Please check the examination details below before entering your candidate information

Candidate surname	Other names

Pearson Edexcel Level 3 GCE

Centre Number | Candidate Number

Time 2 hours

Paper reference **8MA0/01**

Mathematics

Advanced Subsidiary
PAPER 1: Pure Mathematics

You must have:
Mathematical Formulae and Statistical Tables (Green), calculator

Total Marks

Candidates may use any calculator allowed by the regulations of the Joint Council for Qualifications. Calculators must not have the facility for symbolic algebra manipulation, differentiation and integration, or have retrievable mathematical formulae stored in them.

Instructions

- Use **black** ink or ball-point pen.
- If pencil is used for diagrams/sketches/graphs it must be dark (HB or B).
- **Fill in the boxes** at the top of this page with your name, centre number and candidate number.
- Answer **all** questions and ensure that your answers to parts of questions are clearly labelled.
- Answer the questions in the spaces provided – *there may be more space than you need.*
- You should show sufficient working to make your methods clear. Answers without working may not gain full credit.
- Inexact answers should be given to three significant figures unless otherwise stated.

Information

- A booklet 'Mathematical Formulae and Statistical Tables' is provided.
- There are 16 questions in this question paper. The total mark for this paper is 100.
- The marks for **each** question are shown in brackets – *use this as a guide as to how much time to spend on each question.*

Advice

- Read each question carefully before you start to answer it.
- Try to answer every question.
- Check your answers if you have time at the end.
- Good luck with your examination.

Turn over

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Pearson

1. **In this question you should show all stages of your working.**

Solutions relying on calculator technology are not acceptable.

Using algebra, solve the inequality

$$x^2 - x > 20$$

writing your answer in set notation.

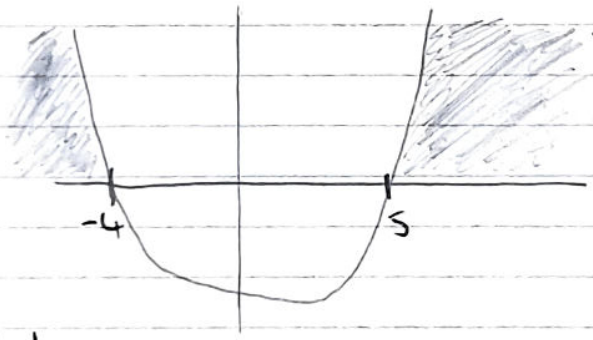
(3)

$x^2 - x > 20$

$\Rightarrow x^2 - x - 20 > 0$

$(x-5)(x+4) > 0$

Critical values $x = 5$ and $x = -4$.



Inequality satisfied when $x < -4$ and $x > 5$

In set notation:

$\{x : x < -4\} \cup \{x : x > 5\}$

Question 1 continued

(Total for Question 1 is 3 marks)

2. **In this question you should show all stages of your working.**

Solutions relying on calculator technology are not acceptable.

Given

$$\frac{9^{x-1}}{3^{y+2}} = 81$$

express y in terms of x, writing your answer in simplest form.

(3)

$$\frac{9^{x-1}}{3^{y+2}} = 81 \Rightarrow \frac{3^{2x-2}}{3^{y+2}} = 3^4$$

$$\Rightarrow 3^{2x-2-(y+2)} = 3^4$$

$$\Rightarrow 2x-2-y-2=4$$

$$\Rightarrow y=2x-8.$$

Question 2 continued

(Total for Question 2 is 3 marks)

3. Find

$$\int \frac{3x^4 - 4}{2x^3}\,dx$$

writing your answer in simplest form.

(4)

$$\int \frac{3x^4-4}{2x^3}\,dx = \int \frac{3x^4}{2x^3} - \frac{4}{2x^3}\,dx$$

$$= \int \frac{3}{2}x - 2x^{-3}\,dx$$

$$= \frac{3}{2} \times \frac{x^2}{2} - 2 \times \frac{x^{-2}}{-2} + c$$

$$\int \frac{3x^4-4}{2x^3}\,dx = \frac{3}{4}x^2 + \frac{1}{x^2} + C$$

Question 3 continued

(Total for Question 3 is 4 marks)

4. [*In this question the unit vectors* **i** *and* **j** *are due east and due north respectively.*]

A stone slides horizontally across ice.

