

Mark Scheme (Results)

November 2021

Pearson Edexcel International GCSE Mathematics A (4MA1) Paper 1H

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General Marking Guidance

- All candidates must receive the same treatment. Examiners must mark the first candidate in exactly the same way as they mark the last.
- Mark schemes should be applied positively. Candidates must be rewarded for what they have shown they can do rather than penalised for omissions.
- Examiners should mark according to the mark scheme not according to their perception of where the grade boundaries may lie.
- There is no ceiling on achievement. All marks on the mark scheme should be used appropriately.
- All the marks on the mark scheme are designed to be awarded. Examiners should always award full marks if deserved, i.e. if the answer matches the mark scheme.
 - Examiners should also be prepared to award zero marks if the candidate's response is not worthy of credit according to the mark scheme.
- Where some judgement is required, mark schemes will provide the principles by which marks will be awarded and exemplification may be limited.
- When examiners are in doubt regarding the application of the mark scheme to a candidate's response, the team leader must be consulted.
- Crossed out work should be marked UNLESS the candidate has replaced it with an alternative response.

• Types of mark

- M marks: method marks
- A marks: accuracy marks
- B marks: unconditional accuracy marks (independent of M marks)

• Abbreviations

- cao correct answer only
- \circ ft follow through
- isw ignore subsequent working

- o SC special case
- o oe or equivalent (and appropriate)
- o dep dependent
- o indep independent
- o awrt answer which rounds to
- o eeoo each error or omission

• No working

If no working is shown then correct answers normally score full marks If no working is shown then incorrect (even though nearly correct) answers score no marks.

• With working

If there is a wrong answer indicated on the answer line always check the working in the body of the script (and on any diagrams), and award any marks appropriate from the mark scheme.

If it is clear from the working that the "correct" answer has been obtained from incorrect working, award 0 marks. If a candidate misreads a number from the question. Eg. Uses 252 instead of 255; method marks may be awarded provided the question has not been simplified. Examiners should send any instance of a suspected misread to review. If there is a choice of methods shown, mark the method that leads to the answer on the answer line; where no answer is given on the answer line, award the lowest mark from the methods shown.

If there is no answer on the answer line then check the working for an obvious answer.

• Ignoring subsequent work

It is appropriate to ignore subsequent work when the additional work does not change the answer in a way that is inappropriate for the question: eg. Incorrect cancelling of a fraction that would otherwise be correct. It is not appropriate to ignore subsequent work when the additional work essentially makes the answer incorrect eg algebra. Transcription errors occur when candidates present a correct answer in working, and write it incorrectly on the answer line; mark the correct answer.

• Parts of questions

Unless allowed by the mark scheme, the marks allocated to one part of the question CANNOT be awarded to another.

Intern	ational G	CSE Maths			
-	-	tions 6, 7, 13, and 18 (where the mark scheme states other a correct method.	wise), the correct an	swer, unless	clearly obtained by an incorrect method, should
	Q	Working	Answer	Mark	Notes
1	(a)		e^{6}	1	B1 cao
	(b)	$x^2 - 3x + x - 3$		2	M1 for any 3 correct terms
					or
					for 4 out of 4 correct terms ignoring signs
					or
					for $x^2 - 2x$
					or
					for $-2x - 3$
		Correct answer scores full marks (unless from	$x^2 - 2x - 3$		A1
		obvious incorrect working)			
					Total 3 marks

2	$30^2 + h^2 = 52^2$ oe or $900 + h^2 = 2704$		3	M1 for applying Pythagoras theorem correctly
	$(h^2 =) 52^2 - 30^2 (=1804)$ or			
	$(h^2 =) 2704 - 900 (= 1804)$			
	$(h=)\sqrt{52^2-30^2} (=\sqrt{1804}) (= 42.47352)$ or			M1 for square rooting
	$(h=)\sqrt{2704-900} (=\sqrt{1804}) (=42.47352)$			
	<i>Correct answer scores full marks (unless from obvious incorrect working)</i>	42.5		A1 awrt 42.5 or allow $2\sqrt{451}$
				Total 3 marks

3 (a)	$54 \div 9 \times 4$ oe or $\frac{4}{9} \times 54$ oe		2	M1 Allow 0.44(44) × 54 or $\frac{24}{54}$
	Correct answer scores full marks (unless from obvious incorrect working	24		A1
(b)	$\frac{"24"+n}{54+n} = \frac{1}{2} \text{ or } \frac{30}{60} \text{ or}$ 54 - "24" (= 30) and "30" - "24" or 2 × "30" - 54		2	M1 ft if "24" < 27 or $\frac{6}{60}$
	Correct answer scores full marks (unless from obvious incorrect working)	6		A1
				Total 4 marks

