GCE

Further Mathematics A

Y542/01: Statistics

Advanced GCE

Mark Scheme for Autumn 2021

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This mark scheme is published as an aid to teachers and students, to indicate the requirements of the examination. It shows the basis on which marks were awarded by examiners. It does not indicate the details of the discussions which took place at an examiners' meeting before marking commenced.

All examiners are instructed that alternative correct answers and unexpected approaches in candidates' scripts must be given marks that fairly reflect the relevant knowledge and skills demonstrated.

Mark schemes should be read in conjunction with the published question papers and the report on the examination.

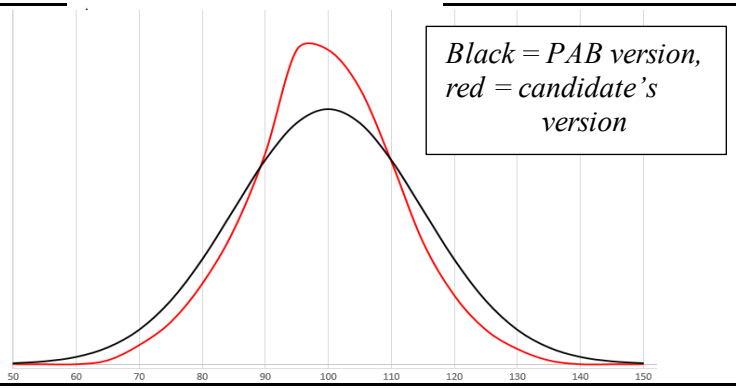
Annotations and abbreviations

Annotation in RM assessor	Meaning
✓and ✗	
BOD	Benefit of doubt
FT	Follow through
ISW	Ignore subsequent working
M0, M1	Method mark awarded 0, 1
A0, A1	Accuracy mark awarded 0, 1
B0, B1	Independent mark awarded 0, 1
SC	Special case
^	Omission sign
MR	Misread
BP	Blank Page
Seen	
Highlighting	
Other abbreviations in mark scheme	**Meaning**
dep*	Mark dependent on a previous mark, indicated by *. The * may be omitted if only one previous M mark
cao	Correct answer only
oe	Or equivalent
rot	Rounded or truncated
soi	Seen or implied
www	Without wrong working
AG	Answer given
awrt	Anything which rounds to
BC	By Calculator
DR	This question included the instruction: In this question you must show detailed reasoning.

Question			Answer	Marks	AO	Guidance	
1	(a)		$y = 52.7 + 0.251x$	**B1***	1.1	a in range [0.250, 0.251]	
				B1*	1.1	b correct to 3 SF	
				depB1	1.1	Completely correct including letters	
				[3]		SC: Correct formulae used for a and b: M1(A1)A1	
1	(b)		This quantity is minimised to find best-fit line	**B1**	2.4	Need "minimised" or "this is its minimum value" OE	
				[1]			
1	(c)		$y' = 11.5 + 0.139x$	**M1**	1.1	Apply inverse formula at least once	
			[$y' = \frac{5}{9} \times$ (*their* $a - 32$) + $\frac{5}{9} \times$ *their* b]	**A1ft**	1.1	All correct, any letters, ft on their y	
				[2]			
2			E(D) = 2×0.1 + 4×0.3 + 6×0.2	**M1**	2.1	NB: a is not needed by this method	Or change 0, 2, 4, 6 to 4, 10, 16, 22 and find a
			= 2.6	**A1**	1.1		
			E(D^2) = 2^2×0.1 + 4^2×0.3 + 6^2×0.2 [= 12.4]	**M1**	1.1	Or $\Sigma(x-\mu)^2p(x)$	
			Var(D) = 12.4 – 2.6^2	**M1**	1.1	$\Sigma p^2 d$ oe gets max M1A1M0M1M1	
			= 5.64	**A1**	1.1		
			Var(3D + 4) = 9×Var(D)	**M1**	3.1a	Allow even if their Var(D) < 0	
			= 50.76	**A1**	3.4	SC: $\Sigma(x-\mu)^2p(x)$: M1A1, a = 0.4 M1	
				[7]		M1(use this formula), A1M1A1	
3	(a)	(i)	P($X \geq 5$) – P($X \geq 11$) = $0.7^4 - 0.7^{10}$	**M1**	3.1b	Allow 1 term wrong at either end	Or $pq^4 + \ldots + pq^9$
			= 0.212	**A1**	3.4	awrt 0.212	
				[2]			
		(ii)	$0.7^{n-1} < \frac{1}{3}$, or 0.103 > 0.1 > 0.072	**M1**	2.1	Solve $0.3 \times 0.7^{n-1} = 0.1$ or < 0.1, allow inequality error	
			$n_{min} = 5$	**A1**	1.1	5 only	
				[2]		**SC**: 5 without sufficient justification: B1	
3	(b)		$\frac{1-p}{p^2} = 42 \Rightarrow 42p^2 + p - 1 = 0$	**M1**	3.1a	Equate correct variance formula to 42	
				A1	1.1	Correct simplified quadratic equation	
			$p = \frac{1}{7}$	**A1**	2.2a		
			Explicitly reject $p = -\frac{1}{6}$	**A1**	2.3	SC: if $-\frac{1}{7}$ and $\frac{1}{6}$, allow A1 for explicitly rejecting $-\frac{1}{7}$	
			E(X) = 7	**A1**	2.1		

