Oxford Cambridge and RSA

# Tuesday 19 October 2021 – Afternoon

## A Level Further Mathematics B (MEI)

**Y431/01** Mechanics Minor

**Time allowed: 1 hour 15 minutes**

**You must have:**
- the Printed Answer Booklet
- the Formulae Booklet for Further Mathematics B (MEI)
- a scientific or graphical calculator

### INSTRUCTIONS
- Use black ink. You can use an HB pencil, but only for graphs and diagrams.
- Write your answer to each question in the space provided in the **Printed Answer Booklet**. If you need extra space, use the lined pages at the end of the Printed Answer Booklet. The question numbers must be clearly shown.
- Fill in the boxes on the front of the Printed Answer Booklet.
- Answer **all** the questions.
- Where appropriate, your answer should be supported with working. Marks might be given for using a correct method, even if your answer is wrong.
- Give your final answers to a degree of accuracy that is appropriate to the context.
- The acceleration due to gravity is denoted by $g\,\text{m}\,\text{s}^{-2}$. When a numerical value is needed use $g = 9.8$ unless a different value is specified in the question.
- Do **not** send this Question Paper for marking. Keep it in the centre or recycle it.

### INFORMATION
- The total mark for this paper is **60**.
- The marks for each question are shown in brackets **[ ]**.
- This document has **8** pages.

### ADVICE
- Read each question carefully before you start your answer.

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Answer **all** the questions.

**1** **(a)** State the dimensions of force. **[1]**

The force $F$ required to keep a car moving at constant speed on a circular track is given by the formula

$$F = \frac{mv^2}{r},$$

where

- $m$ is the constant mass of the car,
- $v$ is the speed of the car,
- $r$ is the radius of the circular track.

**(b)** Verify that the formula is dimensionally consistent. **[2]**

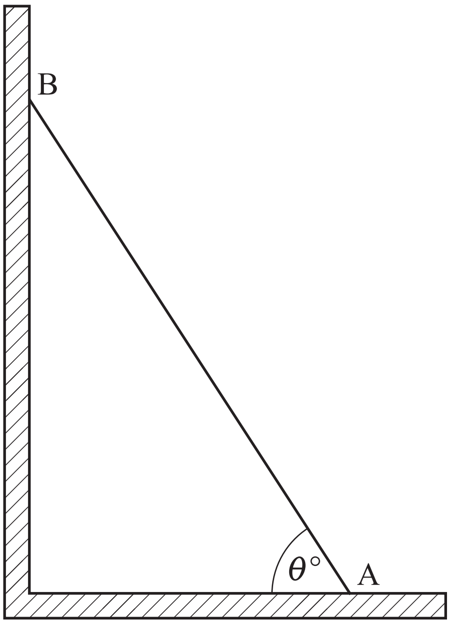
**(c)** Determine the percentage increase in force required to keep a car moving on a circular track if the speed of the car were to increase by 10% and if the track radius were to decrease by 10%. **[2]**

It is proposed that a new unit of force, the *trackforce* (Tr), should be adopted in motor-racing. 1 Tr is defined as the amount of force required to accelerate a mass of 1 ton at a rate of 1 mile per hour per second.

It is given that 1 ton = 1016 kg and 1 mile = 1609 m.

**(d)** Determine the number of newtons that are equivalent to 1 Tr. **[2]**

**2** The diagram shows a uniform beam AB that rests with its end A on rough horizontal ground and its end B against a smooth vertical wall. The beam makes an angle of $\theta°$ with the ground.




The weight of the beam is $W$ N.

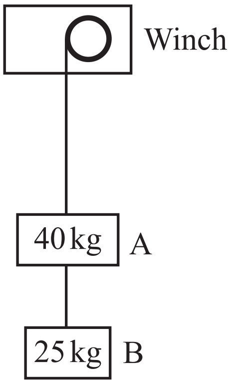
The beam is in limiting equilibrium and the coefficient of friction between the beam and the ground is $\mu$.

