Mark Scheme (Results)

November 2021

Pearson Edexcel GCE
In AS Further Mathematics (8FM0)
Paper 28 Decision Mathematics 2

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General Marking Guidance

- All candidates must receive the same treatment. Examiners must mark the first candidate in exactly the same way as they mark the last.
- Mark schemes should be applied positively. Candidates must be rewarded for what they have shown they can do rather than penalised for omissions.
- Examiners should mark according to the mark scheme not according to their perception of where the grade boundaries may lie.
- There is no ceiling on achievement. All marks on the mark scheme should be used appropriately.
- All the marks on the mark scheme are designed to be awarded. Examiners should always award full marks if deserved, i.e. if the answer matches the mark scheme. Examiners should also be prepared to award zero marks if the candidate's response is not worthy of credit according to the mark scheme.
- Where some judgement is required, mark schemes will provide the principles by which marks will be awarded and exemplification may be limited.
- When examiners are in doubt regarding the application of the mark scheme to a candidate's response, the team leader must be consulted.
- Crossed out work should be marked UNLESS the candidate has replaced it with an alternative response.

EDEXCEL GCE MATHEMATICS
General Instructions for Marking

1. The total number of marks for the paper is 40.

2. The Edexcel Mathematics mark schemes use the following types of marks:

- **M** marks: method marks are awarded for 'knowing a method and attempting to apply it', unless otherwise indicated.
- **A** marks: Accuracy marks can only be awarded if the relevant method (M) marks have been earned.
- **B** marks are unconditional accuracy marks (independent of M marks)
- Marks should not be subdivided.

3. Abbreviations

These are some of the traditional marking abbreviations that will appear in the mark schemes.

- bod – benefit of doubt
- ft – follow through
- the symbol $\surd$ will be used for correct ft
- cao – correct answer only
- cso - correct solution only. There must be no errors in this part of the question to obtain this mark
- isw – ignore subsequent working
- awrt – answers which round to
- SC: special case
- oe – or equivalent (and appropriate)
- dep – dependent
- indep – independent
- dp decimal places
- sf significant figures
- $*$ The answer is printed on the paper
- ⌐ The second mark is dependent on gaining the first mark

4. For misreading which does not alter the character of a question or materially simplify it, deduct two from any A or B marks gained, in that part of the question affected.

5. Where a candidate has made multiple responses and indicates which response they wish to submit, examiners should mark this response.
   If there are several attempts at a question which have not been crossed out, examiners should mark the final answer which is the answer that is the most complete.

6. Ignore wrong working or incorrect statements following a correct answer.

7. Mark schemes will firstly show the solution judged to be the most common response expected from candidates. Where appropriate, alternatives answers are provided in the notes. If examiners are not sure if an answer is acceptable, they will check the mark scheme to see if an alternative answer is given for the method used.

Question	Scheme	Marks	AOs
1(a)	Subtract each entry from a constant (e.g., 72) to convert from maximisation problem to minimisation Add an additional dummy column with any equal values (e.g., 72) to create a square array and input a suitable large number in cell BR	B1 B1	1.1a 3.5c
		(2)	
(b)	e.g. $\begin{pmatrix} & P & Q & R & S & X \\ A & 17 & 16 & 14 & 15 & 72 \\ B & 12 & 11 & 100 & 8 & 72 \\ C & 13 & 12 & 10 & 9 & 72 \\ D & 8 & 6 & 1 & 3 & 72 \\ E & 7 & 4 & 0 & 6 & 72 \end{pmatrix}$ Reduce row A by 14, reduce row B by 8, reduce row C by 9, reduce row D by 1 and no reduction in row E. Reduce column P by 3, reduce column Q by 2, no reductions in columns R and S, reduce column X by 58 (or equivalent) Reducing rows and columns gives e.g., $\begin{pmatrix} & P & Q & R & S & X \\ A & 0 & 0 & 0 & 1 & 0 \\ B & 1 & 1 & 92 & 0 & 6 \\ C & 1 & 1 & 1 & 0 & 5 \\ D & 4 & 3 & 0 & 2 & 13 \\ E & 4 & 2 & 0 & 6 & 14 \end{pmatrix}$ Minimum of three lines required to cover the zeros hence solution is not optimal (augment by 1) e.g., $\begin{pmatrix} & P & Q & R & S & X \\ A & 0 & 0 & 1 & 2 & 0 \\ B & 0 & 0 & 92 & 0 & 5 \\ C & 0 & 0 & 1 & 0 & 4 \\ D & 3 & 2 & 0 & 2 & 12 \\ E & 3 & 1 & 0 & 6 & 13 \end{pmatrix}$ Minimum of four lines required to cover the zeros hence solution is not optimal (augment by 1)	B1 B1 M1 M1	1.1b 2.4 1.1b 1.1b