Question			Answer	Marks	AO	Guidance
				[5]		
4	**(a)**	**(i)**	$\hat{\mu} = \bar{x} = 16.8$	**B1** **[1]**	1.1	Or exact equivalent
		(ii)	$\frac{48398}{160} - 16.8^2$ [= 20.2475] $\times \frac{160}{159}$ = 20.3748…	**M1** **M1** **A1** **[3]**	1.1 1.1 1.1	If single formula used, full marks if correct; M0M1 if wrong but divisor 159 seen anywhere Awrt 20.4, www
4	**(b)**		$\bar{x} \pm z\sqrt{\sigma^2/160}$ $z = 2.576$ (15.88, 17.72)	**M1** **A1** **A1** **[3]**	3.3 1.1 3.4	Any z from Φ^{-1}, 160 needed, allow √ errors Or better, e.g. 2.575829 Both, 4 sf required by question, www (NB: $\sigma^2 = 20.2475$ gives same end-points to 4 SF but this gets M1A1A0)
4	**(c)**	**(i)**	Not needed in **(a)** as E(X) and Var(X) are independent of the distribution	**B1** **[1]**	2.4	Mention at least one of E(X) and Var(X) explicitly, or "not relevant to $\bar{X}$"
		(ii)	Needed in **(b)** as parent distribution not stated to be normal	**B1** **[1]**	2.4	Must make it clear that two distributions are involved. "n is large" etc: B0

Question			Answer	Marks	AO	Guidance
5	(a)		The value of Pearson's pmcc would be changed by (most) such changes. The value of Spearman's r_s would not be changed as the ranks remain unchanged.	**B1** **B1** **[2]**	2.5 2.5	Explain effect on Pearson, *or* not known bivariate normal *or* not testing for *linear* correlation Explain why no effect on Spearman (not "not *likely to be* affected", or "not *much* affected" or "association not correlation"
5	(b)		H_0: no association between ranks of numbers of items H_1: (positive) association between ranks Ranks 1 2 3 4 5 6 7 8 9 4 1 3 2 8 5 9 7 6 $\Sigma d^2 = 38$ $r_s = 1 - \frac{6\Sigma d^2}{9(9^2-1)}$ $= 0.683$ < 0.700 Do not reject H_0. Insufficient evidence of association between rankings of the items	**B1** **M1** **A1** **M1** **A1** **B1** **M1ft** **A1ft** **[8]**	1.1 1.1 1.1 1.2 1.1 1.1 1.1 2.2b	Don't insist on "population" here, but allow use of ρ_s in both, even if no explanation (not just r_s). Context needed, but don't worry about 1- or 2-tailed here Compare TS ($-1 \le$ TS ≤ 1) with 0.7, independent ft on TS provided correct formula used, or on CV 0.600 In context, not too positive. FT on TS only SC: 0.600 (2-tailed): B0 M1A0
6	(a)		H_0: Data consistent with N(100, 15^2) H_1: Data not consistent with N(100, 15^2)	**B1** **[1]**	1.1	Allow: "follows N(100, 15^2)" or "can be modelled by". Parameters not needed. No other alternatives seen!
6	(b)		$P(100 \le X < 110) = 0.2475$ **BC** Expected frequency = 500×0.2475 [= 123.754] $\frac{(129-123.754)^2}{123.754}$ [= 0.222…, **AG**]	 **B1** **M1** **A1** **[3]**	 3.4 2.1 2.2a	 Probability needs to be seen Sufficient working to justify **AG**, needs 123.754 at least