4	$2 \times 0.75 (= 1.5)$ oe or $2 \times 0.75 \times 2 (= 3)$ oe		5	M1 for area of rectangle
	$\pi \times (0.5 \div 2)^2 (= 0.1963)$ or			M1 for area of circle
	$\frac{1}{2} \times \pi \times (0.5 \div 2)^2 (= 0.09817)$			or area of semicircle
	"1.5" - "0.09817" (= 1.4018) or "3" - "0.1963" (= 2.8036)			M1
	"1.4018" $\times 2 \times 250 \div 4$ (= 175.228) or "2.8036" $\times 250 \div 4$ (= 175.228) or "1.4018" $\times 250 \div 4$ (= 87.6)			M1or for 87 – 88
	Correct answer scores full marks (unless from obvious incorrect working)	175		A1 Allow 175 – 176
				Total 5 marks

5	LW = 180 oe (9LW = 1620) or $4L \times (L + W) = 1620 \text{ oe or}$ $5W \times (L + W) = 1620 \text{ oe or}$ 5 4		5	M2 for any two correct equations from (i) $LW = 180$ oe $(9LW = 1620)$ (ii) $4L \times (L + W) = 1620$ oe
	$4L = 5W$ oe $(L = \frac{5}{4}W$ oe or $W = \frac{4}{5}L$ oe)			(iii) $5W \times (L + W) = 1620$ oe (iv) $4L = 5W$ oe $(L = \frac{5}{4}W$ oe or $W = \frac{4}{5}L$ oe)
	$L \times "\frac{4}{5}L" = "180"$ oe or $W \times "\frac{5}{4}W" = "180"$ oe or			(M1 for one correct equation or $1620 \div 9 (= 180)$)M1 for a correct equation in terms of one variableonly
	$4L \times \left(L + \frac{4}{5}L\right) = 1620 \text{ oe or}$			
	$5W \times \left(\frac{5}{4}W + W\right) = 1620 \text{ oe or}$			
	$9L\left("\frac{4}{5}L"\right) = 1620 \text{ oe or } 9\left("\frac{5}{4}W"\right)W = 1620 \text{ oe or}$ $(180)^{2}$			
	$4\left(\left\ \frac{180}{W}\right\ ^{2} + 4(\left\ 180\right\) = 1620 \text{ oe or}$			
	$5("180") + 5\left("\frac{180}{L}"\right)^2 = 1620 \text{ oe}$			
	Correct answer scores full marks (unless from abvious incorrect working)	L = 15 and		A2 for both correct (A1 for one correct)
	obvious incorrect working)	W = 12		Award 4 marks for $L = 12$ and $W = 15$ dep on M3
		=		Total 5 marks

Elir	nination				
6	5a+3p = 1.96 and 3a + 0 or $5a+3p = 196 and 3a + 0$ E.g.	-	M2 for an arithmetical method (must see the calculation to find 0.22 or 0.26 or 0.74 and 0.48 oe) E.g. 6 1(0) = 5.88 (= 0.22) or	5	 M1 for setting up both equations oe Allow the use of apples and pears oe throughout, e.g. 5 apples + 3 pears = 1.96 and 3 apples + 2 pears = 1.22 M1 for a correct method to
	15a+9p = 5.88 15a+10p = 6.1 0 Subtracting (-p=-0.22) E.g. 5a+3p=1.96 and Subtracting	10a+6p = 3.92 9a+6p = 3.66 Subtracting (a = 0.26) 6a+4p = 2.44 oe	6.1(0) - 5.88 (= 0.22) oe or 3.92 - 3.66 (= 0.26) oe or 1.96 - 1.22 (= 0.74) oe and $1.22 - "0.74" (= 0.48)$		eliminate <i>a</i> or <i>p</i> : coefficients of <i>a</i> or <i>p</i> the same and correct operation to eliminate selected variable (condone any one arithmetic error) or to find the cost of 1 apple and 1 pear
	E.g. 5a+3("0.22") = 1.96 or 3a+2("0.22") = 1.22 E.g a+p=0.48 oe	E.g. 5("0.26") + 3p = 196 or 3("0.26") + 2p = 1.22	E.g. $3 \times 0.22 (= 0.66)$ 1.96 - "0.66" (= 1.3(0)) " $1.3(0)" \div 5 (= 0.26)$ or $5 \times 0.26 (= 1.3(0))$ 1.96 - "1.3(0)" (= 0.66) " $0.66" \div 3 (= 0.22)$		M1 (dep on M2) for substituting their value found (must be > 0) of one variable into one of the equations or for repeating above method to find second variable or for third working column allow k(a+p) = k(0.48) or
		or $(a + p =) 0.48 \times 10$ oe or	or Apple and pear is 0.48 oe r $k(a+p) = k(0.48) \times \frac{10}{k}$	4.8(0)	for a complete arithmetical method to find the other value M1 (dep on M3) can be implied by 10(<i>a</i> + <i>p</i>) provided <i>a</i> and <i>p</i> must be > 0 A1 dep M2
					Total 5 marks

