Pearson
Edexcel

Mark Scheme (Results)

November 2021

Pearson Edexcel GCE

In AS Further Mathematics (8FM0)

Paper 21 Further Pure Mathematics 1

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General Marking Guidance

- All candidates must receive the same treatment. Examiners must mark the first candidate in exactly the same way as they mark the last.
- Mark schemes should be applied positively. Candidates must be rewarded for what they have shown they can do rather than penalised for omissions.
- Examiners should mark according to the mark scheme not according to their perception of where the grade boundaries may lie.
- There is no ceiling on achievement. All marks on the mark scheme should be used appropriately.
- All the marks on the mark scheme are designed to be awarded. Examiners should always award full marks if deserved, i.e. if the answer matches the mark scheme. Examiners should also be prepared to award zero marks if the candidate's response is not worthy of credit according to the mark scheme.
- Where some judgement is required, mark schemes will provide the principles by which marks will be awarded and exemplification may be limited.
- When examiners are in doubt regarding the application of the mark scheme to a candidate's response, the team leader must be consulted.
- Crossed out work should be marked UNLESS the candidate has replaced it with an alternative response.

EDEXCEL GCE MATHEMATICS

General Instructions for Marking

1. The total number of marks for the paper is 40.

2. The Edexcel Mathematics mark schemes use the following types of marks:

- **M** marks: method marks are awarded for 'knowing a method and attempting to apply it', unless otherwise indicated.
- **A** marks: Accuracy marks can only be awarded if the relevant method (M) marks have been earned.
- **B** marks are unconditional accuracy marks (independent of M marks)
- Marks should not be subdivided.

3. Abbreviations

These are some of the traditional marking abbreviations that will appear in the mark schemes.

- bod – benefit of doubt
- ft – follow through
- the symbol $\sqrt{}$ will be used for correct ft
- cao – correct answer only
- cso - correct solution only. There must be no errors in this part of the question to obtain this mark
- isw – ignore subsequent working
- awrt – answers which round to
- SC: special case
- oe – or equivalent (and appropriate)
- dep – dependent
- indep – independent
- dp decimal places
- sf significant figures
- $\ast$ The answer is printed on the paper
- The second mark is dependent on gaining the first mark

4. For misreading which does not alter the character of a question or materially simplify it, deduct two from any A or B marks gained, in that part of the question affected.

5. Where a candidate has made multiple responses and indicates which response they wish to submit, examiners should mark this response.
   If there are several attempts at a question which have not been crossed out, examiners should mark the final answer which is the answer that is the most complete.

6. Ignore wrong working or incorrect statements following a correct answer.

7. Mark schemes will firstly show the solution judged to be the most common response expected from candidates. Where appropriate, alternatives answers are provided in the notes. If examiners are not sure if an answer is acceptable, they will check the mark scheme to see if an alternative answer is given for the method used.

Question	Scheme	Marks	AOs
1	$x(x-1) > \dfrac{x-1}{x}$		
	$\dfrac{x^2(x-1)-x-1}{x} > 0$ or $x^3(x-1)-x(x-1) > 0$	M1	2.1
	$\dfrac{(x-1)^2(x+1)}{x} > 0$ or $x(x-1)^2(x+1) > 0$	M1	1.1b
	Critical values 0 and 1	A1	1.1b
	All three critical values $-1,\ 0, 1$	A1	1.1b
	$\{x \in \mathbb{R} : x < -1\} \cup \{x \in \mathbb{R} : 0 < x < 1\} \cup \{x \in \mathbb{R} : x > 1\}$	M1 A1	2.2a 2.5
		(6)	
			(6 marks)

Notes

M1: Gathers terms on one side and puts over a common denominator, or multiplies by x^2 and gathers terms on one side
M1: Factorises numerator into 3 factors or factorises into 4 factors
A1: Identifies the critical values 0 and 1
A1: All 3 correct critical values
M1: Deduces that 1 "inside" inequality and 2 "outside" inequalities are required with critical values in ascending order as shown
A1: Exactly 3 correct intervals using correct notation
Allow e.g. $\{x : x < -1\} \cup \{x : 0 < x < 1\} \cup \{x : x > 1\}$

