

## Centre Number Candidate Number



Pearson Edexcel Level 3 GCE
Monday 18 October 2021 - Afternoon
Paper reference

## 9MA0/31

## Mathematics

## Advanced <br> PAPER 31: Statistics

```
You must have:
Mathematical Formulae and Statistical Tables (Green), calculator
```

Candidates may use any calculator allowed by Pearson regulations. Calculators must not have the facility for symbolic algebra manipulation, differentiation and integration, or have retrievable mathematical formulae stored in them.

## Instructions

- Use black ink or ball-point pen.
- If pencil is used for diagrams/sketches/graphs it must be dark (HB or B ).
- Fill in the boxes at the top of this page with your name, centre number and candidate number.
- Answer all questions and ensure that your answers to parts of questions are clearly labelled.
- Answer the questions in the spaces provided - there may be more space than you need.
- You should show sufficient working to make your methods clear. Answers without working may not gain full credit.
- Values from statistical tables should be quoted in full. If a calculator is used instead of tables the value should be given to an equivalent degree of accuracy.
- Inexact answers should be given to three significant figures unless otherwise stated.


## Information

- A booklet 'Mathematical Formulae and Statistical Tables' is provided.
- The total mark for this part of the examination is 50 . There are 6 questions.
- The marks for each question are shown in brackets - use this as a guide as to how much time to spend on each question.


## Advice

- Read each question carefully before you start to answer it.
- Try to answer every question.
- Check your answers if you have time at the end.

mmerevise.co.uk

1. (a) State one disadvantage of using quota sampling compared with simple random sampling.

In a university $8 \%$ of students are members of the university dance club.
A random sample of 36 students is taken from the university.
The random variable $X$ represents the number of these students who are members of the dance club.
(b) Using a suitable model for $X$, find
(i) $\mathrm{P}(X=4)$
(ii) $\mathrm{P}(X \geqslant 7)$

Only $40 \%$ of the university dance club members can dance the tango.
(c) Find the probability that a student is a member of the university dance club and can dance the tango.

A random sample of 50 students is taken from the university.
(d) Find the probability that fewer than 3 of these students are members of the university dance club and can dance the tango.
a) As quota sampling isn't random it cannot be reliably used to make inferences. b:) The probability of a student being in the club =0.08.

$$
\begin{aligned}
& x \sim B(36,0.08) \\
& \text { so } P(X=4)=0.167387 \ldots \\
& \text { ii) } P(X \geqslant 7)=1-P(X<6)=0.22233 \\
& \text { c) } P(\text { Inc club and dance tanga) }=0.4 \times 0.08=0.032 . \\
& \text { d) } T=\text { Members of the club who can dance tango } \\
& T \sim B(50,0.032)
\end{aligned}
$$

Question 1 continued.
$P(T<3)=0.785081 S$
mmerevise.co.uk
2. Marc took a random sample of 16 students from a school and for each student recorded

- the number of letters, $x$, in their last name
- the number of letters, $y$, in their first name

His results are shown in the scatter diagram on the next page.
(a) Describe the correlation between $x$ and $y$.

Marc suggests that parents with long last names tend to give their children shorter first names.
(b) Using the scatter diagram comment on Marc's suggestion, giving a reason for your answer.

The results from Marc's random sample of 16 observations are given in the table below.

| $x$ | 3 | 6 | 8 | 7 | 5 | 3 | 11 | 3 | 4 | 5 | 4 | 9 | 7 | 10 | 6 | 6 |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| $y$ | 7 | 7 | 4 | 4 | 6 | 8 | 5 | 5 | 8 | 4 | 7 | 4 | 5 | 5 | 6 | 3 |

(c) Use your calculator to find the product moment correlation coefficient between $x$ and $y$ for these data.
(d) Test whether or not there is evidence of a negative correlation between the number of letters in the last name and the number of letters in the first name.

You should

- state your hypotheses clearly
- use a $5 \%$ level of significance
a) Negative Correlation.
b) As the scatter diagram has a negative correlation, it is compatible with Marc's suggestion.

$$
\begin{aligned}
& \text { c) } \sum x=97, \sum y=88, \sum x^{2}=681, \sum y^{2}=520 \\
& \sum x y=502, \quad n=16 \\
& r=\frac{502-(97 \times 88) / 16}{\sqrt{\left(681-\frac{97^{2}}{16}\right) \times\left(520-\frac{88^{2}}{16}\right)}}=-0.54458266 \ldots
\end{aligned}
$$

mmerevise.co.uk

Question 2 continued.



$$
\alpha=0.05
$$

The $5 \%$ significance 1-tail test critical value is

$$
\begin{gathered}
-0.4259 \\
r=-0.544 \ldots<-0.4259
\end{gathered}
$$

Hence there is evidence of negative correlation between length of first names and Lenght of second names.

