## GCE

# Further Mathematics B (MEI) 

Y432/01: Statistics minor

Advanced GCE

Mark Scheme for Autumn 2021

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This mark scheme is published as an aid to teachers and students, to indicate the requirements of the examination. It shows the basis on which marks were awarded by examiners. It does not indicate the details of the discussions which took place at an examiners' meeting before marking commenced.

All examiners are instructed that alternative correct answers and unexpected approaches in candidates' scripts must be given marks that fairly reflect the relevant knowledge and skills demonstrated.

Mark schemes should be read in conjunction with the published question papers and the report on the examination.

## Annotations and abbreviations

| Annotation in scoris | Meaning |
| :--- | :--- |
| $\checkmark$ and $\mathbf{x}$ |  |
| BOD | Benefit of doubt |
| FT | Follow through |
| ISW | Ignore subsequent working |
| M0, M1 | Method mark awarded 0,1 |
| A0, A1 | Accuracy mark awarded 0, 1 |
| B0,B1 | Independent mark awarded 0, 1 |
| E | Explanation mark 1 |
| SC | Special case |
| MR | Omission sign |
| MR | Misread |
| BP | Blank page |
| Highlighting |  |
| Other abbreviations in <br> mark scheme | Meaning |
| E1 |  |
| dep* | Mark for explaining a result or establishing a given result |
| cao | Mark dependent on a previous mark, indicated by *. The ${ }^{*}$ may be omitted if only previous M mark. |
| oe | Correct answer only |
| rot | Or equivalent |
| soi | Rounded or truncated |
| www | Seen or implied |
| AG | Without wrong working |
| awrt | Answer given |
| BC | Anything which rounds to |
| DR | By Calculator |
|  | This indicates that the instruction In this question you must show detailed reasoning appears in the question. |


| Question |  | Answer$\begin{aligned} & k+2 k+5 k+10 k+17 k=1 \\ & 35 k=1 \text { so } k=\frac{1}{35} \end{aligned}$ | Marks <br> M1 <br> A1 <br> $[2]$ | $\begin{gathered} \hline \mathrm{AOs} \\ \hline 2.4 \\ 1.1 \end{gathered}$ | Guidance |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| 1 | (a) |  |  |  | AG |  |
| 1 | (b) | The distribution has (strong) negative skew | $\begin{aligned} & \hline \text { B1 } \\ & {[1]} \end{aligned}$ | 1.1 |  |  |
| 1 | (c) | $E(X)=4 \frac{1}{7}=\frac{29}{7}=4.143 \ldots$ $\operatorname{Var}(X)=1 \frac{23}{245}=\frac{268}{245}=1.094 \ldots$ | B1 <br> B1 <br> [2] | 1.1a <br> 1.1 | BC Accept any equivalent form. <br> BC Accept any equivalent form. | Decimal answers should agree to at least 2 significant figures. |
| 1 | (d) | $\begin{aligned} & E(Y)=10 \frac{5}{7}=\frac{75}{7}=10.714 \\ & \operatorname{Var}(Y)=27 \frac{17}{49}=\frac{1340}{49}=27.347 \end{aligned}$ | B1FT <br> B1FT <br> [2] | 1.1 $1.1$ | BC Accept any equivalent form. FT their $\mathrm{E}(\mathrm{X})$ from (c) <br> BC Accept any equivalent form. FT their $\operatorname{Var}(\mathrm{X})$ from $(\mathbf{c})$ | Decimal answers should agree to at least 2 significant figures. |