Question	Scheme	Marks	AOs
2	$\left(\frac{\mathrm{d}y}{\mathrm{d}x}\right)_1 \approx \frac{(y_2-1)}{0.2}$	B1	1.1b
	$\left(\frac{\mathrm{d}^2y}{\mathrm{d}x^2}\right)_1 \approx \frac{(y_2-2(2)+1)}{0.1^2}$	B1	1.1b
	$\frac{(y_2-2(2)+1)}{0.1^2}+15\left(\frac{(y_2-1)}{0.2}\right)-3(2)^2=2(0.1)\Rightarrow y_2=...$	M1	2.1
	$y_2 \approx \frac{1936}{875}\ \ (2.2125...)$	A1	1.1b
	$\frac{\left(y_3-2\left(\frac{1936}{875}\right)+2\right)}{0.1^2}+15\left(\frac{y_3-2}{0.2}\right)-3\left(\frac{1936}{875}\right)^2=2(0.2)\Rightarrow y_3=...$	M1	2.1
	$y_3 \approx 2.32914...$	A1	1.1b
		(6)	
			(6 marks)

Notes

B1: Correct expression for the first derivative using the given values and the approximation
B1: Correct expression for the second derivative using the given values and the approximation
M1: Uses the approximations for the first and second derivatives, substitutes into the differential equation and obtains a value for y at $x = 0.2$
A1: Correct value for y at $x = 0.2$ (accept the exact value or awrt 2.21)
M1: Completes the process by using their value for y at $x = 0.2$ to obtain a value for y at $x = 0.3$
A1: Correct value for y when $x = 0.3$ (allow awrt 2.33)

Question	Scheme	Marks	AOs
3(a)	$x=0 \Rightarrow D = 2\sin(0)+3\cos(0)+6=6+3=9\,\text{m}$	B1	3.4
		(1)	
(b)	$D = 2\left(\frac{2t}{1+t^2}\right)+3\left(\frac{1-t^2}{1+t^2}\right)+6$	M1	1.1b
	$=\frac{4t+3-3t^2+6+6t^2}{1+t^2}$	M1	1.1b
	$=\frac{3t^2+4t+9}{1+t^2}$ *	A1*	2.1
		(3)	
(c)	$\frac{3t^2+4t+9}{1+t^2}=5 \Rightarrow 3t^2+4t+9=5+5t^2$	M1	3.4
	$t^2-2t-2=0$	A1	1.1b
	$t=\frac{2\pm\sqrt{4+8}}{2} \Rightarrow \frac{x}{6}=\tan^{-1}\left(1+\sqrt{3}\right)$ or $\frac{x}{6}=\tan^{-1}\left(1-\sqrt{3}\right)$	M1	3.4
	$\frac{x}{6}=\tan^{-1}\left(1+\sqrt{3}\right)=1.21... \Rightarrow x=...$	M1	3.1b
	0719 or 07:19 am	A1	3.2a
		(5)	
			(9 marks)

Notes

(a)
B1: Obtains the correct depth of 9 m (must include units)
(b)
M1: Uses the correct formulae to obtain D in terms of t
M1: Correct method to obtain a common denominator
A1*: Collects terms and simplifies to obtain the printed answer with no errors
(c)
M1: Uses $D = 5$ with the model and multiplies up to obtain a quadratic equation in t
A1: Correct 3TQ
M1: Solves their 3TQ in t and proceeds to obtain values of $\frac{x}{6}$ as suggested by the model
M1: A fully correct strategy to identify the required value of x from the positive root of the quadratic equation in t
A1: Correct time. Allow e.g. 439 minutes after midnight.

Question	Scheme	Marks	AOs
4(a)	E.g. $\overrightarrow{AB}=\begin{pmatrix}-25\\9\\5\end{pmatrix},\ \overrightarrow{AC}=\begin{pmatrix}-20\\5\\-4\end{pmatrix}$	M1	1.1b
	$\left\|\overrightarrow{AB}\times\overrightarrow{AC}\right\|=\begin{vmatrix}\mathbf{i}&\mathbf{j}&\mathbf{k}\\-25&9&5\\-20&5&-4\end{vmatrix}=...$	M1	1.1b
	$\text{Area}=\frac{1}{2}\begin{vmatrix}-61\\-200\\55\end{vmatrix}=\frac{1}{2}\sqrt{61^2+200^2+55^2}=108*$	A1*	2.2a
		(3)	
	(a) Alternative:		
	$AB=\sqrt{25^2+9^2+5^2},\ AC=\sqrt{20^2+5^2+4^2},\ BC=\sqrt{5^2+4^2+9^2}$	M1	1.1b
	$AC^2=AB^2+BC^2-2AB\times BC\cos ABC$ $\Rightarrow 441=731+122-2\times\sqrt{731}\times\sqrt{122}\cos ABC$ $\Rightarrow \cos ABC=\frac{731+122-441}{2\times\sqrt{731}\times\sqrt{122}}$	M1	1.1b
	$\text{Area}=\frac{1}{2}\sqrt{731}\sqrt{122}\sin ABC=108*$	A1	2.2a
(b)	A complete attempt to find the volume of the tetrahedron	M1	3.1a
	E.g. $\begin{vmatrix}18&-14&-2\\-7&-5&3\\-2&-9&-6\end{vmatrix}=...$	M1	1.1b
	$=1592$	A1	1.1b
	$V=\frac{1592}{6}$ or e.g. $V=\frac{796}{3}$	A1	1.1b
		(4)	
(c)	$\text{Mass}=\frac{1592}{6}\times0.85\div1000\ \text{(kg)}$	M1	2.1
	$\{=0.2255333...\}=$ awrt 0.226 (kg)	A1	1.1b
		(2)	
			(9 marks)