Question 2 continued.

Question 2 continued.
3. Stav is studying the large data set for September 2015

He codes the variable Daily Mean Pressure, $x$, using the formula $y=x-1010$
The data for all 30 days from Hern are summarised by

$$
\sum y=214 \quad \sum y^{2}=5912
$$

(a) State the units of the variable $x$
(b) Find the mean Daily Mean Pressure for these 30 days.
(c) Find the standard deviation of Daily Mean Pressure for these 30 days.

Stay knows that, in the UK, winds circulate

- in a clockwise direction around a region of high pressure
- in an anticlockwise direction around a region of low pressure

The table gives the Daily Mean Pressure for 3 locations from the large data set on 26/09/2015

| Location | Heathrow | Hurn | Leuchars |
| :--- | :---: | :---: | :---: |
| Daily Mean Pressure | 1029 | 1028 | 1028 |
| Cardinal Wind Direction | NE | $E$ | E |

The Cardinal Wind Directions for these 3 locations on 26/09/2015 were, in random order,
W NE E

You may assume that these 3 locations were under a single region of pressure.
(d) Using your knowledge of the large data set, place each of these Cardinal Wind Directions in the correct location in the table.
Give a reason for your answer.
a) Hectopascal ( ha )
b) $\bar{x}=\bar{y}+1010, \quad \bar{y}=244 / 30$ $\bar{x}=1017.13$
c) As $\quad y=x-1010, \quad \sigma_{x}=\theta y$
mmerevise.co.uk

Question 3 continued.

$$
\theta_{y}=\sqrt{\frac{5912}{30}-\left(\frac{214}{30}\right)^{2}}=12.0905 \ldots=\theta_{x} .
$$

d) As all of these pressures are nearly a standard deviation from the mean, we car consider them high pressures. So the wind is circulating clockwise.
The locations from north to south are Leuchars, Heathrow, Hurn.
Sothe directions of their winds is as written on the table.
4. A large college produces three magazines.

One magazine is about green issues, one is about equality and one is about sports. A student at the college is selected at random and the events $G, E$ and $S$ are defined as follows
$G$ is the event that the student reads the magazine about green issues
$E$ is the event that the student reads the magazine about equality
$S$ is the event that the student reads the magazine about sports
The Venn diagram, where $p, q, r$ and $t$ are probabilities, gives the probability for each subset.

(a) Find the proportion of students in the college who read exactly one of these magazines.

No students read all three magazines and $\mathrm{P}(G)=0.25$
(b) Find
(i) the value of $p$
(ii) the value of $q$

Given that $\mathrm{P}(S \mid E)=\frac{5}{12}$
(c) find
(i) the value of $r$
(ii) the value of $t$
(d) Determine whether or not the events $\left(S \cap E^{\prime}\right)$ and $G$ are independent. Show your working clearly.
a) $0.08+0.09+0.36=0.53$
mmerevise.co.uk

Question 4 continued.
bi) As no student reads all 3 magazines, $P(G \cap E \cap S)=p=0$.
ii) $P(G \cap S)=q$

$$
\begin{aligned}
& P(G)=P(G \cap S)+P(G \cap E)+P(\text { Just } G) \\
& 0.25=q+0.05+0.08 \\
& \Rightarrow q=0.12 .
\end{aligned}
$$

(i) $P(S \mid E)=5 / 12=\frac{P(S \cap E)}{P(E)}$

$$
P(E)=r+p+0.05+0.09=0.14+r
$$

$P(S \cap E)=r+p=r$.

$$
\frac{5}{12}=\frac{r}{0 \cdot 14+r}
$$

$$
\Rightarrow 12 r=5 r+5 \times 0.14
$$

$$
\Rightarrow \quad 7 r=0.7
$$

$$
\Rightarrow \quad r=0.1
$$

$$
\begin{aligned}
& \text { ii) } 0.08+0.05+0.09+q+p+r+0.36+t=1 \\
& q=0.12, p=0, r=0.1 \\
& \Rightarrow \quad 0.8+t=1 \\
& \Rightarrow \quad t=0.2 .
\end{aligned}
$$

tumerevise.co.uk

Question 4 continued.

$$
\text { d) } \begin{aligned}
P\left(S \cap E^{\prime}\right) & =0.36+q=0.36+0.12 \\
& =0.48 \\
P\left(\left(S \cap E^{\prime}\right) \cap G\right) & =q=0.12 \\
P(G) & =0.25 \\
P\left(S \cap E^{\prime}\right) \times P(G) & =0.48 \times 0.25=0.12 \\
& =P\left(\left(S \cap E^{\prime}\right) \cap G\right)
\end{aligned}
$$

