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Candidate surname	Other names								
Centre Number	Candidate Number								
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Pearson Edexcel Level 3 GCE

Time 2 hours Paper reference **8MA0/01**

Mathematics
Advanced Subsidiary
PAPER 1: Pure Mathematics

You must have:
Mathematical Formulae and Statistical Tables (Green), calculator

Total Marks

**Candidates may use any calculator allowed by Pearson regulations.
Calculators must not have the facility for symbolic algebra manipulation,
differentiation and integration, or have retrievable mathematical formulae
stored in them.**

Instructions

- Use **black** ink or ball-point pen.
- If pencil is used for diagrams/sketches/graphs it must be dark (HB or B).
- **Fill in the boxes** at the top of this page with your name, centre number and candidate number.
- Answer **all** questions and ensure that your answers to parts of questions are clearly labelled.
- Answer the questions in the spaces provided
– *there may be more space than you need.*
- You should show sufficient working to make your methods clear.
Answers without working may not gain full credit.
- Inexact answers should be given to three significant figures unless otherwise stated.

Information

- A booklet 'Mathematical Formulae and Statistical Tables' is provided.
- There are 14 questions in this question paper. The total mark for this paper is 100.
- The marks for **each** question are shown in brackets
– *use this as a guide as to how much time to spend on each question.*

Advice

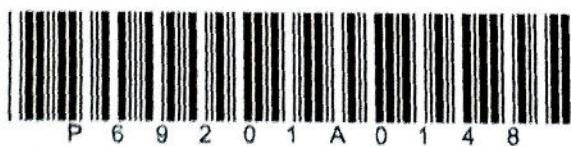
- Read each question carefully before you start to answer it.
- Try to answer every question.
- Check your answers if you have time at the end.

Turn over ➤

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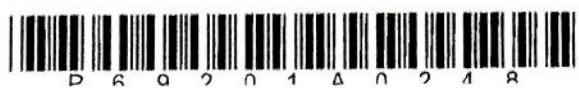
1. Find

$$\int \left(8x^3 - \frac{3}{2\sqrt{x}} + 5 \right) dx$$

giving your answer in simplest form.

(4)

$$\begin{aligned} \int 8x^3 - \frac{3}{2}x^{-\frac{1}{2}} + 5 dx &= \frac{8x^4}{4} - 2 \times \frac{3}{2}x^{\frac{1}{2}} + 5x + C \\ &= 2x^4 - 3x^{\frac{1}{2}} + 5x + C \end{aligned}$$



2.

$$f(x) = 2x^3 + 5x^2 + 2x + 15$$

(a) Use the factor theorem to show that $(x + 3)$ is a factor of $f(x)$.

(2)

(b) Find the constants a , b and c such that

$$f(x) = (x + 3)(ax^2 + bx + c)$$

(2)

(c) Hence show that $f(x) = 0$ has only one real root.

(2)

(d) Write down the real root of the equation $f(x - 5) = 0$

(1)

$$\begin{aligned} (a) \quad f(-3) &= 2(-3)^3 + 5(-3)^2 + 2(-3) + 15 \\ &= (-54) + 45 + (-6) + 15 = 0 \end{aligned}$$

 $\rightarrow (x + 3)$ is a factor

(b)

$$\begin{array}{r} 2x^3 + 5x^2 + 2x + 15 \\ x+3 \overline{)2x^3 + 5x^2 + 2x + 15} \\ \underline{(2x^3 + 6x^2)} \\ -x^2 + 2x + 15 \\ \underline{(-x^2 - 3x)} \\ 5x + 15 \\ \underline{(5x + 15)} \\ 0 \end{array}$$

$a = 2$
$b = -1$
$c = 5$

$$(c) \quad b^2 - 4ac = (-1)^2 - 4(2)(5) = -39 < 0$$

The quadratic has no real roots so $f(x) = 0$ has only 1 real root

$$\begin{aligned} (d) \quad f(x-5) &= (x-5+3)(a(x-5)^2 + b(x-5) + c) \\ &= (x-2)(...) \end{aligned}$$

$$\underline{x = 2}$$



3. The triangle PQR is such that $\vec{PQ} = 3\mathbf{i} + 5\mathbf{j}$ and $\vec{PR} = 13\mathbf{i} - 15\mathbf{j}$

(a) Find \vec{QR} (2)

