

Thursday 21 October 2021 – Afternoon

A Level Further Mathematics B (MEI)

Y435/01 Extra Pure

Time allowed: 1 hour 15 minutes

You must have:

- the Printed Answer Booklet
- the Formulae Booklet for Further Mathematics B (MEI)
- · a scientific or graphical calculator



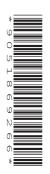
- Use black ink. You can use an HB pencil, but only for graphs and diagrams.
- Write your answer to each question in the space provided in the Printed Answer
 Booklet. If you need extra space use the lined pages at the end of the Printed Answer
 Booklet. The question numbers must be clearly shown.
- Fill in the boxes on the front of the Printed Answer Booklet.
- Answer all the guestions.
- Where appropriate, your answer should be supported with working. Marks might be given for using a correct method, even if your answer is wrong.
- Give your final answers to a degree of accuracy that is appropriate to the context.
- Do not send this Question Paper for marking. Keep it in the centre or recycle it.

INFORMATION

- The total mark for this paper is 60.
- The marks for each question are shown in brackets [].
- This document has 4 pages.

ADVICE

Read each question carefully before you start your answer.



Answer all the questions.

1 In this question you must show detailed reasoning.

A surface S is defined by z = f(x, y) where $f(x, y) = x^3 + x^2y - 2y^2$.

- (a) On the coordinate axes in the Printed Answer Booklet, sketch the section z = f(2, y) giving the coordinates of any turning points and any points of intersection with the axes. [4]
- **(b)** Find the stationary points on *S*.

[7]

- **2** *G* is a group of order 8.
 - (a) Explain why there is no subgroup of G of order 6.

[1]

You are now given that G is a cyclic group with the following features:

- *e* is the identity element of *G*,
- g is a generator of G,
- *H* is the subgroup of *G* of order 4.
- **(b)** Write down the possible generators of H.

[2]

M is the group ($\{0, 1, 2, 3, 4, 5, 6, 7\}$, $+_{8}$) where $+_{8}$ denotes the binary operation of addition modulo 8. You are given that M is isomorphic to G.

(c) Specify all possible isomorphisms between M and G.

[4]

- 3 The matrix **A** is given by $\mathbf{A} = \begin{pmatrix} 3 & 3 & 0 \\ 0 & 2 & 2 \\ 1 & 3 & 4 \end{pmatrix}$.
 - (a) Determine the characteristic equation of A.

[3]

(b) Hence verify that the eigenvalues of **A** are 1, 2 and 6.

[1]

(c) For each eigenvalue of A determine an associated eigenvector.

[4]

(d) Use the results of parts (b) and (c) to find A^n as a single matrix, where n is a positive integer.

[6]

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- 4 The sequence u_0 , u_1 , u_2 , ... satisfies the recurrence relation $u_{n+2} 3u_{n+1} 10u_n = 24n 10$.
 - (a) Determine the general solution of the recurrence relation. [6]
 - (b) Hence determine the particular solution of the recurrence relation for which $u_0 = 6$ and $u_1 = 10$. [3]
 - (c) Show, by direct calculation, that your solution in part (b) gives the correct value for u_2 . [1]

The sequence v_0 , v_1 , v_2 , ... is defined by $v_n = \frac{u_n}{p^n}$ for some constant p, where u_n denotes the particular solution found in part (b).

You are given that v_n converges to a finite non-zero limit, q, as $n \to \infty$.

(d) Determine
$$p$$
 and q . [4]

5 A surface S is defined for $z \ge 0$ by $x^2 + y^2 + 2z^2 = 126$. C is the set of points on S for which the tangent plane to S at that point intersects the x-y plane at an angle of $\frac{1}{3}\pi$ radians.

Show that C lies in a plane, Π , whose equation should be determined. [6]

6 You are given that $q \in \mathbb{Z}$ with $q \ge 1$ and that

$$S = \frac{1}{(q+1)} + \frac{1}{(q+1)(q+2)} + \frac{1}{(q+1)(q+2)(q+3)} + \dots$$

- (a) By considering a suitable geometric series show that $S < \frac{1}{q}$. [3]
- (b) Deduce that $S \notin \mathbb{Z}$. [2]

You are also given that $e = \sum_{r=0}^{\infty} \frac{1}{r!}$.

(c) Assume that $e = \frac{p}{q}$, where p and q are positive integers. By writing the infinite series for e in a form using q and S and using the result from part (b), prove by contradiction that e is irrational.

END OF QUESTION PAPER

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