Initially the stone is at the point $A(-24\mathbf{i} - 10\mathbf{j})$ m relative to a fixed point O.

After 4 seconds the stone is at the point $B(12\mathbf{i} + 5\mathbf{j})$ m relative to the fixed point O.

The motion of the stone is modelled as that of a particle moving in a straight line at constant speed.

Using the model,

(a) prove that the stone passes through O,

(2)

(b) calculate the speed of the stone.

(3)

a) The position vectors are scalar multiples of eachother

$$(-24\underline{i} - 10\underline{j}) = -2(12\underline{i} + 5\underline{j})$$

Hence the vectors $\overrightarrow{AO}$ and $\overrightarrow{OB}$ are parallel, and as the stone is travelling in a straight line $\overrightarrow{AB}$, the stone passes through O as $\overrightarrow{AB}$ does.

b) The distance $AB = \sqrt{(12+24)^2 + (10+5)^2}$

$= 39$m.

The speed of the stone $= \frac{39}{4} = 9.75$ m/s

Question 4 continued

(Total for Question 4 is 5 marks)

5.

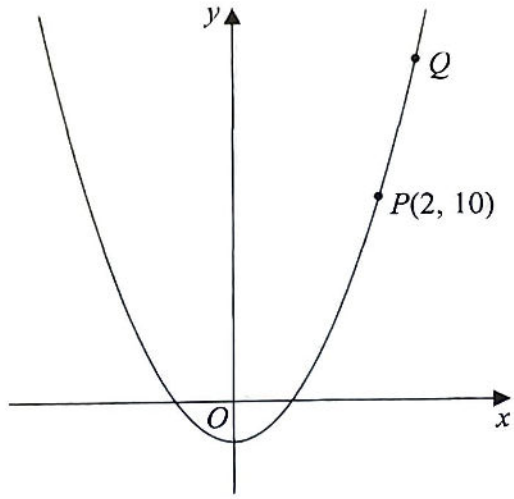


Figure 1

Figure 1 shows part of the curve with equation $y = 3x^2 - 2$

The point $P(2, 10)$ lies on the curve.

(a) Find the gradient of the tangent to the curve at P. **(2)**

The point Q with x coordinate $2 + h$ also lies on the curve.

(b) Find the gradient of the line PQ, giving your answer in terms of h in simplest form. **(3)**

(c) Explain briefly the relationship between part (b) and the answer to part (a). **(1)**

a) $y = 3x^2 - 2$

$\Rightarrow \frac{dy}{dx} = 6x$ At P: $\frac{dy}{dx} = 6 \times 2 = 12$.

Gradient at P = 12.

b) Coordinates at Q: $(2+h, 3(2+h)^2 - 2)$.

$\frac{\Delta y}{\Delta x}$ = Gradient PQ = $\frac{3(2+h)^2 - 2 - 10}{(2+h) - 2} = \frac{12h + 3h^2}{h}$

$= 12 + 3h$.

Question 5 continued

c) As $h \to 0$, the gradient PQ, $12+3h \to 12$.

So as Q gets closer to P, the gradient of the chord tends toward the instantaneous gradient of the curve at P.

(Total for Question 5 is 6 marks)

6. **In this question you should show all stages of your working.**

Solutions relying on calculator technology are not acceptable.

(a) Using algebra, find all solutions of the equation

$$3x^3 - 17x^2 - 6x = 0$$

(3)

(b) Hence find all real solutions of

$$3(y-2)^6 - 17(y-2)^4 - 6(y-2)^2 = 0$$

(3)

a) $3x^3 - 17x^2 - 6x = 0$

$\Rightarrow x(3x^2 - 17x - 6) = 0$

$\Rightarrow x(3x+1)(x-6) = 0$

So the solutions are $x = 0, -1/3, 6$.

b) Let $n = (y-2)^2$ then

$3(y-2)^6 - 17(y-2)^4 - 6(y-2)^2 = 0$ has the solutions

$n = 0, -1/3, 6$ from part (a)

except $n \neq -1/3$ as $n \geq 0$ (as it is squared).

