GCE

Further Mathematics A

Y540/01: Pure Core 1

Advanced GCE

Mark Scheme for Autumn 2021

This mark scheme is published as an aid to teachers and students, to indicate the requirements of the examination. It shows the basis on which marks were awarded by examiners. It does not indicate the details of the discussions which took place at an examiners' meeting before marking commenced.

All examiners are instructed that alternative correct answers and unexpected approaches in candidates' scripts must be given marks that fairly reflect the relevant knowledge and skills demonstrated.

Mark schemes should be read in conjunction with the published question papers and the report on the examination.

Annotations and abbreviations

Annotation in RM assessor	Meaning
✓and ✗	
BOD	Benefit of doubt
FT	Follow through
ISW	Ignore subsequent working
M0, M1	Method mark awarded 0, 1
A0, A1	Accuracy mark awarded 0, 1
B0, B1	Independent mark awarded 0, 1
SC	Special case
^	Omission sign
MR	Misread
BP	Blank Page
Seen	
Highlighting	
Other abbreviations in mark scheme	**Meaning**
dep*	Mark dependent on a previous mark, indicated by *. The * may be omitted if only one previous M mark
cao	Correct answer only
oe	Or equivalent
rot	Rounded or truncated
soi	Seen or implied
www	Without wrong working
AG	Answer given
awrt	Anything which rounds to
BC	By Calculator
DR	This question included the instruction: In this question you must show detailed reasoning.

Question			Answer	Marks	AO	Guidance
1	(a)	(i)	Circle Centre 1–2i, Radius 3	**B1** **B1**	1.1 2.2a	Be generous over circles drawn freehand If the axes are scaled then a mark at (1, -2) will do. For radius, an indication that the radius is 3 will do (e.g. passing through (4, -2) etc if marked will do.)
				[2]		
		(ii)	Straight vertical line $x = \frac{1}{2}$	**B1** **B1**	1.1 2.2a	Can be seen by x = ½ being labelled on the axis and vertical line through it
				[2]		
	(b)		Inside circle And to the left of $x = \frac{1}{2}$	**B1** **B1**	1.1 2.2a	Or their line if it is vertical.
				[2]		

Question			Answer	Marks	AO	Guidance
2	(a)	(i)	$f(0)=\frac{\pi}{4}$	B1	1.1	Not for 45^0
				[1]		
		(ii)	$f'(x)=\frac{1}{1+(1+x)^2} \Rightarrow f'(0)=\frac{1}{2}$	M1 A1	2.1 1.1	Diffn – Must be seen $f'(x)=\frac{1}{1+x^2}$ is M0
				[2]		
		(iii)	$f'(x)=\frac{1}{1+(1+x)^2}=\frac{1}{2+2x+x^2}$ $\Rightarrow f''(x)=\frac{1}{(2+2x+x^2)^2}\times(-1)\times(2+2x)$ $=\frac{-(2+2x)}{(2+2x+x^2)^2}$ $\Rightarrow f''(0)=\left(\frac{-2}{4}\right)=-\frac{1}{2}$	M1 A1 A1	2.1 2.1 2.1	Diffn *their* f'(x) **oe**, e.g. $f''(x)=-\frac{2(1+x)}{\left(1+(1+x)^2\right)^2}$ f''(0) must be seen. The substitution must be seen (implied by $-\frac{2}{4}$) **AG**
				[3]		
	(b)		$f(x)=f(0)+f'(0)x+f''(0)\frac{x^2}{2}$ $=\frac{\pi}{4}+\frac{1}{2}x-\frac{1}{2}\times\frac{x^2}{2}$ $=\frac{\pi}{4}+\frac{x}{2}-\frac{x^2}{4}$	M1 A1	1.1 2.2a	Using the formula and substituting *their* value for f;(0) **ft** their values from (a)
				[2]		

