues. Condone one error.	1.1a	M1	$0=\left(\frac{5}{2}-\lambda\right)\left(\frac{13}{2}-\lambda\right)-\frac{9}{4}$ $0=\lambda^2-9\lambda+14$
	Obtains the correct eigenvalues	1.1b	A1	$\lambda=2$ & $\lambda=7$
	Uses correct equation to find eigenvector for one of their two eigenvalues. PI by a correct eigenvector	1.1a	M1	$\lambda=2:\mathbf{0}=\begin{bmatrix}\frac{1}{2} & \frac{-3}{2}\\ \frac{-3}{2} & \frac{9}{2}\end{bmatrix}\begin{bmatrix}x\\ y\end{bmatrix}$ $\lambda=2:\begin{bmatrix}3\\ 1\end{bmatrix}$
	Obtains a correct eigenvector for one of their two eigenvalues. Allow any scalar multiple	1.1b	A1F	$\lambda=7:\mathbf{0}=\begin{bmatrix}\frac{-9}{2} & \frac{-3}{2}\\ \frac{-3}{2} & \frac{-1}{2}\end{bmatrix}\begin{bmatrix}x\\ y\end{bmatrix}$ $\lambda=7:\begin{bmatrix}-1\\ 3\end{bmatrix}$
	Obtains both correct eigenvectors paired with the corresponding eigenvalue. Allow any scalar multiple	1.1b	A1	
	Total		**5**	

Q	Marking instructions	AO	Marks	Typical solution
11(b)(i)	Compares directions of their eigenvectors eg considers gradients or obtains scalar product	2.1	M1	$\begin{bmatrix}3\\ 1\end{bmatrix}\cdot\begin{bmatrix}-1\\ 3\end{bmatrix}=0$ So the invariant lines are perpendicular.
	Deduces that the invariant lines are perpendicular	2.2a	R1	
	Total		**2**	

Q	Marking instructions	AO	Marks	Typical solution
11(b)(ii)	Deduces that it is a two-way stretch	2.2a	M1	Stretch parallel to $y=\frac{1}{3}x$, SF = 2 Stretch parallel to $y=-3x$, SF = 7
	Describes the transformation fully. Follow through their eigenvalues and corresponding eigenvectors	1.1b	A1F	
	Total		**2**	

	Question total		**9**	

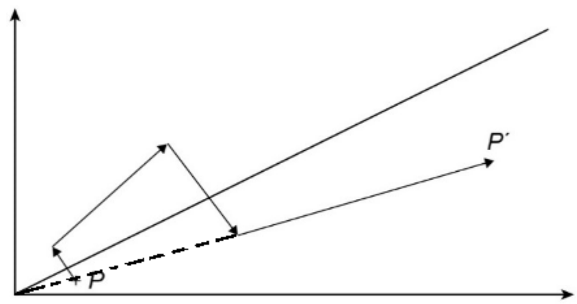
Q	Marking instructions	AO	Marks	Typical solution
12(a)	Draws or describes thin strip(s) under graph – may fit curve exactly or may be rectangular	3.1a	B1	
	Obtains expression for (approximate) volume of a thin disc. Condone expression for volume of a cylinder of radius y or f(x) and of any height	2.4	M1	When a strip is rotated it forms a disc which is approximately cylindrical.
	Obtains expression for (approximate) total volume of discs	2.4	A1	If thickness of disc is δx then volume of the disc is (approximately) $\pi y^2 \delta x$ Total volume of discs is $\sum_{x=a}^{b} \pi y^2 \delta x$
	Completes a correct argument to show the required result, including taking the limit as $\delta x \to 0$	2.1	R1	Volume of solid $= \lim_{\delta x \to 0} \left(\sum_{x=a}^{b} \pi y^2 \delta x \right)$ $= \pi \int_a^b y^2 \, dx$ $= \pi \int_a^b (f(x))^2 \, dx$
	Total		4	