(b) Hence find $|\vec{QR}|$ giving your answer as a simplified surd. (2)

The point S lies on the line segment QR so that $QS:SR = 3:2$

(c) Find \vec{PS} (2)

(a)

$$\begin{aligned}\vec{QR} &= \vec{QP} + \vec{PR} \\ &= -\vec{PQ} + \vec{PR} \\ &= -3\mathbf{i} - 5\mathbf{j} + 13\mathbf{i} - 15\mathbf{j} \\ &= 10\mathbf{i} - 20\mathbf{j}\end{aligned}$$

(b) $|\vec{QR}| = \sqrt{10^2 + 20^2} = 10\sqrt{5}$

(c) $\vec{PS} = \vec{PQ} + \frac{3}{5} \vec{QR}$

$$\begin{aligned}&= 3\mathbf{i} + 5\mathbf{j} + \frac{3}{5} (10\mathbf{i} - 20\mathbf{j}) \\ &= 3\mathbf{i} + 5\mathbf{j} + 6\mathbf{i} - 12\mathbf{j} \\ &= 9\mathbf{i} - 7\mathbf{j}\end{aligned}$$



4.

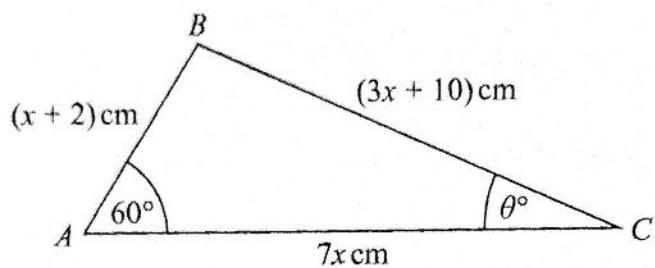


Figure 1

Figure 1 shows a sketch of triangle ABC with $AB = (x + 2)$ cm, $BC = (3x + 10)$ cm, $AC = 7x$ cm, angle $BAC = 60^\circ$ and angle $ACB = \theta^\circ$

(a) (i) Show that $17x^2 - 35x - 48 = 0$

(3)

(ii) Hence find the value of x .

(1)

(b) Hence find the value of θ giving your answer to one decimal place.

(2)

$$\text{(a) (i)} \quad (3x+10)^2 = (x+2)^2 + (7x)^2 - 2(x+2)(7x) \cos 60^\circ$$

$$9x^2 + 60x + 100 = x^2 + 4x + 4 + 49x^2 - (14x^2 + 28x) \times \frac{1}{2}$$

$$34x^2 - 70x - 96 = 0 \rightarrow 17x^2 - 35x - 48 = 0$$

$$\text{(iii)} \quad x = 3 \quad (\text{ignore } x = -0.94 \dots \text{ as } x \text{ can't be negative})$$

$$\text{(b) } \frac{\sin A}{\sin C} = \frac{19}{\sin 60^\circ} \rightarrow \sin A_{CB} = \frac{5 \sin 60^\circ}{19}$$

$$A_{CB} = \theta = 13.2$$

5. The mass, A kg, of algae in a small pond, is modelled by the equation

$$A = pq^t$$

where p and q are constants and t is the number of weeks after the mass of algae was first recorded.

Data recorded indicates that there is a linear relationship between t and $\log_{10} A$ given by the equation

$$\log_{10} A = 0.03t + 0.5$$

- (a) Use this relationship to find a complete equation for the model in the form

$$A = pq^t$$

giving the value of p and the value of q each to 4 significant figures.

(4)

- (b) With reference to the model, interpret

- (i) the value of the constant p ,
- (ii) the value of the constant q .

(2)

- (c) Find, according to the model,

- (i) the mass of algae in the pond when $t = 8$, giving your answer to the nearest 0.5 kg,
- (ii) the number of weeks it takes for the mass of algae in the pond to reach 4 kg.

(3)

- (d) State one reason why this may not be a realistic model in the long term.