Substitutio	on and a second s			
6	5a+3p=1.96 and 3a+2p=1.22 oe or 5a+3p=196 and 3a+2p=122 oe		5	M1 for setting up both equations oe Allow the use of apples and pears oe throughout, e.g. 5 apples + 3 pears = 1.96 and 3 apples + 2 pears = 1.22
	E.g. $3\left(\frac{1.96-3p}{5}\right)+2p=1.22 \text{ or}$ $5\left(\frac{1.22-2p}{3}\right)+3p=1.96 \text{ or}$ $3a+2\left(\frac{1.96-5a}{3}\right)=1.22 \text{ or}$ $5a+3\left(\frac{1.22-3a}{2}\right)=1.96 \text{ or}$			M1 for correctly writing <i>a</i> or <i>p</i> in terms of the other variable and correctly substituting (condone any one arithmetic error)
	p = 0.22 or a = 0.26 E.g. $(a =) \frac{1.96 - 3(0.22)}{5} \text{ or } (a =) \frac{1.22 - 2(0.22)}{3} \text{ or}$ $(p =) \frac{1.96 - 5(0.26)}{3} \text{ or } (p =) \frac{1.22 - 3(0.26)}{2}$ $10 \times "0.26" + 10 \times "0.22"$ Working required	4.8(0)		 M1 (dep on M2) for substituting their value found (must be > 0) of one variable into one of the equations or for repeating above method to find second variable M1 (dep on M3) can be implied by 10(a + p) provided a and p must be > 0 A1 dep M2
				Total 5 marks

7	E.g. $2 \times 2 \times 900 \text{ or } 2^2 \times 900 \text{ or } 2 \times 3 \times 600 \text{ or}$ $2 \times 5 \times 360 \text{ or } 3 \times 3 \times 400 \text{ or } 3^2 \times 400 \text{ or}$ $3 \times 5 \times 240 \text{ or } 5 \times 5 \times 144 \text{ or } 5^2 \times 144$ E.g. E.g. E.g. 2 3600 2 1800 2 900		3	M1 for at least 2 correct stages in prime factorisation which give 2 prime factors – may be in a factor tree or a table or listed eg 2, 2, 900 (see LHS for examples of the amount of work needed for the award of this mark, allow no more than one mistake ft in factor tree or table (eg one mistake with 2 prime factors ft: $3600 = 1800 \times 20 = 2 \times 900 \times 4 \times 5$ or $360 = 2 \times 2 \times 90$)
	E.g. $2 \times 2 \times 2 \times 2 \times 3 \times 3 \times 5 \times 5$ E.g. E.g. E.g. E.g. $2 \times 3 \times 3 \times 5 \times 5$ E.g. $3 \times 5 \times 5 \times 5 \times 5$ E.g. $3 \times 5 \times 5 \times 5 \times 5$ E.g. $3 \times 5 \times 5 \times 5 \times 5 \times 5$ E.g. $3 \times 5 \times $			M1 for 2, 2, 2, 2, 3, 3, 5, 5 or 2^4 , 3^2 , 5^2 or $2^4 + 3^2 + 5^2$ (ignore 1s) (may be a fully correct factor tree or ladder)
	Working required	$2^4 \times 3^2 \times 5^2$		A1 dep on M2 can be any order (allow $2^4 \cdot 3^2 \cdot 5^2$) (SCB1 for $3.6 \times 2^3 \times 5^3$)
				Total 3 marks

8	$\begin{array}{c} 0.22x = 5.48 \text{ oe or} \\ (1\% =) 5.48 \div 22 \ (= 0.24909) \text{ or} \\ 100 \div 22 \ (= 4.54) \end{array}$		M1
	$(x =) 5.48 \div 0.22$ oe or $(100\% =) 5.48 \div 22 \times 100$ or $"0.24909" \times 100$ or $5.48 \times "4.54"$		M1
	Correct answer scores full marks (unless from obvious incorrect working)	24.9	A1 awrt 24.9
			Total 3 marks

8	$0.22x = 5\ 480\ 000$ oe or		M1
ALT	$(1\% =) 5\ 480\ 000 \div 22\ (=\ 249\ 090.9091)$ or		
1	$100 \div 22 (= 4.54)$		
	5 480 000 ÷ "0.22" oe or		M1
	$(100\% =) 5\ 480\ 000 \div 22 \times 100\ or$		
	"249 090.9091…"× 100 or		
	5 480 000 × "4.54…"		
	Correct answer scores full marks (unless from obvious incorrect working)	24 900 000	A1 awrt 24 900 000
			Total 3 marks

