OCR
Oxford Cambridge and RSA

Tuesday 19 October 2021 – Afternoon

A Level Further Mathematics A

Y543/01 Mechanics

Time allowed: 1 hour 30 minutes

You must have:
- the Printed Answer Booklet
- the Formulae Booklet for A Level Further Mathematics A
- a scientific or graphical calculator

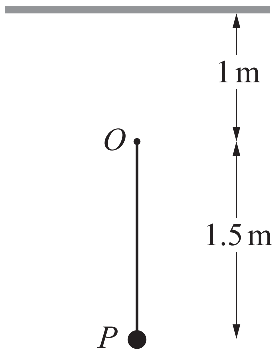
INSTRUCTIONS
- Use black ink. You can use an HB pencil, but only for graphs and diagrams.
- Write your answer to each question in the space provided in the **Printed Answer Booklet**. If you need extra space use the lined pages at the end of the Printed Answer Booklet. The question numbers must be clearly shown.
- Fill in the boxes on the front of the Printed Answer Booklet.
- Answer **all** the questions.
- Where appropriate, your answer should be supported with working. Marks might be given for using a correct method, even if your answer is wrong.
- Give non-exact numerical answers correct to **3** significant figures unless a different degree of accuracy is specified in the question.
- The acceleration due to gravity is denoted by $g\,\mathrm{m\,s^{-2}}$. When a numerical value is needed use $g = 9.8$ unless a different value is specified in the question.
- Do **not** send this Question Paper for marking. Keep in the centre or recycle it.

INFORMATION
- The total mark for this paper is **75**.
- The marks for each question are shown in brackets **[ ]**.
- This document has **8** pages.

ADVICE
- Read each question carefully before you start your answer.

Answer **all** the questions.

1 One end of a light elastic string of natural length 0.6 m and modulus of elasticity 24 N is attached to a fixed point O. The other end is attached to a particle P of mass 0.4 kg. O is a vertical distance of 1 m below a horizontal ceiling. P is held at a point 1.5 m vertically below O and released from rest (see diagram).



Assuming that there is no obstruction to the motion of P as it passes O, find the speed of P when it first hits the ceiling. **[5]**

2 A particle P of mass 2 kg is moving on a large smooth horizontal plane when it collides with a fixed smooth vertical wall. Before the collision its velocity is $(5\mathbf{i}+16\mathbf{j})\,\mathrm{m\,s^{-1}}$ and after the collision its velocity is $(-3\mathbf{i}+\mathbf{j})\,\mathrm{m\,s^{-1}}$.

(a) The impulse imparted on P by the wall is denoted by $\mathbf{I}$ N s.

Find the following.

- The magnitude of $\mathbf{I}$
- The angle between $\mathbf{I}$ and $\mathbf{i}$ **[6]**

(b) Find the loss of kinetic energy of P as a result of the collision. **[3]**

3 A particle P of mass m moves on the x-axis under the action of a force F directed along the axis. When the displacement of P from the origin is x its velocity is v.

(a) By using the fact that the dimensions of the derivative $\frac{\mathrm{d}v}{\mathrm{d}x}$ are the same as those of $\frac{v}{x}$, verify that the equation $F = mv\frac{\mathrm{d}v}{\mathrm{d}x}$ is dimensionally consistent. **[2]**

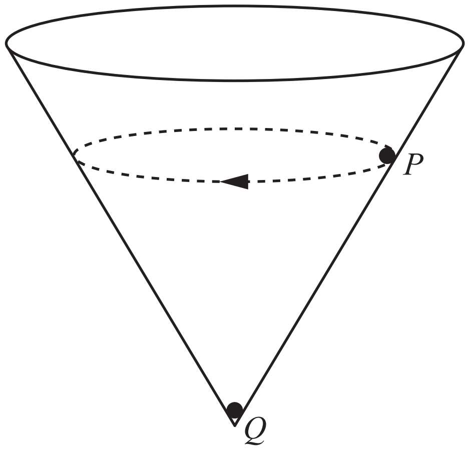
It is given that $v = km^{-\frac{1}{2}}\sqrt{a^2 - x^2}$ where a and k are constants.

(b) Explain why [a] must be the same as [x]. **[1]**

(c) Deduce the dimensions of k. **[2]**

(d) Find an expression for F in terms of x and k. **[3]**

4 A hollow cone is fixed with its axis vertical and its vertex downwards. A small sphere P of mass m kg is moving in a horizontal circle on the inner surface of the cone. An identical sphere Q rests in equilibrium inside the cone (see diagram).



The following modelling assumptions are made.

- P and Q are modelled as particles.
- The cone is modelled as smooth.
- There is no air resistance.

(a) Assuming that P moves with a constant speed $v\,\text{m}\,\text{s}^{-1}$, show that the total mechanical energy of P is $\frac{3}{2}mv^2$ J more than the total mechanical energy of Q. **[6]**

(b) Explain how the assumption that P and Q are both particles has been used. **[1]**

In practice, P will not move indefinitely in a perfectly circular path, but will actually follow an approximately spiral path on the inside surface of the cone until eventually it collides with Q.

(c) Suggest an improvement that could be made to the model. **[1]**

5 A particle P of mass 3 kg moves on the x-axis under the action of a single force acting in the positive x-direction. At time t s, where $t \geqslant 0$, the displacement of P is x m and its velocity is $v\,\mathrm{m\,s^{-1}}$. The magnitude of the force acting is inversely proportional to $(t + 1)^2$.

Initially P is at rest at the point where $x = 1$. When $t = 1$, $v = 2$.