$\Rightarrow (y-2)^2 = 0$ and $(y-2)^2 = 6$

Which gives the solutions

$y = 2$ and $y = 2 \pm \sqrt{6}$.

Question 6 continued

(Total for Question 6 is 6 marks)

7. A parallelogram $PQRS$ has area $50\,\text{cm}^2$

Given

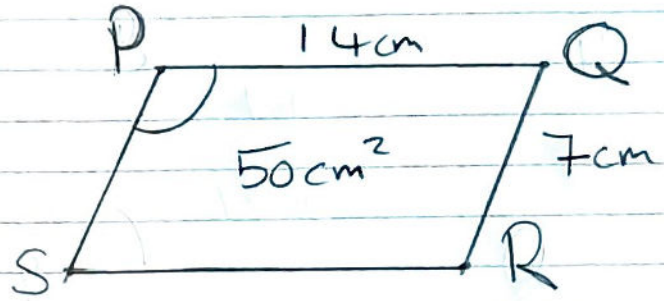
- PQ has length 14 cm
- QR has length 7 cm
- angle SPQ is obtuse

find

(a) the size of angle SPQ, in degrees, to 2 decimal places, **(3)**

(b) the length of the diagonal SQ, in cm, to one decimal place. **(2)**

7a)

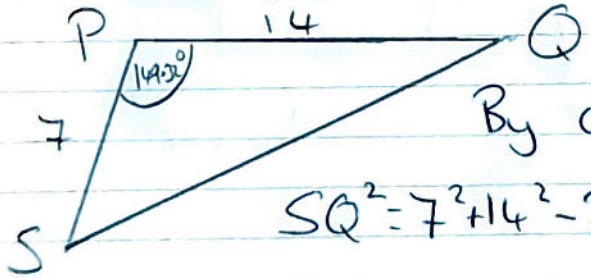


$50 = 7 \times 14 \sin(SPQ)$

$\Rightarrow SPQ = 180 - \arcsin(50/98)$

$SPQ = 149.32°$

b)



By cosine rule:

$SQ^2 = 7^2 + 14^2 - 2 \times 14 \times 7 \cos(149.32...)$

$SQ^2 = 413.57046$

$SQ = 20.3\text{cm}$

Question 7 continued

(Total for Question 7 is 5 marks)

8. $$g(x) = (2 + ax)^8 \qquad \text{where } a \text{ is a constant}$$

Given that one of the terms in the binomial expansion of $g(x)$ is $3402x^5$

(a) find the value of a.

(4)

Using this value of a,

(b) find the constant term in the expansion of

$$\left(1 + \frac{1}{x^4}\right)(2 + ax)^8$$

(3)

a) By the binomial expansion the x^5 term is

$$\binom{8}{5} \times 2^{8-5} (ax)^5$$

$$= \frac{8!}{5!(8-5)!} \times 2^3 \times a^5 x^5$$

$$= 448 a^5 x^5 = 3402 x^5$$

$$\Rightarrow 448a^5 = 3402$$

$$\Rightarrow a^5 = 243/32$$

$$\Rightarrow a = 3/2.$$

b) The first constant is 2^8 (the constant of the expansion $\times 1$) $= 256$

The second constant is the x^4 term of the expansion $\times \frac{1}{x^4}$ term.

$$= {}^8C_4 \times 2^4 a^4 = 70 \times 16 \times 81/16 = 5670$$

The constant term is $256 + 5670 = 5926$.

Question 8 continued

(Total for Question 8 is 7 marks)

9. Find the value of the constant k, $0 < k < 9$, such that

$$\int_k^9 \frac{6}{\sqrt{x}}\,dx = 20$$

(4)

$$\int_k^9 \frac{6}{\sqrt{x}}\,dx = \int_k^9 6x^{-1/2}\,dx = 20$$

$$= \left[12x^{1/2}\right]_k^9 = 20$$

$$= 12\times\sqrt{9} - 12\sqrt{k} = 20$$

$$\Rightarrow 36 - 20 = 12\sqrt{k}$$

$$\Rightarrow \sqrt{k} = 4/3$$

$$\Rightarrow k = 16/9. \quad (\text{as } 0<k<9).$$

Question 9 continued

(Total for Question 9 is 4 marks)

10. A student is investigating the following statement about natural numbers.

"$n^3 - n$ is a multiple of 4"

(a) Prove, using algebra, that the statement is true for all odd numbers.