It is given that the magnitude of the contact force at A is 70 N and the magnitude of the contact force at B is 20 N.

Determine, in any order,

- the value of $W$,
- the value of $\mu$,
- the value of $\theta$. **[7]**

**3** The diagram shows an electric winch raising two crates A and B, with masses 40 kg and 25 kg, respectively. The cable connecting the winch to A, and the cable connecting A to B may both be modelled as light and inextensible. Furthermore, it can be assumed that there are no resistances to motion.




Throughout the entire motion, the power $P$ W developed by the winch is constant.

Crates A and B are both being raised at a constant speed $v\,\text{m s}^{-1}$ when the cable connecting A and B breaks. After the cable between A and B breaks, crate A continues to be raised by the winch. Crate A now accelerates until it reaches a new constant speed of $(v+3)\,\text{m s}^{-1}$.

Determine

- the value of $v$,
- the value of $P$. **[5]**

**4** A child throws a ball of mass $m$ kg vertically upwards with a speed of $7.2\,\text{m}\,\text{s}^{-1}$.
The ball leaves the child's hand at a height of 1.6 m above horizontal ground.

**(a)** Ignoring any possible air resistance, use an energy method to determine the maximum height reached by the ball above the ground. **[2]**

In fact, the ball only reaches a height of 4.1 m above the ground. For the rest of this question you should assume that the air resistance may be modelled as a constant force acting in the opposite direction to the ball's motion.

**(b)** Show that the ball does $0.568m$ J of work against air resistance per metre travelled. **[2]**

**(c)** Calculate the speed of the ball just before it hits the ground. **[3]**

The ball bounces off the ground and first comes instantaneously to rest 2.8 m above the ground.

**(d)** Determine the coefficient of restitution between the ball and the ground. **[3]**

In the first impact between the ball and the ground, the magnitude of the impulse exerted on the ball by the ground is 12 N s.

**(e)** Determine the value of $m$. **[2]**

**5** **Fig. 5.1** shows a solid L-shaped ornament, of uniform density. The ornament is 3 cm thick. The $x$, $y$ and $z$ axes are shown, along with the dimensions of the ornament. The measurements are in centimetres.

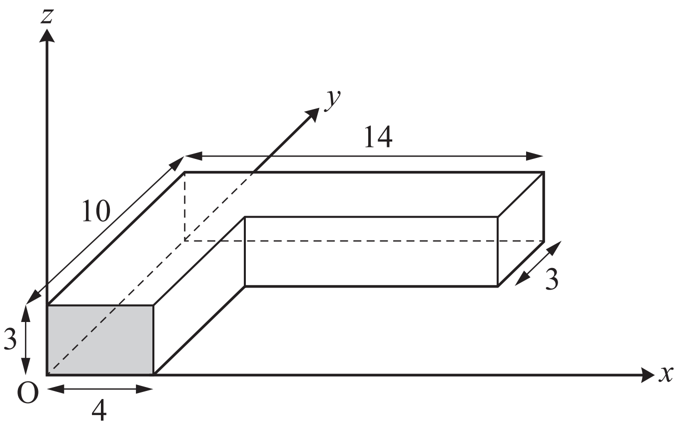



**Fig. 5.1**

**(a)** Determine, with reference to the axes shown, the coordinates of the ornament's centre of mass. **[4]**

**Fig. 5.2** shows the ornament placed so that the shaded face (indicated in **Fig. 5.1**) is in contact with a plane inclined at $\theta°$ to the horizontal, with the 4 cm edge parallel to a line of greatest slope. The surface of the plane is sufficiently rough so that the ornament will not slip down the plane.