Question			Answer	Marks	AO	Guidance
3	(a)		e.g. $\alpha^2+\beta^2+\gamma^2=-5$ means that at least one root is complex But complex roots come in complex pairs so there are 2 complex roots. Given that there are 3 roots and 2 are complex one is real.	B1 B1 B1	2.4 2.4 2.4	
				[3]		
	(b)		$\alpha+\beta+\gamma=3$ $(\alpha+\beta+\gamma)^2=\alpha^2+\beta^2+\gamma^2+2(\alpha\beta+\beta\gamma+\gamma\alpha)$ $9=-5+2(\alpha\beta+\beta\gamma+\gamma\alpha)$ But $k=\alpha\beta+\beta\gamma+\gamma\alpha$ $\Rightarrow k=7$	B1 M1 A1	1.1 3.1a 1.1	Attempt to obtain identity and substitute Condone missing 2 and sign errors
				[3]		
	(c)		$\left(\frac{1}{u}\right)^3-3\left(\frac{1}{u}\right)^2+7\left(\frac{1}{u}\right)-5=0$ $\Rightarrow -5u^3+7u^2-3u+1=0$ **oe**	M1 A1	1.1 1.1	For the substitution "=0" not necessary here but needed for A1 Allow in terms of z Allow **ft** from *their* k in (b)
			Alternate method $\frac{1}{\alpha}+\frac{1}{\beta}+\frac{1}{\gamma}=\frac{7}{5}$, $\frac{1}{\alpha}\frac{1}{\beta}+\frac{1}{\beta}\frac{1}{\gamma}+\frac{1}{\gamma}\frac{1}{\alpha}=\frac{3}{5}$, $\frac{1}{\alpha}\frac{1}{\beta}\frac{1}{\gamma}=\frac{1}{5}$ Answer as above	M1 A1		For calculating the sum, product and sum of product of pairs of reciprocals of α, β, γ
				[2]		

Question			Answer	Marks	AO	Guidance
4	(a)		$\overrightarrow{AB} = \begin{pmatrix} -3 \\ 3 \\ 3 \end{pmatrix}$ **oe**	B1	1.1	**soi**
			Equation of AB is $\begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} 4 \\ 2 \\ 0 \end{pmatrix} + \lambda \begin{pmatrix} -3 \\ 3 \\ 3 \end{pmatrix}$ **oe**	M1	1.1	*their* $\begin{pmatrix} -3 \\ 3 \\ 3 \end{pmatrix}$ **soi** $\mathbf{r} =$ or $\begin{pmatrix} x \\ y \\ z \end{pmatrix} =$ is not required for M1
			$\Rightarrow 4 - x = y - 2 = z$	A1	1.1	Allow equivalent equations e.g. $\Rightarrow 1 - x = y - 5 = z - 3$ from using B
				[3]		

Question			Answer	Marks	AO	Guidance
4	(b)		$\overrightarrow{AB} = \begin{pmatrix} -3 \\ 3 \\ 3 \end{pmatrix}$, $\overrightarrow{OM}$ is $\begin{pmatrix} 4 \\ 2 \\ 0 \end{pmatrix} + \lambda \begin{pmatrix} -3 \\ 3 \\ 3 \end{pmatrix} = \begin{pmatrix} 4-3\lambda \\ 2+3\lambda \\ 3\lambda \end{pmatrix}$	M1	3.1a	Attempt to find general point on AB to get vector CM. Can use (1, 5, 3)
			$\overrightarrow{CM} = \begin{pmatrix} 4-3\lambda \\ 2+3\lambda \\ 3\lambda \end{pmatrix} - \begin{pmatrix} 1 \\ 4 \\ -2 \end{pmatrix} = \begin{pmatrix} 3-3\lambda \\ 3\lambda-2 \\ 3\lambda+2 \end{pmatrix}$	A1	1.1	Allow working throughout that uses e.g. $\begin{pmatrix} -1 \\ 1 \\ 1 \end{pmatrix}$ **ft** *their* vector from (i).
			Perpendicular to $\overrightarrow{AB} \Rightarrow \begin{pmatrix} 3-3\lambda \\ 3\lambda-2 \\ 3\lambda+2 \end{pmatrix} . \begin{pmatrix} -3 \\ 3 \\ 3 \end{pmatrix} = 0$ $\Rightarrow -9+9\lambda+9\lambda-6+9\lambda+6=0$	M1	1.1	Use of dot product to solve
			$\Rightarrow 27\lambda = 9 \Rightarrow \lambda = \frac{1}{3}$. $\Rightarrow$ Coordinates of M are (3, 3, 1)	A1	1.1	Do not accept a vector answer
			Alternative method for last two marks			
			Minimise $\lvert\overrightarrow{CM}\rvert^2 = \left\lVert\begin{pmatrix} 3-3\lambda \\ 3\lambda-2 \\ 3\lambda+2 \end{pmatrix}\right\rVert^2$ $=(3-3\lambda)^2+(3\lambda-2)^2+(3\lambda+2)^2$	M1		Express as a function of λ and minimise the quadratic in λ
			$\Rightarrow \lambda = \frac{1}{3} \Rightarrow$ Coordinates of M are (3, 3, 1)	A1		
				[4]		