Question			Answer	Marks	AO	Guidance
6	(c)		$\Sigma X^2 = 10.5$ $\chi^2(4) = 9.488$ and $10.5 > 9.488$ Reject H_0. Significant evidence that data is not consistent with N(100, 15^2).	**B1** **B1** **M1ft** **A1ft** **[4]**	1.1 1.1 1.1 2.2b	 Like-with-like comparison needed FT on TS or CV here. Needn't be stated if next line right FT on TS (but not CV) if method correct. Wrong CV, e.g. 5.991: B1B0M1A0. No ft on H_0/H_1
6	(d)	(i)	E.g. Too few in $X \geq 110$ or in $X \leq 80$, or too many in others, or data truncated, etc	**B1** **[1]**	3.5b	Any relevant point, needn't refer to values of X^2 "Divide into 5 minute groups": B1. "Data discrete": B0. "The variance" (uncalculated): B0
		(ii)		**B1** **B1** **[2]**	3.3 3.5c	Deal with aspect identified in (i) Basically correct, areas roughly same *Examples:* Uses "data discrete" in (i) B0 More below 100, so translate to left B2 More above 110 so translate to right B2 Divide into 5-minute groups B0 Variance changed, areas not equal B1 Data truncated but worse truncation shown B0
7	(a)		H_0: Two samples are from identical populations H_1: Two samples are from populations with different median ratings. $R_m = 1 + 2 + 3 + 4 + 5 + 9 + 10 + 11$ (= 45) $W = 45$ $8(8 + 8 + 1) - R_m = 91$ $W_{crit} = 49$ Reject H_0. Significant evidence that there is a difference in median ratings/opinions have changed	**B2** **M1** **A1** **B1** **B1** **M1ft** **A1ft** **[8]**	1.1 1.1 1.1 1.1 2.1 1.1 1.1 2.2b	If no reference to "populations", maximum B1 Allow H_0: "identical population medians", H_1: "not identical populations" or "not identical pop medians" "Pupils' opinions have not changed", etc: B2 If omitted, can still get all other marks FT on TS (< 68) or CV FT on TS only. Allow "increased" SC: Sign or paired-sample test, max B2 (hypotheses)
7	(b)		Eliminate the difference between individual pupils' opinions	**B1** **[1]**	3.5b	"Minimises the difference in tastes" B1 (BOD) Scores arbitrary: B1 (etc). Not "more powerful test".
7	(c)		A paired-sample signed-rank test would have been used	**B1**	3.5c	Must mention "paired sample" oe – not just "Wilcoxon

Question			Answer	Marks	AO	Guidance
				[1]		signed rank"
7	(d)		0.025 × 12870 = 322	M1 A1 [2]	3.1a 3.2a	0.05 × 12870 = 643.5 M1 321 or 322 or 643 (from 1-tail), must be integer

Question			Answer	Marks	AO	Guidance
8	(a)		$f(x) = ½$	**B1**	3.3	Stated or implied, e.g. on diagram
			$\int_0^2 \frac{1}{2} a\cos(ax)\mathrm{d}x = 0.3$	**M1**	3.1a	$\int f(x)\ a\cos ax\ \mathrm{d}x$ & equated to 0.3
			$\left[\frac{1}{2}\sin(ax)\right]_0^2$	**B1**	1.1	Correct indefinite integral
			$\frac{1}{2}\sin(2a) = 0.3$	**M1**	2.1	Correct limits, solve
			$a = 0.32175\ldots$	**A1**	1.1	Answer, a.r.t. 0.322 (ignore other answers)
				[5]		
8	(b)		$F(y) = ½\,y \qquad [0 \le y \le 2]$	**M1**	3.1a	Use their $f(y)$ to obtain CDF
				A1	1.1	Correct $F(y)$ (range need not be stated explicitly)
			$P(Y^2 \le m) = P(0 < Y \le \sqrt{m})$	**M1**	2.1	Find CDF of Y^2, allow m^2 instead of $\sqrt{m}$, or $\pm\sqrt{m}$, here
			$= F(\sqrt{m}) \qquad [= ½\sqrt{m}]$	**A1**	1.1	Use $F(y)$ correctly
			$½\sqrt{P_{60}} = 0.6$	**M1**	1.1	Equate to 0.6 and solve, need $\sqrt{m}$ here
			$P_{60} = 1.44$	**A1**	2.2a	1.44 or exact equivalent
				[6]		

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