Q	Marking instructions	AO	Marks	Typical solution
12(b)	Obtains $\dfrac{(x+3)^2}{x(x+1)^2}$	1.1b	B1	$(\mathrm{f}(x))^2 = \dfrac{(x+3)^2}{x(x+1)^2}$
	Expresses $(\mathrm{f}(x))^2$ as partial fractions in correct format	3.1a	M1	$\dfrac{(x+3)^2}{x(x+1)^2} \equiv \dfrac{A}{x} + \dfrac{B}{x+1} + \dfrac{C}{(x+1)^2}$ $x^2+6x+9 = A(x+1)^2 + Bx(x+1) + Cx$
	Obtains correct expression in partial fractions	1.1b	A1	$x = 0 \Rightarrow A = 9$ $x = -1 \Rightarrow C = -4$ Compare x^2-coeff: $B = -8$
	Integrates expression to obtain two logarithmic terms and one algebraic fraction. Condone missing π	1.1a	M1	$\dfrac{(x+3)^2}{x(x+1)^2} \equiv \dfrac{9}{x} - \dfrac{8}{x+1} - \dfrac{4}{(x+1)^2}$ $V = \pi \displaystyle\int_1^2 \dfrac{(x+3)^2}{x(x+1)^2}\,\mathrm{d}x$
	Obtains correct result of integration. Follow through their numerators Condone missing π	1.1b	A1F	$V = \pi \displaystyle\int_1^2 \left(\dfrac{9}{x} - \dfrac{8}{x+1} - \dfrac{4}{(x+1)^2}\right)\mathrm{d}x$ $= \pi\left[9\ln x - 8\ln(x+1) + \dfrac{4}{x+1}\right]_1^2$
	Substitutes limits into their three-term integrated expression for the volume. Must include π	1.1a	M1	$= \pi\left\{\left(9\ln 2 - 8\ln 3 + \dfrac{4}{3}\right) - \left(9\ln 1 - 8\ln 2 + \dfrac{4}{2}\right)\right\}$
	Completes a reasoned argument to show the required result, including correct re-arrangement of log terms	2.1	R1	$= \pi\left(17\ln 2 - 8\ln 3 - \dfrac{2}{3}\right)$ $= \pi\left(\ln\left(\dfrac{2^{17}}{3^8}\right) - \dfrac{2}{3}\right)$
	Total		**7**	

	Question total		**11**	

Q	Marking instructions	AO	Marks	Typical solution
13(a)	States that $m = \tan\theta$ and $\mathbf{A} = \begin{bmatrix} \cos 2\theta & \sin 2\theta \\ \sin 2\theta & -\cos 2\theta \end{bmatrix}$ Condone omission of **A**=	1.2	B1	$\mathbf{A} = \begin{bmatrix} \cos 2\theta & \sin 2\theta \\ \sin 2\theta & -\cos 2\theta \end{bmatrix}$ where $\tan\theta = m$ $\mathbf{A} = \begin{bmatrix} \cos^2\theta - \sin^2\theta & 2\sin\theta\cos\theta \\ 2\sin\theta\cos\theta & \sin^2\theta - \cos^2\theta \end{bmatrix}$ $= \cos^2\theta \begin{bmatrix} 1-\tan^2\theta & 2\tan\theta \\ 2\tan\theta & \tan^2\theta - 1 \end{bmatrix}$ $= \dfrac{1}{\sec^2\theta} \begin{bmatrix} 1-m^2 & 2m \\ 2m & m^2-1 \end{bmatrix}$ $= \dfrac{1}{\tan^2\theta + 1} \begin{bmatrix} 1-m^2 & 2m \\ 2m & m^2-1 \end{bmatrix}$ $= \left(\dfrac{1}{m^2+1}\right) \begin{bmatrix} 1-m^2 & 2m \\ 2m & m^2-1 \end{bmatrix}$
	Replaces $\sin 2\theta$ and $\cos 2\theta$ in the matrix with expressions in terms of trigonometric ratios of θ or in terms of m	3.1a	M1	
	Deduces that one element of the matrix can be expressed as $\cos^2\theta\left(1-\tan^2\theta\right)$ or $\cos^2\theta\left(1-m^2\right)$ or $2\tan\theta\left(\cos^2\theta\right)$ or $2m\cos^2\theta$ or uses $\cos 2\theta = \dfrac{1-m^2}{1+m^2}$ or $\sin 2\theta = \dfrac{2m}{1+m^2}$	2.2a	M1	
	Uses $\cos^2\theta = \dfrac{1}{m^2+1}$	1.1a	M1	
	Completes a reasoned argument to obtain the required result Must include **A**=…	2.1	R1	
	Total		**5**	