Question			Answer	Marks	AO	Guidance
4	(c)		$CM^2 = 2^2 + 1^2 + 3^2 = 14$ $AB^2 = 3^2 + 3^2 + 3^2 = 27$	B1 B1	1.1 1.1	B1 for each distance **ft** *their M* **ft** *their* AB
			$\Rightarrow \text{Area} = \frac{1}{2}\left\|\overrightarrow{\text{AB}}\right\|.\left\|\overrightarrow{\text{CM}}\right\|$ $= \frac{3}{2}\sqrt{42}$	M1 A1	3.1a 1.1	Formula for area **ft** *their* M
			Alternative method 1 $\text{Area} = \frac{1}{2}\left\|\overrightarrow{AB} \times \overrightarrow{BC}\right\| = \frac{1}{2}\left\|\begin{pmatrix}3\\-3\\-3\end{pmatrix} \times \begin{pmatrix}0\\1\\5\end{pmatrix}\right\| = \frac{1}{2}\left\|\begin{pmatrix}-12\\-15\\3\end{pmatrix}\right\|$ $= \frac{1}{2}\sqrt{12^2 + 15^2 + 3^2} = \frac{1}{2}\sqrt{378} = \frac{3}{2}\sqrt{42}$	M1 M1 A1 A1		Formula for area Cross product
			Alternative method 2 $\text{Area} = \frac{1}{2}\left\|\overrightarrow{AB}\right\|\left\|\overrightarrow{BC}\right\|\sin\theta$ where $\overrightarrow{AB}.\overrightarrow{BC} = \left\|\overrightarrow{AB}\right\|\left\|\overrightarrow{BC}\right\|\cos\theta$ $\Rightarrow \cos\theta = \frac{-12}{\sqrt{27}\sqrt{26}} = \frac{-4}{\sqrt{78}}$ $\Rightarrow \sin\theta = \sqrt{1 - \frac{8}{39}} = \frac{\sqrt{31}}{\sqrt{39}}$ $\Rightarrow \text{Area} = \frac{1}{2}\sqrt{27}\sqrt{26}\frac{\sqrt{31}}{\sqrt{39}} = \frac{3}{2}\sqrt{42}$	M1 A1 M1 A1		For use of dot product, formula for area Pythagoras to find $\sin\theta$
				[4]		