| Question |  | Answer | Marks | AOs | Guidance |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| 2 | (a) | $a$ is the independent variable since the values of $a$ are not subject to random variation | B1 | 2.4 | B1: values of $a$ are controlled B0: $d$ is dependent on $a$ | Explanation required |
|  |  |  | [1] |  |  |  |
| 2 | (b) | $\boldsymbol{d}=-1.104 \boldsymbol{a}+197.1$ | $\begin{aligned} & \text { M1 } \\ & \text { A1 } \end{aligned}$ | $\begin{aligned} & \hline 3.3 \\ & 1.1 \end{aligned}$ | For either -1.104(a) or 197.1 BC | $\begin{aligned} & y=-1.104 x+197.1 \\ & \text { scores M1 A0 } \end{aligned}$ |
|  |  |  | [2] |  |  |  |
| 2 | (c) | estimate $=130.9(\mathrm{~m})$ | B1FT | 1.1 | FT from (b) if the value is plausible from the scatter diagram. | Accept 130 as rounded to 2 significant figures. |
| 2 | (d) | Because this would be extrapolation and it is possible that the relationship is different for young children | $\begin{aligned} & \text { B1 } \\ & \text { B1 } \\ & {[2]} \\ & \hline \end{aligned}$ | $\begin{gathered} \hline 2.2 b \\ 2.4 \end{gathered}$ | For 'extrapolation' <br> B1: a 5-year-old child may not be able to read yet | B0 for comment about child not being able to drive |
| 2 | (e) | $\begin{aligned} & \text { Residual }=150-(-1.104 \ldots \times 40+197.1 \ldots) \\ & =-3.0 \end{aligned}$ | $\begin{gathered} \text { M1 } \\ \text { A1FT } \\ {[2]} \end{gathered}$ | $\begin{aligned} & 1.1 \\ & 1.1 \end{aligned}$ | Subtraction other way around scores M1 only <br> Allow - 2.9 (using 1.104 and 197.1) FT from (b) |  |
| 2 | (f) | Because the values of $a$ are non-random so it makes no sense to try to predict them. | $\begin{aligned} & \text { B1 } \\ & {[1]} \\ & \hline \end{aligned}$ | 3.2b | Should show understanding of a purpose of a regression line being to make predictions. |  |


| Question |  | Answer | Marks | AOs | Guidance |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| 3 | (a) | The sample must be random | $\begin{gathered} \hline \text { B1 } \\ {[1]} \end{gathered}$ | 1.2 |  |  |
| 3 | (b) | $\begin{aligned} & \text { E8: } \frac{23 \times 29}{120}=5.5583 \\ & \text { C13: } \frac{(28-33.1417)^{2}}{33.1417} \\ & \quad=0.7977 \end{aligned}$ | $\begin{aligned} & \text { B1 } \\ & \text { M1 } \\ & \text { A1 } \\ & {[3]} \end{aligned}$ | $\begin{gathered} 1.1 \\ 1.1 \mathrm{a} \\ 1.1 \end{gathered}$ |  |  |
| 3 | (c) | $\mathrm{H}_{0}$ : No association between age and smoking (status) <br> $\mathrm{H}_{1}$ : Some association between age \& smoking (status) <br> Degrees of freedom $=2$ <br> Critical value $=5.991$ <br> Test statistic $=3.3642+0.6964+\ldots+0.2792=6.4801$ $6.4801>5.991$ <br> There is sufficient evidence at the $5 \%$ level to suggest that there is association between age and smoking (status) |  | 3.4 <br> 3.3 <br> 1.1 <br> 1.1 <br> 2.2b <br> 3.5a | Both hypotheses needed <br> Use of 'correlation' in place of 'association' is B0 <br> or $\chi_{2}^{2}(6.4801)=0.9608$ <br> or p -value $=0.0392$ <br> FT their value of C13 <br> or $0.9608>0.95$ or $0.0392<0.05$ <br> Correct test and critical values required Use of 'correlation' in place of 'association' is $\mathbf{A 0}$ | Comparing their test and critical values leading to a conclusion. Conclusion in context |
| 3 | (d) | For 16-34 year olds the contribution of 3.3642 suggests that more are smokers than would be expected. <br> For 35-59 year olds things are (approximately) as expected if there were no association. <br> For people aged 60 and over the contribution of 1.1775 suggests that fewer are smokers than would be expected. | E1 <br> E1 <br> E1 <br> [3] | $2.3$ <br> 3.5a <br> 3.2a | Max of 2 marks out of 3 if no contributions are mentioned. <br> Allow equivalent statements about non-smokers | Should take each age group in turn and discuss status Max 2 marks if done differently |