Hence they are independent.
mmerevise.co.uk
Question 4 continued.
5. The heights of females from a country are normally distributed with

- a mean of 166.5 cm
- a standard deviation of 6.1 cm

Given that $1 \%$ of females from this country are shorter than $k \mathrm{~cm}$,
(a) find the value of $k$
(b) Find the proportion of females from this country with heights between 150 cm and 175 cm

A female, from this country, is chosen at random from those with heights between 150 cm and 175 cm
(c) Find the probability that her height is more than 160 cm

The heights of females from a different country are normally distributed with a standard deviation of 7.4 cm

Mia believes that the mean height of females from this country is less than 166.5 cm Mia takes a random sample of 50 females from this country and finds the mean of her sample is 164.6 cm
(d) Carry out a suitable test to assess Mia's belief.

You should

- state your hypotheses clearly
- use a $5 \%$ level of significance
a) Let $F \sim N\left(\mid 66.5,6.1^{2}\right)$ $P(F<k)=0.01$
$\Rightarrow \quad \frac{k-166.5}{6.1}=-2.3263$
$\Rightarrow K=152.3 \mathrm{~cm}$.
b) $P(150<F<175)=0.914840 \ldots$

Question 5 continued.

$$
\text { c) } \begin{array}{rl} 
& P(F>160 \mid 150<F<175) \\
= & \frac{P(160<F<175)}{P(150<F<175)} \\
P & P(160<F<175)=0.7749487 \ldots \\
\Rightarrow & P(F 60 \mid 150<F<175)=\frac{0.7749487 \ldots}{0.914840 \ldots} \\
= & 0.84708 \cdots
\end{array}
$$

d) Null hypothesis: $H_{0} \mu=166.5$.
$M$ : as hypothesis: $H_{1}: \mu<166 \cdot 5$.
Let $X=$ height from this country

$$
\begin{aligned}
& \bar{x} \sim N\left(166.5,\left(\frac{7.4}{\sqrt{50}}\right)^{2}\right) \\
& P(\bar{x}<164.6)=0.03472<0.05
\end{aligned}
$$ so we can reject $H_{0}$.

$\Rightarrow$ There is evidence to support Mia's belief.

Question 5 continued.

Question 5 continued.
mmerevise.co.uk
6. The discrete random variable $X$ has the following probability distribution

| $x$ | $a$ | $b$ | $c$ |
| :---: | :---: | :---: | :---: |
| $\mathrm{P}(X=x)$ | $\log _{36} a$ | $\log _{36} b$ | $\log _{36} c$ |

where

- $\quad a, b$ and $c$ are distinct integers $(a<b<c)$
- all the probabilities are greater than zero
(a) Find
(i) the value of $a$
(ii) the value of $b$
(iii) the value of $c$

Show your working clearly.

The independent random variables $X_{1}$ and $X_{2}$ each have the same distribution as $X$
(b) Find $\mathrm{P}\left(X_{1}=X_{2}\right)$
ai) $\log _{36}(a)+\log _{36}(b)+\log _{36}(c)=1$. $\Rightarrow \log _{36}(a b c)=1$. $\Rightarrow a b c=36$.
As all the probabilities ave greater than 0 , 1<a,b,c.
Prime factors of $36=2 \times 2 \times 3 \times 3$.
As we reed 3 distinct integers that multiply to 36 we $36^{\text {got }}=2 \times 3 \times 6$

$$
\Rightarrow a=2, b=3, c=6(\text { as } a<b=c) .
$$

mmerevise.co.uk

Question 6 continued.
b) 3 ways they can be equal:

$$
\begin{aligned}
& P\left(x_{1}=x_{2}=a\right)=\left(\log _{36} a\right)^{2},\left[P\left(x_{1}=a\right) \times P\left(x_{2}=a\right)\right] \\
& P\left(x_{1}=x_{2}=b\right)=\left(\log _{36} b\right)^{2},\left[P\left(x_{1}=b\right) \times P\left(x_{2}=b\right)\right] \\
& P\left(x_{1}=x_{2}=c\right)=\left(\log _{36} c\right)^{2},\left[P\left(x_{1}-c\right) \times P\left(x_{2}=c\right)\right] \\
& \text { So } P\left(x_{1}=x_{2}\right)=\left(\log _{36} a\right)^{2}+\left(\log _{36} b\right)^{2}+\left(\log _{36} c\right)^{2} \\
&=0.381401 \ldots
\end{aligned}
$$

Question 6 continued.