Question			Answer	Marks	AO	Guidance
5			Take $z=\cos\theta+\mathrm{i}\sin\theta \Rightarrow z^{-1}=\cos\theta-\mathrm{i}\sin\theta$	M1	2.1	Use of z and de Moivre Sight of i is necessary
			$\Rightarrow z-\frac{1}{z}=2\mathrm{i}\sin\theta$			
			$\left(z-\frac{1}{z}\right)^5=32\mathrm{i}\sin^5\theta=z^5-5z^3+10z-\frac{10}{z}+\frac{5}{z^3}-\frac{1}{z^5}$	A1	1.1	Both sides
			$\Rightarrow 32\mathrm{i}\sin^5\theta=\left(z^5-\frac{1}{z^5}\right)-5\left(z^3-\frac{1}{z^3}\right)+10\left(z-\frac{1}{z}\right)$	M1	1.1	Attempt conversion into sin **soi**
			$=2\mathrm{i}\sin 5\theta-10\mathrm{i}\sin 3\theta+20\mathrm{i}\sin\theta$			
			$\Rightarrow \sin^5\theta=\frac{5}{8}\sin\theta-\frac{5}{16}\sin 3\theta+\frac{1}{16}\sin 5\theta.$			
			i.e. $A=\frac{5}{8},\ B=-\frac{5}{16},\ C=\frac{1}{16}$	A1	2.2a	All three stated
			Alternative method 1			Sight of i in the denominator is necessary
			$(\sin\theta)^5=\left(\frac{\mathrm{e}^{\mathrm{i}\theta}-\mathrm{e}^{-\mathrm{i}\theta}}{2\mathrm{i}}\right)^5$	M1		
			$=\frac{1}{(2\mathrm{i})^5}\left(\mathrm{e}^{5\mathrm{i}\theta}-5\mathrm{e}^{3\mathrm{i}\theta}+10\mathrm{e}^{\mathrm{i}\theta}-10\mathrm{e}^{-\mathrm{i}\theta}+5\mathrm{e}^{-3\mathrm{i}\theta}-\mathrm{e}^{-5\mathrm{i}\theta}\right)$	A1		
			$=\frac{1}{32\mathrm{i}}\left(\left(\mathrm{e}^{5\mathrm{i}\theta}-\mathrm{e}^{-5\mathrm{i}\theta}\right)-5\left(\mathrm{e}^{3\mathrm{i}\theta}-\mathrm{e}^{-3\mathrm{i}\theta}\right)+10\left(\mathrm{e}^{\mathrm{i}\theta}-\mathrm{e}^{-\mathrm{i}\theta}\right)\right)$	M1		Collection to convert back
			$=\frac{1}{16}(\sin 5\theta-5\sin 3\theta+10\sin\theta)$			
			$\Rightarrow \sin^5\theta=\frac{5}{8}\sin\theta-\frac{5}{16}\sin 3\theta+\frac{1}{16}\sin 5\theta$			
			$\Rightarrow A=\frac{5}{8},\ B=-\frac{5}{16},\ C=\frac{1}{16}$	A1		All three stated

			Alternative method 2 $z=\cos\theta+i\sin\theta$ $\Rightarrow z^5=\cos5\theta+i\sin5\theta$ $=(\cos\theta+i\sin\theta)^5$	**M1**		De Moivre
			$=\cos^5\theta+5i\cos^4\theta\sin\theta-10\cos^3\theta\sin^2\theta-10i\cos^2\theta\sin^3\theta+5\cos\ \theta\sin^4\theta+i\sin^5\theta$ $\Rightarrow\sin5\theta=5\cos^4\theta\sin\theta-10\cos^2\theta\sin^3\theta+\sin^5\theta$ $=5\left(1-\sin^2\theta\right)^2\sin\theta-10\left(1-\sin^2\theta\right)\sin^3\theta+\sin^5\theta$ $=5\sin\theta-10\sin^3\theta+5\sin^5\theta-10\sin^3\theta+10\sin^5\theta+\sin^5\theta$ $=5\sin\theta-20\sin^3\theta+16\sin^5\theta$ $z^3=\cos3\theta+i\sin3\theta$ $=(\cos\theta+i\sin\theta)^5=\cos^3\theta+3i\cos^2\theta\sin\theta-3\cos\theta\sin^2\theta-i\sin^3\theta$ $\Rightarrow\sin3\theta=3\cos^2\theta\sin\theta-\sin^3\theta$ $=3\sin\theta-4\sin^3\theta$			
			$\Rightarrow\sin5\theta-5\sin3\theta=-10\sin\theta+16\sin^5\theta$	**A1**		for both
			$\Rightarrow16\sin^5\theta=10\sin\theta-5\sin3\theta+\sin5\theta$	**M1**		Eliminate $\sin^3\theta$
			$\Rightarrow A=\dfrac{10}{16}=\dfrac{5}{8},\ B=-\dfrac{5}{8},\ C=\dfrac{1}{16}$	**A1**		All three stated
				[4]		

Question			Answer	Marks	AO	Guidance
6			For AB, $V=\pi\times1^2\times4=12.566...(\ )$			4π
			For BC, $V=\int_a^b \pi x^2\mathrm{dy}=\pi\int_4^9\left(37-(y-10)^2\right)\mathrm{d}y$	**M1**	3.3	Split into two parts and use formulae An integral and an attempt at the volume of a cylinder must be seen
			$=356.05....$	**A1**	1.1	Integration – ignore limits **BC** $\frac{340}{3}\pi$
			$\Rightarrow$ Total $V=356.05...+12.566...=368.61...$ $=369\ \left(\mathrm{cm}^3\right)$ to 3 sf	**A1**	3.4	Units are not required $\frac{352}{3}\pi$
				[3]		