9 (i)	$-7+3 \le 2x < 5+3 \text{ oe or} \frac{-7}{2} \le x-\frac{3}{2} < \frac{5}{2} \text{ oe or} -7+3 \le 2x \text{ oe and } 2x < 5+3 \text{ oe} \text{ or } (x =) -2 \text{ or } (x =) 4$		3	M1 or one side of the inequality correct, i.e., $x \ge -2$ oe or $x < 4$ Condone = rather than \le or $<$ or any other sign for the M marks.
	$\frac{-7+3}{2} \le x < \frac{5+3}{2} \text{ or}$ $\frac{-7}{2} + \frac{3}{2} \le x < \frac{5}{2} + \frac{3}{2}$ or $\frac{-7+3}{2} \le x \text{ oe and } x < \frac{5+3}{2}$ or $(x =) -2 \text{ and } (x =) 4$			M1
	Correct answer scores full marks (unless from obvious	$-2 \leq x < 4$		A1 allow $x \ge -2$ and $x < 4$
	incorrect working)		ļ	Allow [-2, 4)
(ii)	$\begin{array}{c ccccccccccccccccccccccccccccccccccc$		2	M1 ft for drawing a line from -2 to 4 or (indep) for a closed circle or [at -2 or (indep) for an open circle or) or [at 4 Only allow a follow through for a double ended inequality in (i)
		Correct		A1 ft for correct diagram
		diagram		Only allow a follow through for a double ended inequality in (i)
				Total 5 marks

10	$0.0027 = \frac{5.4}{(V)}$ oe		5	M1 for correctly using density = $\frac{\text{mass}}{\text{volume}}$
	$(V=)\frac{5.4}{0.0027} = 2000$			M1 for correctly rearranging for V
	$\pi \times 10^2 \times h = 2000 \mathrm{oe}$			M1ft their 2000 for $\pi \times 10^2 \times h =$ their V
	$(h=)\frac{2000}{\pi \times 10^2}$ oe (= 6.3661)			M1ft their 2000 dep on previous M1 for correctly rearranging for h
	<i>Correct answer scores full marks (unless from obvious incorrect working)</i>	6.4		A1 awrt 6.4
				Total 5 marks

11	(a)		(12), 36, 64, 76, 86, 90	1	B1
	(b)			2	M1 ft from table for at least 5 points plotted correctly (± 0.5 squares) at end of interval
					or ft from (CF) table for all 6 points plotted consistently (± 0.5 squares) within each interval in the freq table at the correct height
			Correct cf diagram		A1 accept curve or line segments accept graph that is not joined to (25, 0)
	(c)	E.g. reading at 42 minutes and reading at 52 minutes		2	M1 for correct use of 42 and 52, ft from a cum freq graph provided method is shown – e.g. a line vertically drawn to the graph from readings of 42 and 52 on the Time axis to meet the graph and then a horizontal line to the CF axis (even if wrongly read scale) or clear marks on the graph and CF axis that correspond to the correct readings or correct values from the CF axis
		Correct answer scores full marks (unless from obvious incorrect working)	25 – 29		A1 ft Accept a single value in range 25 to 29 or ft from their cumulative frequency graph provided method is shown
					Total 5 marks

12 (a)	$\tan 20 = \frac{100}{d} \text{ oe or } \tan (90 - 20) = \frac{d}{100} \text{ oe or}$ $\frac{d}{\sin (90 - 20)} = \frac{100}{\sin 20} \text{ oe}$		3	M1
	$(d =) \frac{100}{\tan 20} (= 274.747) \text{ or } (d =) 100 \times \tan(90 - 20) (= 274.747) \text{ or}$ $(d =) \frac{100}{\sin 20} \times \sin(90 - 20) (= 274.747)$			M1
	<i>Correct answer scores full marks (unless from obvious incorrect working)</i>	275		A1 awrt 275
(b)	$\tan 25 = \frac{100+h}{275} \text{ oe or } \tan 25 = \frac{y}{275} \text{ oe or}$ $275 \times \tan 25 (=128) \text{ or}$ $\tan(90-25) = \frac{275}{100+h} \text{ oe or } \tan(90-25) = \frac{275}{y} \text{ oe or}$ $\frac{100+h}{\sin 25} = \frac{275}{\sin(90-25)} \text{ or } 128.1 - 128.2$ (y is the height of cliff and radio mast)		3	M1 ft part (a) Allow $(hyp =)\sqrt{100^2 + 275^2}$ or $(=\sqrt{85486.321} = 292.380)$ $(hyp =)\frac{100}{\sin 20} \times \sin 90 (= 292.380)$
	$(h=)275 \times \tan 25 - 100 = 28.1169 \text{ or}$ $(h=)\frac{275}{\tan 90 - 25} - 100 (= 28.1169) \text{ or}$ $(h=)\frac{275}{\sin(90 - 25)} \times \sin 25 - 100 (= 28.1169)$			M1 ft part (a) $(h=)\frac{"292.380"}{\sin(90-25)} \times \sin(25-20)$ (= 28.1169)
	Correct answer scores full marks (unless from obvious incorrect working)	28.1		A1 Accept answers in the range 28 – 28.2 Total 5 marks