| Question |  | Answer | Marks | AOs | Guidance |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| 4 | (a) | Because (the grouping of points on) the scatter diagram appears to be very roughly elliptical, the distribution may be bivariate Normal. | $\begin{aligned} & \text { E1 } \\ & \text { E1 } \\ & {[2]} \\ & \hline \end{aligned}$ | $\begin{gathered} \hline 3.5 \mathrm{a} \\ 2.4 \end{gathered}$ | For elliptical <br> For full answer (dep. on first mark) "the data is bivariate Normal" is $\mathbf{E 0}$ |  |
| 4 | (b) | $\begin{aligned} & S_{x y}=12317.2-\frac{1}{10} \times 351.9 \times 350.0=0.7 \\ & S_{x x}=12384.5-\frac{1}{10} \times 351.9^{2}=1.139 \\ & S_{y y}=12251.2-\frac{1}{10} \times 350.0^{2}=1.2 \\ & r=\frac{S_{x y}}{\sqrt{S_{x x} S_{y y}}}=\frac{0.7}{\sqrt{1.139 \times 1.2}} \\ & =0.60 \end{aligned}$ | M1 <br> M1 <br> M1 <br> A1 <br> [4] | 1.1a 1.1 <br> 3.3 <br> 1.1 | For either $S_{x x}$ or $S_{y y}$ <br> For general form including square root <br> SC2 for correct value without any intermediate calculations | Detailed reasoning required. $(0.598750 \ldots)$ |
| 4 | (c) | $\begin{aligned} & \mathrm{H} 0: \rho=0 \\ & \mathrm{H}_{1}: \rho>0 \end{aligned}$ <br> where $\rho$ is the population product moment correlation coefficient between $x$ and $y$ <br> For $n=10,5 \%$ critical value (one tailed) $=0.5494$ <br> Since $0.60>0.5494$ the result is significant. <br> There is sufficient evidence (at the $5 \%$ level) to suggest that there is positive correlation between directly measured and satellite measured salinity level. | $\begin{gathered} \hline \text { B1 } \\ \text { B1 } \\ \\ \text { B1 } \\ \text { M1 } \\ \text { A1FT } \\ {[5]} \end{gathered}$ | 3.3 <br> 2.5 <br> 3.4 <br> 1.1 <br> 2.2b | For both hypotheses <br> For defining $\rho$ <br> For critical value <br> For comparison of test statistic and critical value leading to a conclusion FT for conclusion in context | $\mathrm{H}_{0}$ : no correlation in the population $\mathrm{H}_{1}$ : positive correlation in the population scores first B1 |
| 4 | (d) | It means that one can be more confident that the correlation is genuine, rather than simply the result of random variation. | $\begin{aligned} & \hline \text { E1 } \\ & {[1]} \end{aligned}$ | 2.4 |  |  |
| 4 | (e) | The test shows that there is almost certainly some real correlation in the population. <br> However, it is uninformative/of little use since the effect size is so small. | $\begin{gathered} \hline \text { E1 } \\ \text { E1 } \\ {[2]} \\ \hline \end{gathered}$ | $\begin{aligned} & 3.5 \mathrm{a} \\ & 2.2 \mathrm{~b} \end{aligned}$ |  |  |