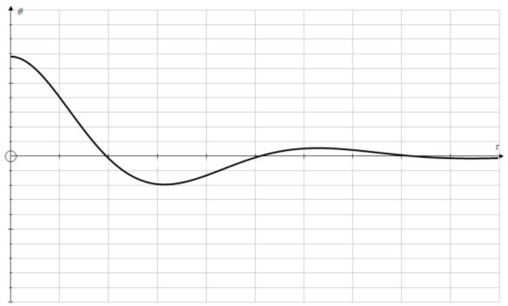
Question			Answer	Marks	AO	Guidance
7	(a)		$\begin{pmatrix} r=0\Rightarrow \sin 3\theta=0 \\ \Rightarrow 3\theta=0,\ \pi \end{pmatrix}\Rightarrow \theta=0,\ \dfrac{\pi}{3}$	B1	1.1	Both required Don't give if any extras within range. Ignore values outside range
				[1]		
	(b)		$\left[\sin\dfrac{3\pi}{6},\dfrac{\pi}{6}\right]$ i.e. $\left[1,\dfrac{\pi}{6}\right]$	B1 B1	1.1 1.1	For r For θ
				[2]		
	(c)		**DR** $\text{Area}=\dfrac{1}{2}\int_0^{\pi/3} r^2\,\mathrm{d}\theta=\dfrac{1}{2}\int_0^{\pi/3}\sin^2 3\theta\,\mathrm{d}\theta$	M1	1.1	Correct use of formula – ignore limits
			$=\dfrac{1}{4}\int_0^{\pi/3}(1-\cos 6\theta)\,\mathrm{d}\theta$	M1*	3.1a	Attempt to use double angle formula (Could be wrong way round, 2 missing or sign wrong)
			$=\dfrac{1}{4}\left[\theta-\dfrac{1}{6}\sin 6\theta\right]_0^{\pi/3}$	DepM1	1.1	Integrate *their integrand*
			$=\dfrac{1}{4}\left(\dfrac{\pi}{3}-0\right)=\frac{1}{12}\pi$	A1	1.1	Use correct limits, must be seen
				[4]		
	(d)		$\sin 3\theta=3\sin\theta-4\sin^3\theta$ $\Rightarrow r=\dfrac{3y}{r}-4\left(\dfrac{y}{r}\right)^3$	M1	1.1	Using triple angle formula and $y=r\sin\theta$
			$\Rightarrow r^4=3r^2y-4y^3$ $\left(x^2+y^2\right)^2=3y\left(x^2+y^2\right)-4y^3$ **oe** e.g. $\left(x^2+y^2\right)^2=3x^2y-y^3$	A1	1.1	isw
				[2]		

Question			Answer	Marks	AO	Guidance
8	(a)		$y = 4\sinh x + 3\cosh x$			
			$\Rightarrow \frac{\mathrm{d}y}{\mathrm{d}x} = 4\cosh x + 3\sinh x$	M1	1.1	Diffn (Hyperbolics or exponentials)
			$= 0$ when $4\cosh x + 3\sinh x = 0$			
			$\Rightarrow 4\left(\frac{\mathrm{e}^x + \mathrm{e}^{-x}}{2}\right) + 3\left(\frac{\mathrm{e}^x - \mathrm{e}^{-x}}{2}\right) = 0$	M1	2.1	Set = 0 and use exponential forms – can change to exponentials before diffn.
			$\Rightarrow \mathrm{e}^{2x} = -\frac{1}{7}$	A1	2.4	Conclusion with justification
			which is not possible as $\mathrm{e}^{2x} > 0$ so no turning points			
			Alternative method			
			$y = 4\sinh x + 3\cosh x$			
			$\Rightarrow \frac{\mathrm{d}y}{\mathrm{d}x} = 4\cosh x + 3\sinh x$	M1		Differentiate
			$= 0$ when $\tanh x = -\frac{4}{3}$	M1		Set = 0 and use formula for tanh
			But $\lvert\tanh x\rvert < 1$ for all x.			
			So there are no values of x for which $\frac{\mathrm{d}y}{\mathrm{d}x} = 0$	A1		Conclusion with justification
			So no turning points			
				[3]		