13	15.5 or 16.5 or 24.5 or 25.5 or 125 or 135		3	B1 Accept 16.49 for 16.5
				25.49 for 25.5 134.9 for 135
	$\frac{(YZ)}{\sin(125)} = \frac{16.5}{\sin(24.5)}$ oe			$\frac{(YZ)}{UB_1} = \frac{UB_1}{UB_1}$ oe where
				$\frac{16 < UB_1 \le 16.5 \text{ and}}{16 \le UB_1 \le 16.5 \text{ and}}$
				$125 \le LB_2 < 130$ and $24.5 \le LB_3 < 25$
	Working required	32.6		A1 Accept 32.5 to 32.6 from correct working
				Total 3 marks

14	(a)(i)		b – a	1	B1 oe
	(ii)	E.g. $(KI = KJ + JI =)$		2	M1ft (i) for any valid correct path (oe) in
		2(b-a) + 2b oe			capitals or lower case letters
		Correct answer scores full marks (unless from	$4\mathbf{b} - 2\mathbf{a}$		A1 oe simplified
		obvious incorrect working)			
	(iii)	E.g. $(LD = LF + FE + ED =)$		2	M1ft (i) for any valid correct path (oe) in
		(b - a) + (b - a) - a oe			capitals or lower case letters
		Correct answer scores full marks (unless from	2 b – 3 a		A1 oe simplified
		obvious incorrect working)			
	(b)	$(GHIJKL =) 6 \times 5 \times 2^2 (= 120)$ or		3	M1
		$(GABH =) 5 \times 2^2 - 5 (= 15) \text{ or } 3 \times 5 (= 15) \text{ or}$			
		(Number of triangles in shaded region =)			
		$(6 \times 4) - 6 (= 18)$			
		" 120 " – (6 × 5) or			M1
		$6 \times "15"$ or			
		"18" × 5			
		Correct answer scores full marks (unless from	90		A1
		obvious incorrect working)			
					Total 8 marks

15 (a	1)	$\frac{\frac{3}{9}}{\frac{2}{9}, \frac{4}{9}, \frac{3}{9}}$	2	 B1 for lower 1st game branch probability B1ft for all values correct on 2nd game branches
(t	($\frac{2}{9} \times \frac{3}{9}$) or $\left(\frac{3}{9} \times \frac{2}{9}\right)$ or $\left(\frac{4}{9} \times \frac{4}{9}\right)$ oe or		3	M1 ft from their tree diagram for one correct product from <i>WL</i> or <i>L W</i> or <i>DD</i> (allow probabilities to 2 dp truncated or rounded)
	$\left(\frac{2}{9} \times \frac{3}{9}\right) + \left(\frac{3}{9} \times \frac{2}{9}\right) + \left(\frac{4}{9} \times \frac{4}{9}\right) \text{ oe}$			M1 ft for a fully correct method
	Correct answer scores full marks (unless from obvious incorrect working)	$\frac{28}{81}$		A1 Allow 0.345 (2 dp truncated or rounded) or 34.5% (2 sf truncated or rounded)
(c	$\left(\frac{2}{9} \times \frac{4}{9} \times \frac{3}{9}\right) \text{ or } \left(\frac{4}{9} \times \frac{4}{9} \times \frac{4}{9}\right)$		3	M1ft from their tree diagram for any combination of <i>WLD</i> or <i>DDD</i> (allow probabilities to 2 dp truncated or rounded)
	$6 \times \left(\frac{2}{9} \times \frac{4}{9} \times \frac{3}{9}\right) + \left(\frac{4}{9} \times \frac{4}{9} \times \frac{4}{9}\right)$			M1ft for a fully correct method
	Correct answer scores full marks (unless from obvious incorrect working)	$\frac{208}{729}$		A1 Allow 0.285 (2 dp truncated or rounded) or 28.5% (2 sf truncated or rounded)
				Total 8 marks