Q	Marking instructions	AO	Marks	Typical solution
13(b)	Obtains BA (condone AB) or Uses the fact that **B** =3$\mathbf{I}$	1.1a	M1	**BA**= $\left(\frac{1}{m^2+1}\right)\begin{bmatrix}3 & 0\\0 & 3\end{bmatrix}\begin{bmatrix}1-m^2 & 2m\\2m & m^2-1\end{bmatrix}$ $=\left(\frac{1}{m^2+1}\right)\begin{bmatrix}3(1-m^2) & 6m\\6m & 3(m^2-1)\end{bmatrix}$
	Squares their BA or AB or 3$\mathbf{IA}$ and simplifies	1.1a	M1	(**BA**)^2 = $\left(\frac{1}{m^2+1}\right)^2\begin{bmatrix}3(1-m^2) & 6m\\6m & 3(m^2-1)\end{bmatrix}\begin{bmatrix}3(1-m^2) & 6m\\6m & 3(m^2-1)\end{bmatrix}$
	Completes a reasoned argument to show the required result, using BA not AB or Uses and states the fact that **A**^{2} = $\mathbf{I}$ because it is a reflection Condone missing (**BA**)^2=	2.1	R1	$=\left(\frac{1}{m^2+1}\right)^2\begin{bmatrix}9(1-m^2)^2+36m^2 & 0\\0 & 36m^2+9(m^2-1)^2\end{bmatrix}$ $=\left(\frac{1}{m^2+1}\right)^2\begin{bmatrix}9+18m^2+9m^4 & 0\\0 & 9+18m^2+9m^4\end{bmatrix}$ $=\begin{bmatrix}9 & 0\\0 & 9\end{bmatrix}$ $=9\mathbf{I}$
	Total		**3**	

Q	Marking instructions	AO	Marks	Typical solution
13(c)(i)	Uses the fact that B represents an enlargement, scale factor 3 or that A represents a reflection in $y = mx$	1.2	M1	
	Draws four correct lines on the diagram (the arrows and the dashed line are not needed), and labels P' Condone A and B reversed	1.1b	A1	
	Total		**2**	

Q	Marking instructions	AO	Marks	Typical solution
13(c)(ii)	Explains how at least one of their lines represents a relevant transformation.	2.4	E1	The lines on the diagram show the effect of A, then B, then A again, then B again, on the point *P*. The end point is the result of transforming *P* by an enlargement of scale factor 9. Centre of enlargement *O*
	Explains how their lines represent the result of the transformations.	2.4	E1	
	Total		**2**	