| Question |  |  | Answer | Marks | AOs | Guidance |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| 5 | (a) | (i) | $\begin{aligned} \mathrm{P}(X \geq 10) & =1-0.9161 \\ & =0.0839 \end{aligned}$ | $\begin{aligned} & \hline \text { M1 } \\ & \text { A1 } \\ & {[2]} \\ & \hline \end{aligned}$ | $\begin{gathered} \hline \text { 3.1b } \\ 1.1 \end{gathered}$ | Or $\mathrm{P}(X \geq 10)=1-\mathrm{P}(X \leq 9)$ BC (0.083924...) |  |
| 5 | (a) | (ii) | $\begin{aligned} & \text { Poisson (60) } \\ & \mathrm{P}(X<50)=0.0844 \end{aligned}$ | $\begin{aligned} & \text { M1 } \\ & \text { A1 } \\ & {[2]} \\ & \hline \end{aligned}$ | $\begin{aligned} & \hline 3.3 \\ & 1.1 \end{aligned}$ | $\begin{aligned} & \text { soi } \\ & \text { BC }(0.084406 \ldots) \end{aligned}$ |  |
| 5 | (b) |  | $\begin{aligned} & 0.9161^{20} \\ & =0.1733 \end{aligned}$ | $\begin{gathered} \text { M1 } \\ \text { A1FT } \\ {[2]} \\ \hline \end{gathered}$ | $\begin{gathered} \hline \text { 3.1a } \\ 1.1 \end{gathered}$ | soi or 0.1732 from calculator value in (a) |  |
| 5 | (c) |  | $\begin{aligned} & \text { Use of } \operatorname{Var}(X)=\mathrm{E}(X) \\ & \operatorname{Var}(X)=\mathrm{E}\left(X^{2}\right)-(\mathrm{E}(X))^{2} \quad \mu=12-\mu^{2} \\ & \mu=3 \\ & \mathrm{P}(X<5)=0.8153 \end{aligned}$ | $\begin{aligned} & \text { M1 } \\ & \text { M1 } \\ & \text { A1 } \\ & \text { A1 } \\ & {[4]} \end{aligned}$ | $\begin{gathered} \hline 3.1 \mathrm{~b} \\ 2.2 \mathrm{a} \\ 1.1 \\ 1.1 \end{gathered}$ | For equation BC (0.815263...) |  |


| Question |  |  | Answer | Marks | AOs | Guidance |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| 6 | (a) | (i) | Uniform distribution on $\{1,2, \ldots, n\}$ $\mathrm{P}\left(X \leq \frac{1}{4} n\right)=\frac{1}{4}$ | $\begin{aligned} & \text { B1 } \\ & {[1]} \\ & \hline \end{aligned}$ | 3.3 |  |  |
| 6 | (a) | (ii) | $\begin{aligned} \mathrm{P}\left(X \leq \frac{1}{4} n\right) & =\frac{k}{4 k+1} \\ & =\frac{\frac{1}{4}(n-1)}{n} \\ & =\frac{n-1}{4 n} \end{aligned}$ | M1 <br> M1 <br> A1 <br> [3] | $\begin{aligned} & 2.2 \mathrm{a} \\ & 1.1 \\ & 1.1 \end{aligned}$ |  | Single fraction required for A1 |
| 6 | (b) |  | $\begin{aligned} & \mathrm{E}(X)=51 \quad \operatorname{Var}(X)=850 \\ & \mathrm{SD}(X)=29.1 \text { So require } \mathrm{P}(21.9<X<80.1) \\ & \mathrm{P}(22 \leq X \leq 80)=\frac{59}{101} \end{aligned}$ | $\begin{aligned} & \text { M1 } \\ & \text { M1 } \\ & \text { A1 } \\ & {[3]} \\ & \hline \end{aligned}$ | $\begin{gathered} \hline \text { 3.1b } \\ \text { 1.1a } \\ 1.1 \end{gathered}$ | For either For required interval |  |

OCR (Oxford Cambridge and RSA Examinations)<br>The Triangle Building<br>Shaftesbury Road<br>Cambridge<br>CB2 8EA<br>OCR Customer Contact Centre<br>Education and Learning<br>Telephone: 01223553998<br>Facsimile: 01223552627<br>Email: general.qualifications@ocr.org.uk<br>www.ocr.org.uk

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