(4)

(b) Use a counterexample to show that the statement is not always true.

(1)

a) If n is odd then $n = 2k+1$ for an integer, k.

$n^3 - n = (2k+1)^3 - (2k+1)$

$= 8k^3 + 12k^2 + 6k + 1 - (2k+1)$

$= 8k^3 + 12k^2 + 4k = 4(2k^3 + 3k^2 + k)$

As k is an integer $4 \times (2k^3 + 3k^2 + k)$ is a multiple of 4. Therefor if n is odd then $n^3 - n$ is a multiple of 4.

b) Let $n = 2$:

$2^3 - 2 = 8 - 2 = 6$ which is not a multiple of 4.

Question 10 continued

Question 10 continued

Question 10 continued

(Total for Question 10 is 5 marks)

11. The owners of a nature reserve decided to increase the area of the reserve covered by trees.

Tree planting started on 1st January 2005.

The area of the nature reserve covered by trees, A km^2, is modelled by the equation

$$A = 80 - 45e^{ct}$$

where c is a constant and t is the number of years after 1st January 2005.

Using the model,

(a) find the area of the nature reserve that was covered by trees just before tree planting started.

(1)

On 1st January 2019 an area of 60 km^2 of the nature reserve was covered by trees.

(b) Use this information to find a complete equation for the model, giving your value of c to 3 significant figures.

(4)

On 1st January 2020, the owners of the nature reserve announced a long-term plan to have 100 km^2 of the nature reserve covered by trees.

(c) State a reason why the model is not appropriate for this plan.

(1)

a) When $t=0$ $e^{ct}=1$

$A = 80 - 45 \times 1 = 35$ km^2.

b) $t = 14$ years. So

$A = 80 - 45e^{14c} = 60$

$\Rightarrow \quad 45e^{14c} = 20$

$\Rightarrow \quad c = \frac{1}{14}\ln\left(\frac{20}{45}\right)$

$c = -0.0579235\ldots$

$A = 80 - 45e^{-0.0579t}$ (c to 3 sig fig).

c) The maximum area found by the model is 80 km^2, as $t \to \infty$.

Question 11 continued

(Total for Question 11 is 6 marks)

12. **In this question you should show all stages of your working.**

Solutions relying entirely on calculator technology are not acceptable.

(i) Solve, for $0 < \theta \leqslant 450°$, the equation

$$5\cos^2\theta = 6\sin\theta$$

giving your answers to one decimal place.

(5)

(ii) (a) A student's attempt to solve the question

"Solve, for $-90° < x < 90°$, the equation $3\tan x - 5\sin x = 0$"

is set out below.

$$3\tan x - 5\sin x = 0$$
$$3\frac{\sin x}{\cos x} - 5\sin x = 0$$
$$3\sin x - 5\sin x\cos x = 0$$
$$3 - 5\cos x = 0$$
$$\cos x = \frac{3}{5}$$
$$x = 53.1°$$

Identify two errors or omissions made by this student, giving a brief explanation of each.

(2)

The first four positive solutions, in order of size, of the equation

$$\cos(5\alpha + 40°) = \frac{3}{5}$$

are α_1, α_2, α_3 and α_4

(b) Find, to the nearest degree, the value of α_4

(2)

i) As $\cos^2\theta = 1 - \sin^2\theta$

Then $5\cos^2\theta = 6\sin\theta$

$\Rightarrow 5(1 - \sin^2\theta) = 6\sin\theta$

$\Rightarrow 5 - 5\sin^2\theta = 6\sin\theta$

$\Rightarrow 5\sin^2\theta + 6\sin\theta - 5 = 0$

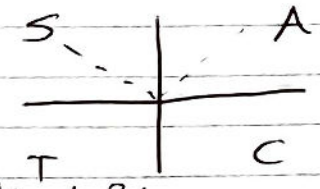
Question 12 continued

$\Rightarrow$ We can use the quadratic formula for $ax^2+bx+c=0$ where $a=5$, $b=6$, $c=-5$ and $x=\sin\theta$.