15 (a) ALT		$\frac{\frac{3}{9}}{\frac{2}{9}, \frac{4}{9}, \frac{3}{9}}$	2	B1 for lower 1 st game branch probability B1ft for all values correct on 2 nd game branches
(b)	1 and $\left(\frac{2}{9} \times \frac{2}{9}\right)$ or $\left(\frac{4}{9} \times \frac{2}{9}\right)$ or $\left(\frac{4}{9} \times \frac{3}{9}\right)$ or $\left(\frac{3}{9} \times \frac{3}{9}\right)$ oe		3	M1ft from their tree diagram for 1 and one correct product from WW, DW, DL or LL (allow probabilities to 2 dp truncated or rounded)
	$1 - \left[\left(\frac{2}{9} \times \frac{2}{9}\right) + 2\left(\frac{4}{9} \times \frac{2}{9}\right) + 2\left(\frac{4}{9} \times \frac{3}{9}\right) + \left(\frac{3}{9} \times \frac{3}{9}\right) \right] \text{oe}$			M1ft for a fully correct method
	Correct answer scores full marks (unless from obvious incorrect working)	$\frac{28}{81}$		A1 Allow 0.345 (2 dp truncated or rounded) or 34.5% (2 sf truncated or rounded)
(c)	$1 \text{ and } \left(\frac{2}{9} \times \frac{2}{9} \times \frac{2}{9}\right) \text{ or } \left(\frac{2}{9} \times \frac{2}{9} \times \frac{4}{9}\right) \text{ or } \left(\frac{2}{9} \times \frac{2}{9} \times \frac{3}{9}\right) \text{ or } \left(\frac{2}{9} \times \frac{4}{9} \times \frac{4}{9}\right) \text{ or } \left(\frac{2}{9} \times \frac{3}{9} \times \frac{3}{9}\right) \text{ or } \left(\frac{4}{9} \times \frac{3}{9} \times \frac{3}{9}\right) \text{ or } \left(\frac{4}{9} \times \frac{3}{9} \times \frac{3}{9}\right) \text{ or } \left(\frac{3}{9} \times \frac{3}{9} \times \frac{3}{9}\right) \text{ oe }$		3	M1ft from their tree diagram for 1 and one correct product from WWW or WWD or WWL or WDD or WLL or DDL or DLL or LLL (allow probabilities to 2 dp truncated or rounded)
	$1 - \begin{bmatrix} \left(\frac{2}{9} \times \frac{2}{9} \times \frac{2}{9}\right) + 3\left(\frac{2}{9} \times \frac{2}{9} \times \frac{4}{9}\right) + 3\left(\frac{2}{9} \times \frac{2}{9} \times \frac{3}{9}\right) + 3\left(\frac{2}{9} \times \frac{4}{9} \times \frac{4}{9}\right) \\ + 3\left(\frac{2}{9} \times \frac{3}{9} \times \frac{3}{9}\right) + 3\left(\frac{4}{9} \times \frac{4}{9} \times \frac{3}{9}\right) + 3\left(\frac{4}{9} \times \frac{3}{9} \times \frac{3}{9}\right) + 3\left(\frac{4}{9} \times \frac{3}{9} \times \frac{3}{9}\right) + \left(\frac{3}{9} \times \frac{3}{9} \times \frac{3}{9}\right) \end{bmatrix} $ oe			M1ft for a fully correct method
	Correct answer scores full marks (unless from obvious incorrect working)	$\frac{208}{729}$		A1 Allow 0.285 (2 dp truncated or rounded) or 28.5% (2 sf truncated or rounded)
				Total 8 marks

16 (a)	(11 - x) + (x) + (18 - x) + 3 = 25 oe or (11 - x) + (x) + (18 - x) + 3 + 7 = 25 + 7 oe or x + y + z = 25 - 3 and $x + z = 11$ and $x + y = 18$ oe where $y = M \cap E \cap F'$ and $z = M \cap F \cap E'$		2	M1 for setting up a correct equation
	Correct answer scores full marks (unless from obvious incorrect working)	7		A1 (allow 7 in the Venn diagram if no answer is given in (a))
(b)	$\begin{array}{c c} & & & \\ & & & & \\ & & & & \\ & & & \\ & & & & & \\ & & & & \\ & & & & \\ & & & & & \\ & &$		2	B2 ft for $18 - x$, x and $11 - x$ dep on M1 in part (a) and (x < 12) (NB 0, 1 and 4 are fixed) for 6 correct remaining values (B1 ft for 4 or 5 correct remaining values) Allow just <i>E</i> to be blank if other sections are populated with a number
(c)	$\frac{3+"11"}{25}$ or $\frac{3+(18-"7")}{25}$ or 0.56 oe	$\frac{14}{25}$	1	B1ft for $18 - x$, <i>x</i> and $11 - x$ oe
				Total 5 marks