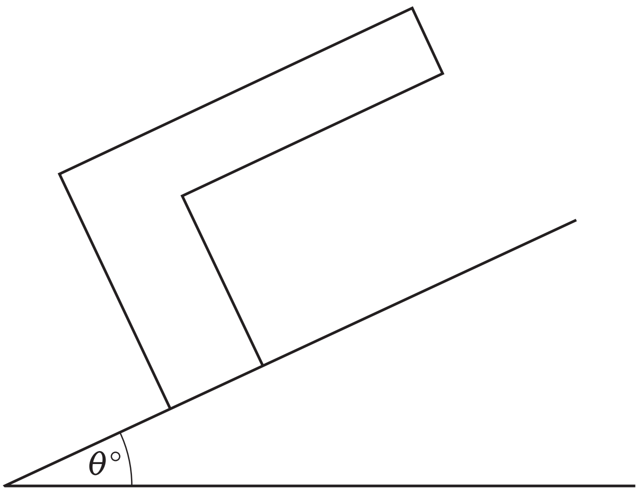



**Fig. 5.2**

**(b)** Determine the minimum and maximum possible values of $\theta$ for which the ornament does not topple. **[3]**

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The ornament is now placed with its shaded face in contact with a rough horizontal surface. A force of magnitude $P$ N, acting parallel to the planes of the L-shaped faces, is applied to one of the edges of the ornament, as shown in **Fig. 5.3**. The force is inclined at an angle of 30° to the horizontal. The coefficient of friction between the ornament and the surface is $\mu$.

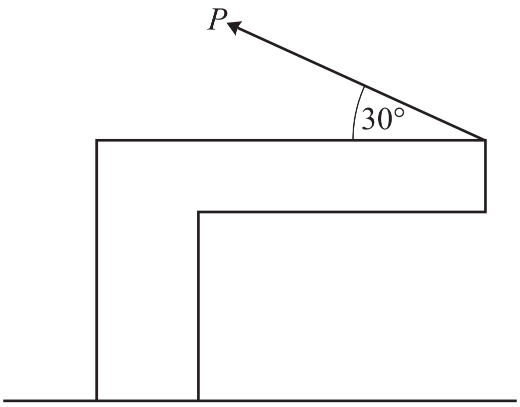



**Fig. 5.3**

The value of $P$ is gradually increased until the ornament is on the point of toppling but does not slide.

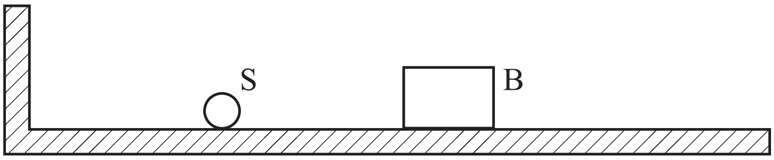
**(c)** Determine the minimum value of $\mu$. **[7]**

**(d)** Explain how your answer to part **(c)** would change if the angle between $P$ and the horizontal was less than 30°. **[2]**

**6** A block rests on a horizontal surface. The coefficient of friction between the block and the surface is $\mu$.

**(a)** Show that if the block is given an initial speed of $v\,\mathrm{m\,s^{-1}}$, it will move a distance of $\frac{v^2}{2\mu g}$ before coming to rest. **[3]**

Block B rests on the same horizontal surface as a sphere S. On the other side of S is a vertical wall, as shown below. The mass of B is 8 times the mass of S.




S is projected directly towards B with speed $u\,\mathrm{m\,s^{-1}}$ and hits B.
It is given that

- the coefficient of restitution between S and B is 0.8,
- collisions between S and the wall are perfectly elastic,
- the wall is perpendicular to the direction of motion of S and B.

Furthermore, you should model the contact between B and the surface as **rough** and model the contact between S and the surface as **smooth**.

**(b)** Determine, in terms of $u$, expressions for

- the speed of S
- the speed of B

immediately after the first collision between S and B. In each case stating the corresponding direction of motion. **[5]**

It is given that B has sufficient time to come to rest before each subsequent collision with S.

Let $x_n$ be the distance B moves after the $n$th impact between S and B.

**(c)** Explain why $x_{n+1} = \frac{9}{25}x_n$. **[2]**

**(d)** Given that $u = 11.2$ and the coefficient of friction between B and the surface is $\frac{1}{7}$, show that B will travel a total distance that cannot exceed 2.8 m. **[3]**

**END OF QUESTION PAPER**

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