This gives us

$$\sin\theta = \frac{-6 \pm \sqrt{6^2-4\times5\times-5}}{2\times5}$$

$$\Rightarrow \sin\theta = \frac{-3\pm\sqrt{34}}{5}$$



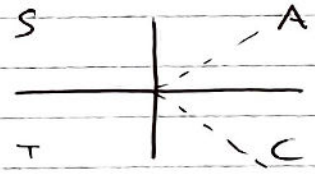
$\Rightarrow \theta = 34.484\ldots, 145.515\ldots, 394.484\ldots$

$\theta = 34.5^\circ, 145.5^\circ, 394.5^\circ$ (1dp).

ii a) • They cancel out $\sin x$ on line 4 and so miss the solution $\sin x = 0 \Rightarrow x = 0$.

• They do not find the solutions to $\cos x = 3/5$ in the range, they miss the solution $x = -53.1^\circ$.

ii b) By a cast diagram we know there are solutions for $\cos(5\alpha+40) = 3/5$ at



$5\alpha_1+40 = 53.1^\circ$
$5\alpha_2+40 = 360^\circ - 53.1^\circ$
$5\alpha_3+40 = 360^\circ + 53.1^\circ$
$5\alpha_4+40 = 720^\circ - 53.1^\circ$

$\Rightarrow 5\alpha_4 = 720 - 53.1 - 40 = 626.9$
$\Rightarrow \alpha_4 = 125^\circ$ (nearest degree).

Question 12 continued

Question 12 continued

(Total for Question 12 is 9 marks)

13.

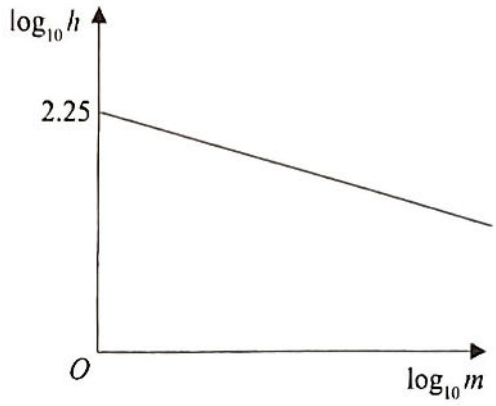


Figure 2

The resting heart rate, h, of a mammal, measured in beats per minute, is modelled by the equation

$$h = pm^q$$

where p and q are constants and m is the mass of the mammal measured in kg.

Figure 2 illustrates the linear relationship between $\log_{10} h$ and $\log_{10} m$

The line meets the vertical $\log_{10} h$ axis at 2.25 and has a gradient of -0.235

(a) Find, to 3 significant figures, the value of p and the value of q.

(3)

A particular mammal has a mass of 5 kg and a resting heart rate of 119 beats per minute.

(b) Comment on the suitability of the model for this mammal.

(3)

(c) With reference to the model, interpret the value of the constant p.

(1)

a) As the relationship between $\log_{10} h$ and $\log_{10} m$ is linear we can write it as $y = mx + c$.

$\Rightarrow \quad \log_{10} h = -0.235 \log_{10} m + 2.25.$

$\Rightarrow \quad h = 10^{-0.235 \log_{10} m} \times 10^{2.25}$

$h = m^{-0.235} \times 10^{2.25}$

$\Rightarrow q = -0.235$ and $p = 178$ (3 sig fig).

Question 13 continued

b) If $m = 5kg$ then the model predicts

$h = 178 \times 5^{-0.235} = 121.9...$ beats per minute.

This is accurate ~~to~~ to the measured heart rate within 2 significant figures. So the model is suitable.

c) p would be the resting heart rate in bpm of a mammal with a mass of 1kg.

Question 13 continued

Question 13 continued

(Total for Question 13 is 7 marks)

14. A curve C has equation $y = f(x)$ where

$$f(x) = -3x^2 + 12x + 8$$

(a) Write $f(x)$ in the form

$$a(x + b)^2 + c$$

where a, b and c are constants to be found.

(3)

The curve C has a maximum turning point at M.

(b) Find the coordinates of M.