17 (a)	6y(y-1) + 5(y-1) or y(6y+5) - 1(6y+5)		2	M1 for $(6y \pm 5)(y \pm 1)$ or $(6y \pm 1)(y \pm 5)$ or $(ay + 5)(by - 1)$ where $ab = 6$ or 5b - a = -1 or $(6y + p)(y + q)$ where $pq = -5$ or 6q + p = -1 Condone use of a different letter to y
	Correct answer scores full marks (unless from obvious incorrect working)	(6y + 5)(y - 1)		A1 oe
(b)	8w - fw = 2f + 3 oe		3	M1 for multiplying by denominator and expanding in a correct equation
	8w-3 = 2f + fw oe or -2f - fw = 3 - 8w oe			M1 for gathering terms in f on one side and other terms the other side in a correct equation ft their equation dep on 2 terms in f and two other terms
	Correct answer scores full marks (unless from obvious incorrect working)	$f = \frac{8w - 3}{2 + w}$		A1 oe accept $f = \frac{3 - 8w}{-2 - w}$ oe
(c)	$4(x^2-2x)+7 \text{ or } 4\left(x^2-2x+\frac{7}{4}\right) \text{ oe}$		3	M1
	$4[(x-1)^2-1^2]+7$ or $4[(x-1)^2-1^2+\frac{7}{4}]$ or			M1 for a complete method
	Correct answer scores full marks (unless from obvious incorrect working)	$4(x-1)^2+3$		A1 allow $a = 4, b = -1$ and $c = 3$
				Total 8 marks

17 (c)	$ax^2+2bax+b^2a+c$		3	M1 for correctly expanding $a(x+b)^2 + c$
ALT				to give $ax^2 + 2bax + b^2a + c$
	$2ba = -8$ and $b^2a + c = 7$			M1 for a complete method (equating
				coefficients)
	Correct answer scores full marks (unless from obvious incorrect working)	$4(x-1)^2+3$		A1 allow $a = 4$, $b = -1$ and $c = 3$

18	E.g.		3	M1 for selecting 2 correct recurring
	y = 0.4x and $10y = 4.x$			decimal expressions and then a
	(10y - y = 4.x - 0.4 oe)			demonstration to subtract
	or			(If recurring dots not shown then allow
	10y = 4.x and $100y = 4x.x$			each expression to 1 dp
	(100y - 10y = 4x - 4 oe)			e.g. $y = 0.4x$ and $100y = 4x.x$ as a
	or			pair and $100y - y$ or $4x.x 0.4x$)
	100y = 4x.x and $1000y = 4xx.x$			or an answer of $y = \frac{4x - 4}{90}$ oe
	(1000y - 100y = 4xx - 4x oe)			of an answer of $y = \frac{90}{90}$ of
	E.g.			M1 for a correct subtraction with correct
	$9y = 4\frac{x}{10} - \frac{4}{10} = \frac{40 + x - 4}{10}$ oe or			expressions simplified
	90y = 40 + x - 4 oe or			
	900y = 400 + 10x + x - 40 - x oe			
	Working required	36+x		A1 dep on M2 oe
		90		
				Total 3 marks

19 (a)	E.g. $x + y + x + y + x = 100$ oe or $3x + 2y = 100$ oe $\left(y = \frac{100 - 3x}{2}\right)$ E.g. E.g. E.g. $x^2 = \left(\frac{x}{2}\right)^2 + h^2$ and $\left(=\frac{1}{2} \times x \times x \times \frac{\sqrt{3}}{2}\right)$ $\left(=\frac{x^2\sqrt{3}}{4}\right)$ $\left(=\frac{x^2\sqrt{3}}{4}\right)$		3	M1 for a correct equation for the perimeter of the shape or for a correct expression for the area of triangle <i>CED</i>
	$x''\left(\frac{100-3x}{2}\right)''+''\frac{x^2\sqrt{3}}{4}''$ oe			M1 for the area of the shape in terms of x only
	E.g. $x\left(\frac{200-6x}{4}\right) + \frac{x^2\sqrt{3}}{4}$ or $\frac{x}{4}(200-6x+x\sqrt{3})$ or $\frac{200x-6x^2}{4} + \frac{x^2\sqrt{3}}{4}$ or $\frac{x}{4}(200x-6x^2+x^2\sqrt{3})$	Shown		A1 for the area given in correct form with full working shown (at least one intermediate step before the answer)
(b) (i)	$\left(\frac{\mathrm{d}R}{\mathrm{d}x}\right) = \int 50 - \frac{3}{2} \times 2 \times x + 2 \times \frac{x\sqrt{3}}{4} = 0 \text{ oe}$		2	M1 for differentiation of correct expression with 2 out of 3 terms correct and equated to 0 (can be implied by subsequent working)
	<i>Correct answer scores full marks (unless from obvious incorrect working)</i>	$\frac{100}{6-\sqrt{3}}$		A1 for a correct expression
(ii)		Correct reason	1	B1 for correct reason <i>R</i> is a quadratic with negative coefficient of x^2 E.g. the graph of <i>R</i> is \cap shaped or (allow $\frac{d^2 R}{dx^2} < 0$ oe)
				Total 6 marks