(1)

$$(a) p = 10^{0.5} = 3.162 \quad A = 3.162 \times 1.072^t$$

$$q = 10^{0.03} = 1.072$$

(b) (i) The initial mass (in kg) of algae (in the pond)

(ii) The ratio of algae from one week to the next

(c) (i) 5.5 kg

$$(ii) 4 = 3.162 \times 1.072^t \rightarrow t = 3.4 \text{ weeks}$$

(a) The weather may affect the rate of growth



6. (a) Find the first 4 terms, in ascending powers of x , of the binomial expansion of

$$\left(3 - \frac{2x}{9}\right)^8$$

giving each term in simplest form.

(4)

$$f(x) = \left(\frac{x-1}{2x}\right)\left(3 - \frac{2x}{9}\right)^8$$

- (b) Find the coefficient of x^2 in the series expansion of $f(x)$, giving your answer as a simplified fraction.

(2)

$$(a) {}^8C_0 3^8 \left(-\frac{2x}{9}\right)^0 + {}^8C_1 3^7 \left(-\frac{2x}{9}\right)^1 +$$

$${}^8C_2 3^6 \left(-\frac{2x}{9}\right)^2 + {}^8C_3 3^5 \left(-\frac{2x}{9}\right)^3$$

$$= 6561 - 3888x + 1008x^2 - \frac{448}{3}x^3 + \dots$$

$$(b) \text{ coefficient of } x^2 \text{ is } \frac{1}{2} \times 1008 - \frac{1}{2} \times \left(-\frac{448}{3}\right)$$

$$= \frac{1136}{3}$$

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7. (a) Factorise completely $9x - x^3$

(2)

The curve C has equation

$$y = 9x - x^3$$

- (b) Sketch C showing the coordinates of the points at which the curve cuts the x -axis.

(2)

The line l has equation $y = k$ where k is a constant.

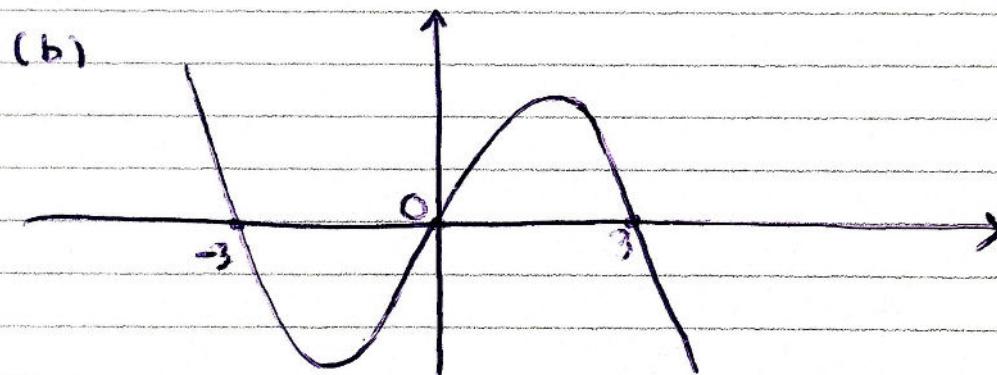
Given that C and l intersect at 3 distinct points,

- (c) find the range of values for k , writing your answer in set notation.

Solutions relying on calculator technology are not acceptable.

(3)

(a) $x(9-x^2) \rightarrow x(3-x)(3+x)$



(c) $\frac{dy}{dx} = 9 - 3x^2 = 0 \quad x = \pm\sqrt{3}$

$$y = 9(\pm\sqrt{3}) - (\pm\sqrt{3})^3 = \pm 6\sqrt{3}$$

$$\{k \in \mathbb{R} : -6\sqrt{3} < k < 6\sqrt{3}\}$$



8.

In this question you must show all stages of your working.

Solutions relying entirely on calculator technology are not acceptable.

The air pressure, P kg/cm², inside a car tyre, t minutes from the instant when the tyre developed a puncture is given by the equation

$$P = k + 1.4e^{-0.5t} \quad t \in \mathbb{R} \quad t \geq 0$$

where k is a constant.

Given that the initial air pressure inside the tyre was 2.2 kg/cm²

(a) state the value of k .

(1)

From the instant when the tyre developed the puncture,

(b) find the time taken for the air pressure to fall to 1 kg/cm²

Give your answer in minutes to one decimal place.

(3)

(c) Find the rate at which the air pressure in the tyre is decreasing exactly 2 minutes from the instant when the tyre developed the puncture.