	e.g., $\begin{pmatrix} & P & Q & R & S & X \\ A & 0 & 0 & 2 & 2 & 0 \\ B & 0 & 0 & 93 & 0 & 5 \\ C & 0 & 0 & 2 & 0 & 4 \\ D & 2 & 1 & 0 & 1 & 11 \\ E & 2 & 0 & 0 & 5 & 12 \end{pmatrix}$	M1	1.1b
	Minimum of five lines required to cover the zeros hence solution is optimal	B1	2.4
	(£) 262	A1	2.2a
		(7)	

(9 marks)

Notes:

(a)

B1: Valid statement regarding converting a maximisation problem to a minimisation problem

B1: Explain the need to add a dummy column to create a square array and input a large value in cell BR (oe)

(b) – No mark in this part if array is not square

B1: Mark awarded when both steps complete (subtraction, addition of extra column and large value in cell BR)

B1: Correct statements regarding row and column reduction (either by giving correct numerical reductions or correct general statement)

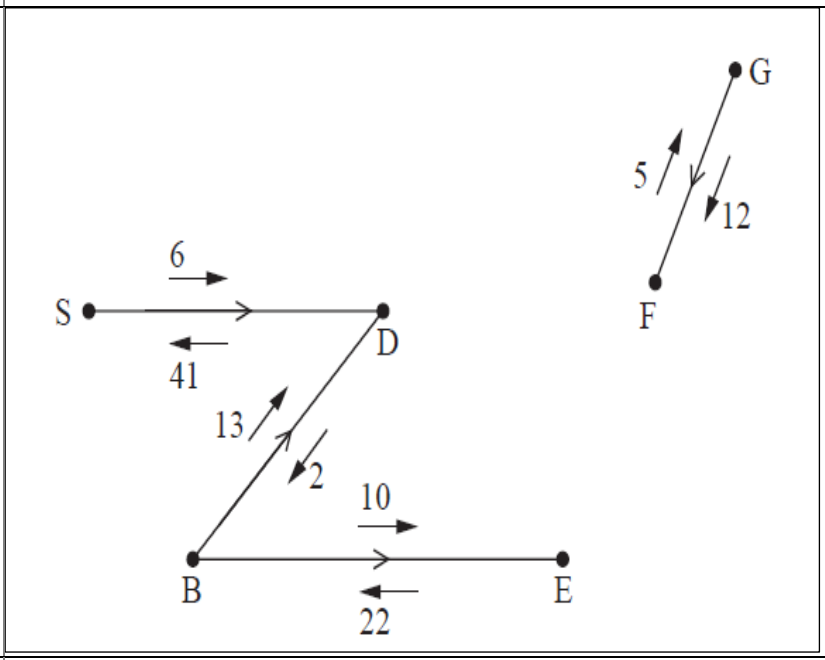
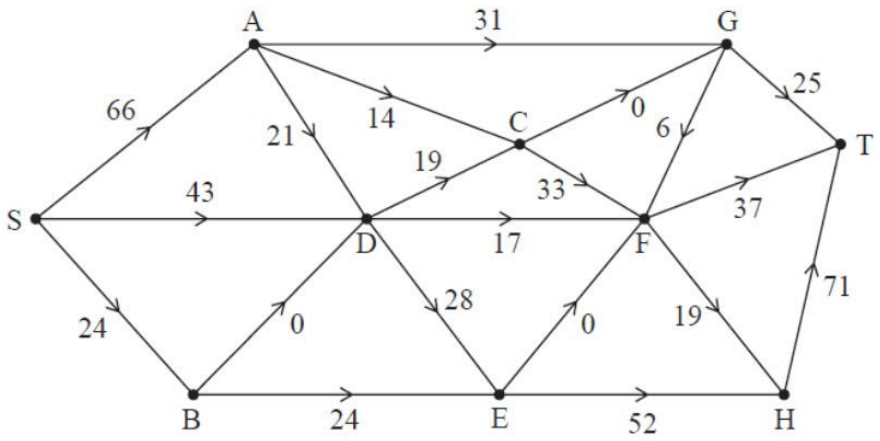
M1: Simplifying the initial matrix by reducing rows and then columns