	(b)		$y = 4\sinh x + 3\cosh x = 5$			
			$\Rightarrow 4\left(\frac{e^x - e^{-x}}{2}\right) + 3\left(\frac{e^x + e^{-x}}{2}\right) = 5$	M1	3.1a	Use of exponentials
			$\Rightarrow 7e^x - e^{-x} = 10 \Rightarrow 7e^{2x} - 10e^x - 1 = 0$	M1	3.1a	equation of the form $ae^{2x} + be^x + c = 0$ (for non-zero a, b and c)
			$e^x = \frac{10 \pm \sqrt{100+28}}{14} = \frac{5+\sqrt{32}}{7}$ or $\frac{5-\sqrt{32}}{7}$	A1	1.1	Two roots for e^x
			But $e^x > 0$ so cannot $= \frac{5-\sqrt{32}}{7}$	A1	2.3	One rejected plus reason
			So the only root is $e^x = \frac{5+\sqrt{32}}{7}$			
			$\Rightarrow x = \ln\left(\frac{5+4\sqrt{2}}{7}\right)$	A1	1.1	Ignore inclusion of 2nd root
			Alternative method (see appendix for full working)			
			$4\sinh x + 3\cosh x = 5 \Rightarrow 4\sinh x = 5 - 3\cosh x$	M1		Use Pythagoras
			$\therefore 16\sinh^2 x = 16(\cosh^2 x - 1) = 25 - 30\cosh x + 9\cosh^2 x$			
			$7\cosh^2 x + 30\cosh x - 41 = 0$	M1		Quadratic in cosh (or sinh)
			$\cosh x \geq 1 \Rightarrow \cosh x = \frac{-15+16\sqrt{2}}{7}$	A1		Two roots
			$\Rightarrow x = \cosh^{-1}\frac{-15+16\sqrt{2}}{7} = \pm\ln\left(\frac{-15+16\sqrt{2}+4\sqrt{43-30\sqrt{2}}}{7}\right)$			
			But the negative root does not work in the original equation since LHS would be negative while RHS would be positive (but equal when squared).	A1		One rejected plus reason
			$\therefore x = \ln\left(\frac{-15+16\sqrt{2}+4\sqrt{43-30\sqrt{2}}}{7}\right)$	A1		
				[5]		

Question			Answer	Marks	AO	Guidance
9	(a)		$\begin{pmatrix} 2 & 1 \\ -1 & 0 \end{pmatrix}\begin{pmatrix} x \\ kx \end{pmatrix} = \begin{pmatrix} 2x+kx \\ -x \end{pmatrix}$	M1 A1	3.1a 1.1	
			same line $\Rightarrow -x = k(2x+kx)$ for all $x(\neq 0)$ $\Rightarrow -1 = k(2+k) \Rightarrow k^2+2k+1=0$	M1	2.1	
			$\Rightarrow k=-1$ (i.e. $y=-x$)	A1	1.1	Value of k can be implied by the correct equation
				[4]		
	(b)		$\begin{pmatrix} 2 & 1 \\ -1 & 0 \end{pmatrix}\begin{pmatrix} x \\ -x \end{pmatrix} = \begin{pmatrix} 2x-x \\ -x \end{pmatrix} = \begin{pmatrix} x \\ -x \end{pmatrix}$ so each point maps to itself and it is a line of invariant points	B1	2.4	Must have a reason e.g. it is sufficient to test one point other than (0, 0)
				[1]		

Question			Answer	Marks	AO	Guidance
10			$\frac{1}{(2r-1)(2r+1)}=\frac{A}{2r-1}+\frac{B}{2r+1}$	M1	3.1a	partial fractions
			$\Rightarrow A(2r+1)+B(2r-1)=1$	M1	1.1	Allow any method to determine A and B
			$\Rightarrow A-B=1,\ A+B=0$ $\Rightarrow A=\frac{1}{2}, B=-\frac{1}{2}$	A1	1.1	Both values
			$\Rightarrow \sum_{r=1}^{n}\frac{1}{(2r-1)(2r+1)}=\frac{1}{2}\left(\begin{array}{l}\left(\frac{1}{1}-\frac{1}{3}\right)+\left(\frac{1}{3}-\frac{1}{5}\right)+\ldots\\+\left(\frac{1}{2n-3}-\frac{1}{2n-1}\right)+\left(\frac{1}{2n-1}-\frac{1}{2n+1}\right)\end{array}\right)$	M1 M1	3.1a 2.1	Use of differences Deal with subtraction
			$=\frac{1}{2}\left(1-\frac{1}{2n+1}\right)$ **oe**	A1	1.1	
			$\frac{1}{2}\left(1-\frac{1}{2n+1}\right)\geqslant 0.49$ $\Rightarrow n\geqslant 0.98n+0.49$ $\Rightarrow n\geqslant\frac{0.49}{0.02}=24.5$	M1	3.1a	Use of inequality on *their* formula
			$\Rightarrow n=25$	A1	3.2a	No marks for a purely numerical solution.
				[8]		