Q	Marking instructions	AO	Marks	Typical solution
13(d)	Expresses **BA** correctly in terms of m or calculates $\mathbf{B}^{-1}\mathbf{C}$ correctly	1.1b	B1	**C**= **BA** $\begin{bmatrix} \frac{12}{5} & \frac{9}{5} \\ \frac{9}{5} & -\frac{12}{5} \end{bmatrix} = \left(\frac{1}{m^2+1}\right)\begin{bmatrix} 3(1-m^2) & 6m \\ 6m & 3(m^2-1) \end{bmatrix}$
	Sets up an equation in m	3.1a	M1	$\frac{9}{5} = \frac{6m}{m^2+1}$ $3m^2 - 10m + 3 = 0$
	Finds a correct solution of their correct equation in m (condone other solution(s) not rejected)	1.1b	A1	$m = 3$ or $m = \frac{1}{3}$ $m = 3$ gives $\frac{3(1-m^2)}{m^2+1} = -\frac{12}{5} \neq \frac{12}{5}$ so discard $m = 3$
	Uses a rigorous argument to obtain the required result, including clear reason for choosing $m = \frac{1}{3}$	2.1	R1	Correct value is $m = \frac{1}{3}$
	Total		**4**	

	Question total		**16**	

Q	Marking instructions	AO	Marks	Typical solution
14(a)	Obtains two correct values	3.1b	B1	$a=40$ $b=13$ $c=1.95$ $d=1$
	Obtains four correct values Condone "40%" and "1%" Do not accept -1.95	3.1b	B1	
	Total		**2**	

Q	Marking instructions	AO	Marks	Typical solution
14(b)	Differentiates one equation	3.1a	M1	$13y=0.4x-\dot{x}\quad(1)$ $y=\frac{2}{65}x-\frac{1}{13}\dot{x}$ $\dot{y}=\frac{2}{65}\dot{x}-\frac{1}{13}\ddot{x}$
	Substitutes for $\dot{y}$, or for x and $\dot{x}$, in the other equation to eliminate one variable	3.1a	M1	
	Forms a correct simplified second order differential equation	1.1b	A1	Sub into (2) $\frac{2}{65}\dot{x}-\frac{1}{13}\ddot{x}=0.01x-1.95$ $5\ddot{x}-2\dot{x}+0.65x=126.75$ CF: $5m^2-2m+0.65=0$ $m=0.2\pm0.3\text{i}$ PI: $x=195$ $\therefore x=Ae^{0.2t}\cos(0.3t)+Be^{0.2t}\sin(0.3t)+195$
	Obtains roots of their auxiliary equation	1.1a	M1	
	Uses a valid method to find a particular integral for their DE	2.2a	M1	$y=\frac{2}{65}x-\frac{1}{13}\dot{x}$ $\therefore y=\frac{1}{65}Ae^{0.2t}\cos(0.3t)+\frac{3}{130}Ae^{0.2t}\sin(0.3t)$ $+\frac{1}{65}Be^{0.2t}\sin(0.3t)-\frac{3}{130}Be^{0.2t}\cos(0.3t)+6$
	States general solution for either x or y with their non-zero particular integral	1.1b	A1F	
	States general solutions for both x and y CAO	1.1b	A1	When $t=0, x=1755$ and $y=30$ so $1755=A+195$ and $30=\frac{A}{65}-\frac{3B}{130}+6$ $\Rightarrow A=1560, B=0$
	Uses initial conditions to find a value for each constant	3.4	M1	$x=1560e^{0.2t}\cos(0.3t)+195$ $y=24e^{0.2t}\cos(0.3t)+36e^{0.2t}\sin(0.3t)+6$
	Writes correct solutions for both x and y	1.1b	A1	
	Total		**9**	

Q	Marking instructions	AO	Marks	Typical solution
14(c)	Investigates values of x for $t > 5$ PI by t = 5.38, x = 0	3.2a	M1	Using calculator: When t = 5.38, x = 0 At this time $y \approx 108$ so there are still birds of prey The rabbits die out first. So the conservationists' plan succeeds.
	Obtains a time when x = 0 or x < 0, and states that the rabbits die out	3.2a	A1	
	Obtains a positive value of y for a value of t for which $x \leq 0$ and uses their correct answers to show that the rabbits die out first. Condone no investigation of values of y between $t = 5$ and their 5.38	3.5a	E1	
	Total		**3**	

	Question total		**14**	

	Paper total		**100**	