M1: Develop an improved solution – need to see one double covered +e; one uncovered –e; and one single covered unchanged. 3 lines needed to 4 lines needed

M1: Develop an improved solution – need to see one double covered +e; one uncovered –e; and one single covered unchanged. 4 lines needed to 5 lines needed (so getting to the optimal table)

B1: Correct statements regarding the minimum number of lines to cover zeros

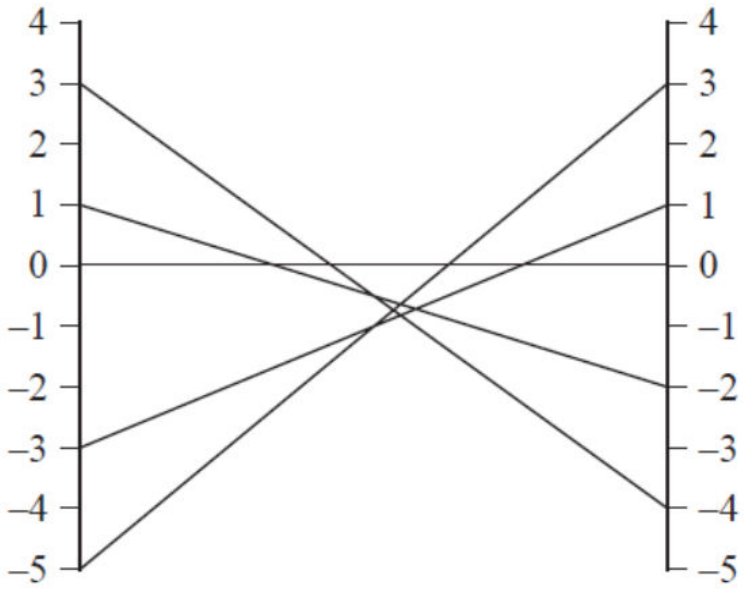
A1: cso on final table (so must have scored all previous marks in this part) + deduction of the correct total earnings

Question	Scheme	Marks	AOs
2(a)	125	B1	1.1b
		(1)	
(b)	Value of cut = 31 + 8 + 37 + 19 + 56 = 151	B1	1.1b
		(1)	
(c)		M1 A1	1.1b 1.1b
		(2)	
(d)	e.g., SADFHT – 4; SDBEHT – 2; SACFT – 1; SAGFEHT - 1	M1 A1 A1	1.1b 1.1b 1.1b
		(3)	
(e)	e.g., 	B1	1.1b
		(1)	

(f)	Use of max-flow min-cut theorem Identification of cut through GT, FT, FH, EF, DE, BD and SB Value of flow = 133 Therefore it follows that flow is maximal	M1 A1 A1	2.1 3.1a 2.2a
		(3)	
			(11 marks)

Notes:

(a)
B1: cao
(b)
B1: cao
(c)
M1: Two numbers on each arc and at least two arcs or four numbers correct (so correct numbers with the correct arrows).
A1: cao (do give bod since they might well cross these numbers out.)
(d)
M1: One flow augmenting route found from S to T
A1: Two correct routes + flow values
A1: cso – increasing the flow by 8
(e)
B1: cao
(f)
M1: Construct argument based on max-flow min-cut theorem (e.g. attempt to find a cut through saturated arcs)
A1: Use appropriate process of finding a minimum cut – cut + value correct
A1: Correct deduction that the flow is maximal

Question	Scheme	Marks	AOs
3(a)	Row minima: –3, –1, –3, –1 max is –1 Column maxima: 4, 5 min is 4 Row(maximin) ≠ Col(minimax) therefore game is not stable	M1 A1	1.1b 2.4
		(2)	
(b)	If A plays option Q, B's gains are $-4p + 3(1-p) = 3 - 7p$ If A plays option R, B's gains are $-2p + 1(1-p) = 1 - 3p$ If A plays option S, B's gains are $3p + (-5)(1-p) = -5 + 8p$ If A plays option T, B's gains are $p + (-3)(1-p) = -3 + 4p$  $3 - 7p = -3 + 4p \Rightarrow p = 6/11$ B should play option X with probability 6/11 and option Y with probability 5/11	M1 A1 M1dep A1 A1 A1ft	1.1b 1.1b 1.1b 1.1b 1.1b 3.2a
		(6)	
(c)	V(B) = 3 – 7(6/11) = –9/11 ⇒ V(A) = 9/11	B1	2.2a
		(1)	
(d)	If A plays their option Q with probability q and their option T with probability $1-q$ then $4q + (-1)(1-q) = \frac{9}{11}$ $q = \frac{4}{11} \Rightarrow$ A should play option Q with probability 4/11, options R and S never and option T with probability 7/11	M1 A1	3.1b 3.2a
		(2)	
			(11 marks)

Notes:

(a)

M1: Finding row minimums and column maximums – condone one error

A1: Row maximin (–1) = col minimax (4) so unstable (dependent on correct row minimums and column maximums)

(b)

M1: Setting up four expressions in terms of p

A1: All four expressions correct

M1dep: Axes correct, at least two lines correctly drawn for their expressions (dependent on previous M mark)

A1: Correct graph

A1: Using the graph to obtain the correct probability expressions leading to the correct value of p

A1ft: Interpret their value of p in the context of the question – must refer to play and name options

(c)

B1: cao

(d):

M1: Setting up a linear equation with their V(A) and the two options used in **(b)**

A1: Interpret the correct value of q in the context of the question – must refer to play and name options

Question	Scheme	Marks	AOs
4(a)	$r = 0.25$	B1	3.4
		(1)	
(b)	(aux equation $m - 1.0025 = 0 \Rightarrow$) complementary fn is $A(1.0025)^n$	B1	1.1b
	Consider a trial solution of the form $u_n = \lambda$ so $\lambda - 1.0025\lambda = -x$ $\Rightarrow \lambda = ...$	M1	1.1b
	General solution is $u_n = A(1.0025)^n + 400x$	A1	1.1b
	$n = 1,\ u_1 = 155000 \Rightarrow A = ...$	M1	3.4
	$u_n = (155000 - 400x)(1.0025)^{n-1} + 400x$	A1	1.1b
		(5)	
(c)	Applying $u_{360} = 0$	B1	3.1b
	$155000(1.0025)^{359} + 400x(1 - 1.0025^{359}) = 0 \Rightarrow x = ...$	M1	1.1b
	$x = 654.61$	A1	2.2a
		(3)	
			(9 marks)

Notes:

(a)

B1: cao

(b)

B1: cao

M1: Substituting their trial solution into the recurrence relation in an attempt to find their λ (which if correct is $400x$)

A1: cao for the general solution

M1: Using the conditions in the model to calculate A (which if correct is $(155000 - 400x)(1.0025)^{-1}$)

A1: cao for the particular solution (oe)

(c)

B1: Applying $u_{360} = 0$ to their general solution (dependent on both M marks in **(b)**)

M1: Dependent on previous B mark – solving their linear equation to find x

A1: cao