Question			Answer	Marks	AO	Guidance
11	(a)	(i)	For SHM $\lambda = 0$	**B1**	3.3	
				[1]		
	(a)	(ii)	The door should close, but in SHM the motion continues indefinitely	**B1**	3.5b	
				[1]		
	(b)		Over- or critical- damping implies $\lambda^2 - 12 \geqslant 0$ So $\lambda \geqslant 2\sqrt{3}$	**M1** **A1**	3.3 3.4	Consider discriminant with $\geq$ or $>$ **Ignore** $\lambda \leqslant -2\sqrt{3}$
				[2]		
	(c)		e.g. 	**B1**	3.4	Graph of under-damped system. Start anywhere non-zero on θ-axis with zero gradient. Each peak must be lower than before At least two peaks (not including start point) The graph must look as though it is approaching the t axis
				[1]		

Appendix

8(b) Alternate solution

$4\sinh x + 3\cosh x = 5 \Rightarrow 4\sinh x = 5 - 3\cosh x$

$\therefore 16\sinh^2 x = 16(\cosh^2 x - 1) = 25 - 30\cosh x + 9\cosh^2 x$

$7\cosh^2 x + 30\cosh x - 41 = 0$

$$\cosh x = \frac{-30 \pm \sqrt{30^2 - 4\times 7\times -41}}{2\times 7} = \frac{-30 \pm \sqrt{2048}}{14}$$

$$\cosh x \geq 1 \Rightarrow \cosh x = \frac{-30 + 32\sqrt{2}}{14} = \frac{-15+16\sqrt{2}}{7}$$

$$\Rightarrow x = \cosh^{-1}\frac{-15+16\sqrt{2}}{7} = \pm\ln\left(\frac{-15+16\sqrt{2}}{7} + \sqrt{\left(\frac{-15+16\sqrt{2}}{7}\right)^2 - 1}\right)$$

$$= \pm\ln\left(\frac{-15+16\sqrt{2}}{7} + \sqrt{\frac{225+512-480\sqrt{2}}{49} - \frac{49}{49}}\right)$$

$$= \pm\ln\left(\frac{-15+16\sqrt{2}}{7} + \sqrt{\frac{688-480\sqrt{2}}{49}}\right) = \pm\ln\left(\frac{-15+16\sqrt{2}+4\sqrt{43-30\sqrt{2}}}{7}\right)$$

But the negative root does not work in the original equation since LHS would be negative while RHS would be positive (but equal when squared).

$$\therefore x = \ln\left(\frac{-15+16\sqrt{2}+4\sqrt{43-30\sqrt{2}}}{7}\right)$$

NB $\left(5-3\sqrt{2}\right)^2 = 25+18-30\sqrt{2}$

$= 43-30\sqrt{2}$ and $5-3\sqrt{2} > 0$

$$\therefore x = \ln\left(\frac{-15+16\sqrt{2}+4\left(5-3\sqrt{2}\right)}{7}\right) = \ln\left(\frac{5+4\sqrt{2}}{7}\right)$$

Question 2(a)(ii) Alternative solution

$y=\tan^{-1}(1+x)\Rightarrow 1+x=\tan y$

$\Rightarrow 1=\sec^2 y.\dfrac{\mathrm{d}y}{\mathrm{d}x}$

$\Rightarrow \dfrac{\mathrm{d}y}{\mathrm{d}x}=\dfrac{1}{\sec^2 y}=\dfrac{1}{1+\tan^2 y}=\dfrac{1}{1+(1+x)^2}$

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