Notes

(a)
M1: Attempts to find 2 edges of the required triangle
M1: Uses the correct process of the vector product for 2 appropriate vectors

A1*: Deduces the correct area with no errors but condone sign slips on the components provided the work is otherwise correct e.g. allow $\text{Area} = \frac{1}{2}\begin{vmatrix} -61 \\ -200 \\ -55 \end{vmatrix} = \frac{1}{2}\sqrt{61^2 + 200^2 + 55^2} = 108*$

Alternative

M1: Attempts lengths of all 3 sides

M1: Applies the cosine rule to find one of the angles of the triangle

A1*: Deduces the correct area with no errors

(b)

M1: See scheme

M1: Uses appropriate vectors in an attempt at the scalar triple product

A1: Correct numerical expression for the scalar triple product (allow $\pm$)

A1: Correct volume

(c)

M1: A correct method for changing their units for their volume and for finding the mass in kg

A1: Correct answer

Question	Scheme	Marks	AOs
5(a)	$\frac{\mathrm{d}y}{\mathrm{d}x}=\frac{2a}{2ap}=\frac{1}{p}$ or $y=2\sqrt{a}\sqrt{x}\Rightarrow\frac{\mathrm{d}y}{\mathrm{d}x}=\frac{\sqrt{a}}{\sqrt{x}}=\frac{1}{p}$ or $2y\frac{\mathrm{d}y}{\mathrm{d}x}=4a\Rightarrow\frac{\mathrm{d}y}{\mathrm{d}x}=\frac{2a}{y}=\frac{1}{p}$	B1	1.1b
	$y-2ap=-p\left(x-ap^2\right)$	M1	2.1
	$2aq-2ap=-p\left(aq^2-ap^2\right)$ $pq^2+2q-2p-p^3=0$	A1	1.1b
	$\left(q-p\right)\left(pq+p^2+2\right)=0\Rightarrow q=...$	M1	3.1a
	$q=\frac{-p^2-2}{p}$ *	A1*	1.1b
		(5)	
(b)	$PQ^2=\left(ap^2-aq^2\right)^2+\left(2ap-2aq\right)^2$	M1	1.1b
	$=a^2\left(p-q\right)^2\left(p+q\right)^2+4a^2\left(p-q\right)^2$ $=a^2\left(p-q\right)^2\left[\left(p+q\right)^2+4\right]$ $=a^2\left(2p+\frac{2}{p}\right)^2\left[\left(-\frac{2}{p}\right)^2+4\right]$	M1 A1	2.1 1.1b
	$=\frac{4a^2}{p^2}\left(p^2+1\right)^2\frac{4}{p^2}\left(p^2+1\right)=\frac{16a^2}{p^4}\left(p^2+1\right)^3$	A1 A1	1.1b 1.1b
		(5)	
			(10 marks)

Notes

(a)

B1: Deduces the correct tangent gradient

M1: Correct strategy for the equation of the normal

A1: Correct equation in terms of p and q

M1: Applies a correct strategy for finding q in terms of p. E.g. uses the fact that $q = p$ is known and uses inspection or long division to find the other root

A1*: Correct proof with no errors

Alternative:

B1: As above

M1A1: $\frac{2aq-2ap}{aq^2-ap^2}\times\frac{1}{p}=-1$

M1: Finds gradient of PQ and uses product of gradients $= -1$

A1: Correct equation

M1A1: As above

(b)

M1: Applies Pythagoras correctly to find PQ^2

M1: Uses their q in terms of p to obtain an expression in terms of p only

A1: Correct expression in any form in terms of p only

A1: $k = 16$ or $n = 3$

A1: $k = 16$ and $n = 3$