20	$\left(\frac{-6+5}{2}, \frac{2+3}{2}\right) = \left(-\frac{1}{2}, \frac{5}{2}\right)$ oe		7	M1 for finding the midpoint of <i>AB</i>
	$\frac{2-3}{-6-5} \left(= \frac{-1}{-11} = \frac{1}{11} \right) \text{oe}$			M1 for finding the gradient of <i>AB</i>
	$\frac{1}{11} = -1$ or $(m =) -11$			M1ft their gradient of <i>AB</i> (indep) for the correct use of $m_1 \times m_2 = -1$
	$"\frac{5}{2}" = "-11" \left("-\frac{1}{2}"\right) + c \text{ oe or } y - "\frac{5}{2}" = "-11" \left(x - "-\frac{1}{2}"\right)$			M1 for an expression that gives the y value at C
	and			
	$(y=)$ "-11"(-1)-3(=8) or $(y=)$ "-11" $\left(-1-$ "- $\frac{1}{2}$ " $\right)$ +" $\frac{5}{2}$ "(=8)			
See alt methods	(Perp =) $\sqrt{\left(8 - \frac{5}{2}\right)^2 + \left(-1 - \frac{1}{2}\right)^2} \left(=\frac{\sqrt{122}}{2}\right)$ and			M1
	$(AB=)\sqrt{3-2^2+5-6^2} (=\sqrt{122})$			
	(Area of triangle =) $\frac{1}{2} \times \sqrt{122} \times \frac{\sqrt{122}}{2}$			M1 for a complete method
	Correct answer scores full marks (unless from obvious	30.5		A1 oe
	incorrect working)			Allow answers in the range $30.4 - 30.5$
				Total 7 marks

Q20 Alternative ways of finding the area of the triangle (final 2 M marks)			
Alt1	$(11 \times 6) - (0.5 \times 1 \times 11) - (0.5 \times 5 \times 6) - (0.5 \times 5 \times 6)$		M1 for any 3 correct triangles
	$(11 \times 6) - (0.5 \times 1 \times 11) - (0.5 \times 5 \times 6) - (0.5 \times 5 \times 6)$		M1 for a complete method
	Correct answer scores full marks (unless from obvious incorrect working)		
Alt 2	$(AC = BC =)\sqrt{5^2 + 6^2} (= \sqrt{61})$		M1 for AC is perp to BC
	(Area of triangle =) $\frac{1}{2} \times \sqrt{61} \times \sqrt{61}$		M1 for a complete method
	Correct answer scores full marks (unless from obvious incorrect working)		
Alt 3	$\sqrt{\left(8 - \frac{5}{2}\right)^2 + \left(-1 - \frac{1}{2}\right)^2} \left(= \frac{\sqrt{122}}{2}\right) \text{ and } (AM =) \sqrt{\left(-6 - \frac{1}{2}\right)^2 + \left(2 - \frac{5}{2}\right)^2} \left(= \frac{\sqrt{122}}{2}\right)$		M1 for the height of the triangle, AM and BM where M is the midpoint of AB
	or $(BM =) \sqrt{\left(5\frac{1}{2}\right)^2 + \left(3 - \frac{5}{2}\right)^2} \left(= \frac{\sqrt{122}}{2}\right)$		
	(Area of triangle =) $2 \times \frac{1}{2} \times \frac{\sqrt{122}}{2} \times \frac{\sqrt{122}}{2}$		M1 for a complete method
	Correct answer scores full marks (unless from obvious incorrect working)		
Alt 4	$(AC = BC =)\sqrt{5^2 + 6^2} (=\sqrt{61}) \text{ and } (AB =)\sqrt{3 - 2^2 + 5 - 6^2} (=\sqrt{122})$		M1 for finding <i>AC</i> , <i>BC</i> and <i>AB</i>
	$\sqrt{\frac{\sqrt{122} + 2\sqrt{61}}{2}} \left(\frac{\sqrt{122} + 2\sqrt{61}}{2} - \sqrt{122}\right) \left(\frac{\sqrt{122} + 2\sqrt{61}}{2} - \sqrt{61}\right) \left(\frac{\sqrt{122} + 2\sqrt{61}}{2} - \sqrt{61}\right)$		M1 for applying Heron's formula
	Correct answer scores full marks (unless from obvious incorrect working)		