Give your answer in kg/cm² per minute to 3 significant figures.

(2)

$$(a) 2.2 = k + 1.4e^{-0.5 \cdot 0} \rightarrow k = 0.8$$

$$(b) 1 = 0.8 + 1.4e^{-0.5t} \rightarrow 1.4e^{-0.5t} = 0.2$$

$$-0.5t = \ln\left(\frac{0.2}{0.8}\right) \rightarrow t = 3.9 \text{ minutes}$$

$$(c) \frac{dP}{dt} = -0.7e^{-0.5t}$$

$$\text{when } t = 2 \quad -0.7e^{-0.5 \times 2} = 0.258 \text{ kg/cm}^2 \text{ per minute}$$



9. (a) Given that $p = \log_3 x$, where $x > 0$, find in simplest form in terms of p ,

$$(i) \log_3\left(\frac{x}{9}\right)$$

$$(ii) \log_3(\sqrt{x})$$

(2)

(b) Hence, or otherwise, solve

$$2\log_3\left(\frac{x}{9}\right) + 3\log_3(\sqrt{x}) = -11$$

giving your answer as a simplified fraction.

Solutions relying on calculator technology are not acceptable.

(4)

$$(a) (i) \log_3\left(\frac{x}{9}\right) = \log_3 x - \log_3 9 = p - 2$$

$$(ii) \log_3(\sqrt{x}) = \log_3(x^{\frac{1}{2}}) = \frac{1}{2}\log_3 x = \frac{1}{2}p$$

$$(b) 2(p-2) + 3\left(\frac{1}{2}p\right) = -11$$

$$2p - 4 + \frac{3}{2}p = -11$$

$$\frac{7}{2}p = -7 \rightarrow p = -2$$

$$\log_3 x = -2 \rightarrow x = 3^{-2} = \frac{1}{9}$$



10.

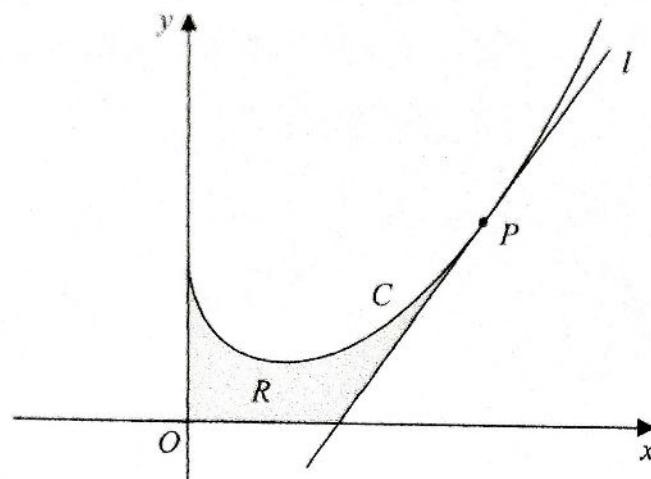


Figure 2

In this question you must show all stages of your working.

Solutions relying on calculator technology are not acceptable.

Figure 2 shows a sketch of part of the curve C with equation

$$y = \frac{1}{3}x^2 - 2\sqrt{x} + 3 \quad x \geq 0$$

The point P lies on C and has x coordinate 4

The line l is the tangent to C at P .

(a) Show that l has equation

$$13x - 6y - 26 = 0 \quad (5)$$

The region R , shown shaded in Figure 2, is bounded by the y -axis, the curve C , the line l and the x -axis.

(b) Find the exact area of R .

$$(a) \quad y = \frac{1}{3}x^2 - 2\sqrt{x} + 3 \rightarrow \frac{dy}{dx} = \frac{2}{3}x - x^{-\frac{1}{2}} \quad (5)$$

$$x=4 \quad y = \frac{1}{3}(4^2) - 2(4) + 3 = \frac{13}{3} \quad P(4, \frac{13}{3})$$

$$\frac{dy}{dx} \Big|_{x=4} : \frac{2}{3} \times 4 - 4^{-\frac{1}{2}} = \frac{13}{6}$$

$$y = \frac{13}{6}x + c \rightarrow \frac{13}{3} = \frac{13}{6} \times 4 + c \rightarrow c = -\frac{13}{6}$$