(2)

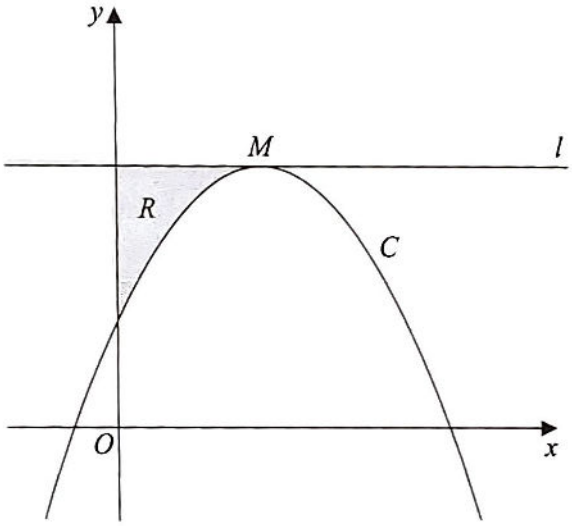


Figure 3

Figure 3 shows a sketch of the curve C.

The line l passes through M and is parallel to the x-axis.

The region R, shown shaded in Figure 3, is bounded by C, l and the y-axis.

(c) Using algebraic integration, find the area of R.

(5)

a) $f(x) = -3x^2 + 12x + 8$

$= -3(x^2 + 4x) + 8$

$= -3[(x-2)^2 - 4] + 8$

$= -3(x-2)^2 + 20$

b) By (a), $M = (2, 20)$

Question 14 continued

c) The line L is $y=20$. The area under the line L between the y-axis and M is $20 \times 2 = 40$.

So the area of R is 40 - the area under the curve C between $x=0$ and $x=2$.

$$R = 40 - \int_0^2 -3x^2 + 12x + 8 \, dx$$

$$R = 40 - \left[-x^3 + 6x^2 + 8x\right]_0^2$$

$$R = 40 - \left(-2^3 + 6 \times 2^2 + 8 \times 2\right)$$

$$R = 40 - 32 = 8.$$

Question 14 continued

Question 14 continued

(Total for Question 14 is 10 marks)

15.

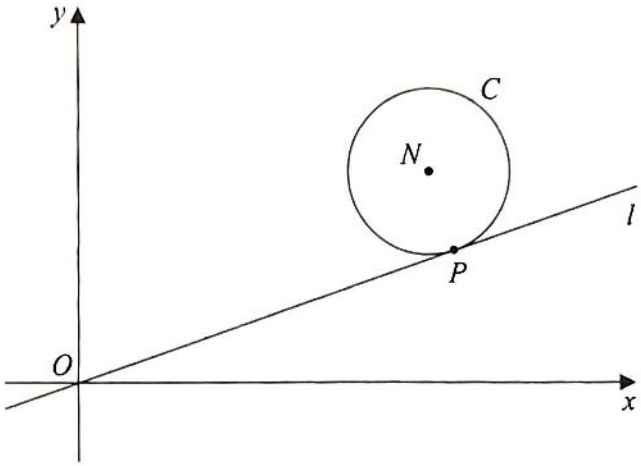


Figure 4

Figure 4 shows a sketch of a circle C with centre $N(7, 4)$

The line l with equation $y = \frac{1}{3}x$ is a tangent to C at the point P.

Find

(a) the equation of line PN in the form $y = mx + c$, where m and c are constants,

(2)

(b) an equation for C.

(4)

The line with equation $y = \frac{1}{3}x + k$, where k is a non-zero constant, is also a tangent to C.

(c) Find the value of k.

(3)

a) PN is perpendicular to l, so $m = -3$, and as N is on the line we have point $(7, 4)$.

$\Rightarrow y - 4 = -3(x - 7)$
$\Rightarrow y = -3x + 25$

b) The radius of C, r = length NP.

P is the intersect of $y = \frac{1}{3}x$ an $y = -3x + 25$

Question 15 continued

$\Rightarrow$ At P: $\frac{1}{3}x = -3x + 25$

$\Rightarrow$ $x = -9x + 75$

$\Rightarrow$ $10x = 75 \Rightarrow x = 7.5$.

$y = \frac{1}{3} \times 7.5 = 2.5$. $P(7.5, 2.5)$.