(a) Show that $\dfrac{\mathrm{d}v}{\mathrm{d}t} = \dfrac{k}{3(t+1)^2}$ where k is a constant. **[1]**

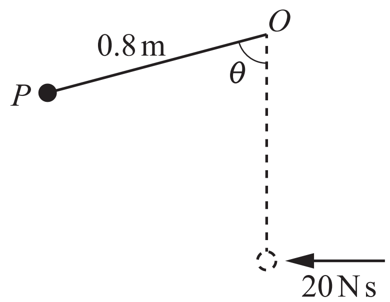
(b) Find an expression for v in terms of t. **[4]**

(c) Find an expression for x in terms of t. **[2]**

As t increases, v approaches a limiting value, v_T.

(d) Determine how far P is from its initial position at the instant when v is 95% of v_T. **[5]**

6 A particle P of mass 4 kg is attached to one end of a light inextensible string of length 0.8 m. The other end of the string is attached to a fixed point O. P is at rest vertically below O when it experiences a horizontal impulse of magnitude 20 N s. In the subsequent motion the angle the string makes with the downwards vertical through O is denoted by θ (see diagram).

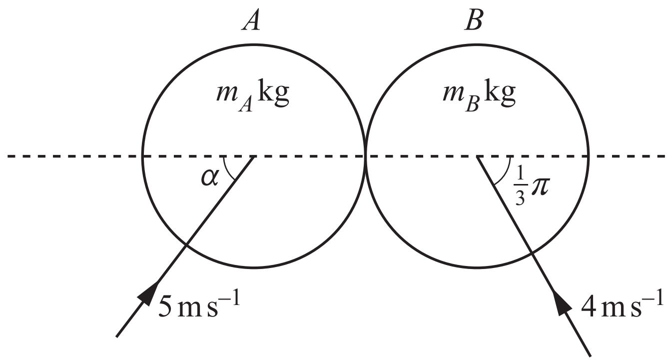


(a) Find the magnitude of the acceleration of P at the first instant when $\theta = \frac{1}{3}\pi$ radians. **[7]**

(b) Determine the value of θ at which the string first becomes slack. **[3]**

7 Two smooth circular discs A and B of masses m_A kg and m_B kg respectively are moving on a horizontal plane. At the instant before they collide the velocities of A and B are as follows, as shown in the diagram below.

- The velocity of A is $5\,\text{m}\,\text{s}^{-1}$ at an angle of α to the line of centres, where $\tan\alpha = \frac{4}{3}$.
- The velocity of B is $4\,\text{m}\,\text{s}^{-1}$ at an angle of $\frac{1}{3}\pi$ radians to the line of centres.



The direction of motion of B after the collision is perpendicular to the line of centres.

(a) Show that $\frac{3}{2} \leqslant \frac{m_B}{m_A} \leqslant 4$. **[6]**

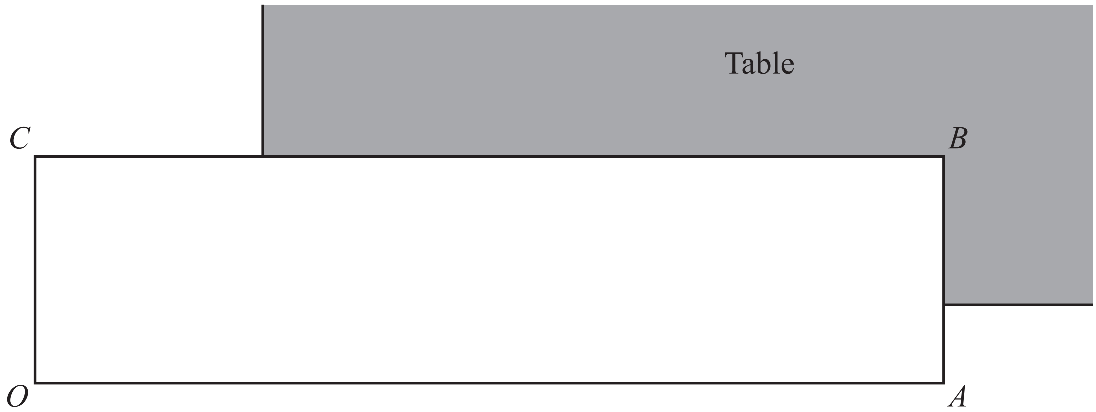
(b) Given that $m_A = 2$ and $m_B = 6$, find the total loss of kinetic energy as a result of the collision. **[4]**

8 A rectangular lamina of mass M has vertices at the origin O (0, 0), A $(24a, 0)$, B $(24a, 6a)$ and C $(0, 6a)$, where a is a positive constant. A small object P of mass m is attached to the lamina at the point (x, y). The centre of mass of the system consisting of the lamina and P is at the point $(\overline{x}, \overline{y})$. P is modelled as a particle and the lamina is modelled as being uniform.

(a) Show that $\overline{x} = \dfrac{12Ma+mx}{M+m}$. **[1]**

(b) Find a corresponding expression for $\overline{y}$. **[1]**

The lamina, with P no longer attached, is placed on a horizontal rectangular table, with its sides parallel to the edges of the table, and partly overhanging the edges of the table, as shown in the diagram. The corner of the table is at the point $(6a, 2a)$.



When P is placed on the lamina at O, the lamina topples over one of the edges of the table.

(c) Show that $m > \frac{1}{2}M$. **[4]**

The lamina is now put back on the table in the same position as before. P is placed at the point $(12a, 6a)$ on the smooth upper surface of the lamina, and is projected towards O. At a subsequent instant during the motion, P is at the point $(12ak, 6ak)$ where $0 < k < 1$.

(d) Assuming that the lamina has not yet toppled, find, in terms of M and m, the value of k for which the centre of mass of the system lies on the table edge parallel to OC. **[3]**

(e) For the case $m = \frac{3}{2}M$, determine which table edge the lamina topples over. **[4]**

END OF QUESTION PAPER

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