$$6y - 13x + 26 = 0 \quad \text{or} \quad 13x - 6y - 26 = 0$$



Question 10 continued

$$(b) \int \frac{x^2}{3} - 2x + 3 \, dx = \frac{x^3}{9} - \frac{4}{3}x^{\frac{3}{2}} + 3x + C$$

$$y=0 \rightarrow x=2$$

$$\text{Area of } R: \left[\frac{x^3}{9} - \frac{4}{3}x^{\frac{3}{2}} + 3x \right]_0^4 - \frac{1}{2}(4-2) \times \frac{13}{3}$$

$$= \frac{76}{9} - \frac{13}{3} = \frac{31}{9}$$



11.

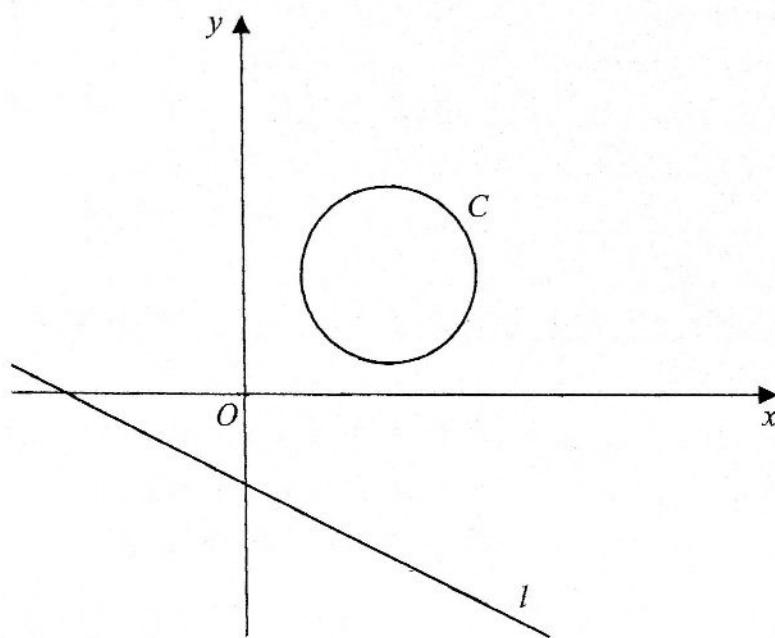


Figure 3

Figure 3 shows the circle C with equation

$$x^2 + y^2 - 10x - 8y + 32 = 0$$

and the line l with equation

$$2y + x + 6 = 0$$

(a) Find

- (i) the coordinates of the centre of C ,
- (ii) the radius of C .

(3)

(b) Find the shortest distance between C and l .

(5)

(a) (i) $(x-5)^2 + (y-4)^2 - 25 - 16 + 32 = 0$

$$(x-5)^2 + (y-4)^2 = 9$$

Centre

Coordinates: $(5, 4)$

(ii) Radius: $\sqrt{9} = 3$

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Question 11 continued

$$(b) \quad 2y + x + 6 = 0 \quad y = -\frac{1}{2}x - 3$$

$$-\frac{1}{2} \rightarrow 2 \quad m = 2$$

$$y = 2x + c \quad 4 = 2 \times 5 + c \rightarrow c = -6$$

$$\text{intersection: } 2(2x-6) + x + 6 = 0$$

$$4x - 12 + x + 6 = 0$$

$$5x = 6 \rightarrow x = \frac{6}{5}$$

$$y = 2\left(\frac{6}{5}\right) - 6 = -\frac{18}{5} \quad \left(\frac{6}{5}, -\frac{18}{5}\right)$$

Distance from centre of circle to intersection:

$$\sqrt{\left(5 - \frac{6}{5}\right)^2 + \left(4 + \frac{18}{5}\right)^2} = \frac{19\sqrt{5}}{5}$$

$$\text{Distance required: } \frac{19\sqrt{5}}{5} - 3 \quad \text{or } 5.50$$



P 6 0 2 0 1 4 0 3 5 4 8

12. A company makes drinks containers out of metal.

The containers are modelled as closed cylinders with base radius r cm and height h cm and the capacity of each container is 355 cm^3

The metal used

- for the circular base and the curved side costs 0.04 pence/cm²
- for the circular top costs 0.09 pence/cm²

Both metals used are of negligible thickness.