Length $PN = \sqrt{(7.5-7)^2 + (4-2.5)^2} = \sqrt{\frac{5}{2}}$

$C: (x-7)^2 + (y-4)^2 = 5/2$.

c) When $y = \frac{1}{3}x + k$ satisfies the equation for C,

$(x-7)^2 + (\frac{1}{3}x + k - 4)^2 = 5/2$.

$x^2 - 14x + 49 + \frac{1}{9}x^2 + \frac{k}{3}x - \frac{4}{3}x + \frac{k}{3}x + k^2 - 4k$
$- \frac{4}{3}x - 4k + 16 = 5/2$

$\Rightarrow \frac{10}{9}x^2 + \left(\frac{2}{3}k - \frac{50}{3}\right)x + k^2 - 8k + \frac{125}{2} = 0$.

This quadratic must have only one solution, as a tangent meets a circle only once.

$\Rightarrow b^2 - 4ac = 0$.

$\Rightarrow \left(\frac{2}{3}k - \frac{50}{3}\right)^2 - 4 \times \left(\frac{10}{9}\right) \times \left(k^2 - 8k + \frac{125}{2}\right) = 0$.

$\frac{4}{9}k^2 - \frac{200}{9}k + \frac{2500}{9} - \frac{40}{9}k^2 + \frac{320}{9}k - \frac{2500}{9} = 0$.

$\Rightarrow 4k^2 - 200k - 40k^2 + 320k = 0$
$-36k^2 + 120k = 0$

Question 15 continued

$k=0$ is the case for line l.

$\Rightarrow \quad -36k+120=0$

$\Rightarrow \quad k=\frac{120}{36}=\frac{10}{3}$ for the non-zero constant.

equation $y=\frac{1}{3}x+\frac{10}{3}$

Question 15 continued

(Total for Question 15 is 9 marks)

16. The curve C has equation $y = f(x)$ where

$$f(x) = ax^3 + 15x^2 - 39x + b$$

and a and b are constants.

Given

- the point (2, 10) lies on C
- the gradient of the curve at (2, 10) is –3

(a) (i) show that the value of a is –2

(ii) find the value of b.

(4)

(b) Hence show that C has no stationary points.

(3)

(c) Write f(x) in the form $(x - 4)Q(x)$ where $Q(x)$ is a quadratic expression to be found.

(2)

(d) Hence deduce the coordinates of the points of intersection of the curve with equation

$$y = f(0.2x)$$

and the coordinate axes.

(2)

ai) $\frac{dy}{dx} = 3ax^2 + 30x - 39$

$\frac{dy}{dx} = -3$ when $x = 2 \Rightarrow 3a \times 4 + 60 - 39 = -3$

$12a = -24$

$a = -2$.

ii) As $f(2) = 10$

$\Rightarrow (-2) \times 2^3 + 15 \times 2^2 - 39 \times 2 + b = 10$

$\Rightarrow -34 + b = 10$

$\Rightarrow b = 44$.

b) $f'(x) = -6x^2 + 30x - 39$

$30^2 - 4 \times -6 \times -39 = -36$.

As $b^2 - 4ac < 0$, $f'(x)$ has no real roots and so the gradient of C is never 0, hence no stantonary points.

Question 16 continued

c)

$$
\begin{array}{r}
-2x^2+7x-11 \\
x-4\,\overline{)\,-2x^3+15x^2-39x+44} \\
\underline{-2x^3+8x^2} \\
7x^2-39x \\
\underline{7x^2-28x} \\
-11x+44 \\
-11x+44
\end{array}
$$

$f(x) \equiv (x-4)(-2x^2+7x-11)$.

d) When $x=0$, $f(0) = f(0.2\times 0) = 44$.

$(0, 44)$.

When $y=0$

$(0.2x-4)(-2\times(0.2x)^2+7(0.2x)-11)=0$

$\Rightarrow 0.2x-4=0 \Rightarrow x=20$, $(20,0)$

$f(0.2x)=(0.2x-4)(-0.08x^2+1.4x-11)=0$.

As $1.4^2-4\times 0.08\times -11<0$, $x=20$ is the only solution to $f(0.2x)=0$.

$(0,44)$ & $(20,0)$

Question 16 continued

(Total for Question 16 is 11 marks)

TOTAL FOR PAPER IS 100 MARKS