(a) Show that the total cost, C pence, of the metal for one container is given by

$$C = 0.13\pi r^2 + \frac{28.4}{r} \quad (4)$$

(b) Use calculus to find the value of r for which C is a minimum, giving your answer to 3 significant figures. (4)

(c) Using $\frac{d^2C}{dr^2}$ prove that the cost is minimised for the value of r found in part (b). (2)

(d) Hence find the minimum value of C , giving your answer to the nearest integer. (2)

$$(a) V = \pi r^2 h = 355 \rightarrow h = \frac{355}{\pi r^2}$$

$$C = 0.04(\pi r^2 + 2\pi r h) + 0.09(\pi r^2)$$

$$C = 0.13\pi r^2 + 0.08\pi r h = 0.13\pi r^2 + 0.08\pi r \left(\frac{355}{\pi r^2} \right)$$

$$C = 0.13\pi r^2 + \frac{28.4}{r}$$

$$(b) \frac{dC}{dr} = 0.26\pi r - \frac{28.4}{r^2}$$

$$\frac{dC}{dr} = 0 \Rightarrow r^3 = \frac{28.4}{0.26\pi} \Rightarrow r = \sqrt[3]{\frac{28.4}{0.26\pi}} = 3.26$$

$$(c) \frac{d^2C}{dr^2} = 0.26\pi + \frac{56.8}{r^3} = 0.26\pi + \frac{56.8}{3.26^3}$$

$$= 2.45 > 0 \rightarrow \text{minimum cost}$$



Question 12 continued

$$(a) C = 0.13\pi (3.26)^2 + \frac{28.4}{3.26}$$

$$C = 15$$

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13.

In this question you must show all stages of your working.

Solutions relying entirely on calculator technology are not acceptable.

(a) Show that

$$\frac{1}{\cos \theta} + \tan \theta \equiv \frac{\cos \theta}{1 - \sin \theta} \quad \theta \neq (2n+1)90^\circ \quad n \in \mathbb{Z} \quad (3)$$

Given that $\cos 2x \neq 0$ (b) solve for $0 < x < 90^\circ$

$$\frac{1}{\cos 2x} + \tan 2x = 3 \cos 2x$$

giving your answers to one decimal place.

(5)

$$(a) \frac{1}{\cos \theta} + \tan \theta = \frac{1 + \sin \theta}{\cos \theta} \quad (5)$$

$$\frac{1 + \sin \theta}{\cos \theta} \times \frac{1 - \sin \theta}{1 - \sin \theta} = \frac{1 - \sin^2 \theta}{\cos \theta (1 - \sin \theta)} = \frac{\cos^2 \theta}{\cos \theta (1 - \sin \theta)}$$

$$= \frac{\cos \theta}{1 - \sin \theta} \text{ L.H.S}$$

$$(b) \frac{1 + \sin 2x}{\cos 2x} = 3 \cos 2x \rightarrow 1 + \sin 2x = 3 \cos^2 2x$$

$$1 + \sin 2x = 3(1 - \sin^2 2x)$$

$$3 \sin^2 2x + \sin 2x - 2 = 0 \quad (\text{quadratic } a=3, b=1, c=-2)$$

$$\sin 2x = \frac{-1 \pm \sqrt{1^2 + 4(3)(-2)}}{2} = \frac{2}{3} \text{ and } -1$$

\Rightarrow not included

$$2x = 41.8^\circ, 149^\circ, 138^\circ, 227^\circ$$

$$x = 20.9^\circ, 69.1^\circ$$



14. (i) A student states

"if x^2 is greater than 9 then x must be greater than 3"

Determine whether or not this statement is true, giving a reason for your answer.

(1)

(ii) Prove that for all positive integers n ,

$$n^3 + 3n^2 + 2n$$

is divisible by 6

(3)

(i) The statement is not true because

when $x = -4$ $x^2 = 16 > 9$ which is > 9 but
 $x < 3$

$$(ii) n^3 + 3n^2 + 2n = n(n^2 + 3n + 2) = n(n+1)(n+2)$$

this is the product of 3 consecutive integers, as

$n(n+1)(n+2)$ is a multiple of 2 and a multiple of 3, it must be a multiple of 6.

