OCR
Oxford Cambridge and RSA

Wednesday 14 October 2020 – Afternoon

AS Level Mathematics A

H230/02 Pure Mathematics and Mechanics

Printed Answer Booklet

Time allowed: 1 hour 30 minutes

You must have:
- Question Paper H230/02 (inside this document)
- a scientific or graphical calculator

Please write clearly in black ink. **Do not write in the barcodes.**

Centre number | | | | | | Candidate number | | | | |

First name(s) ______________________________

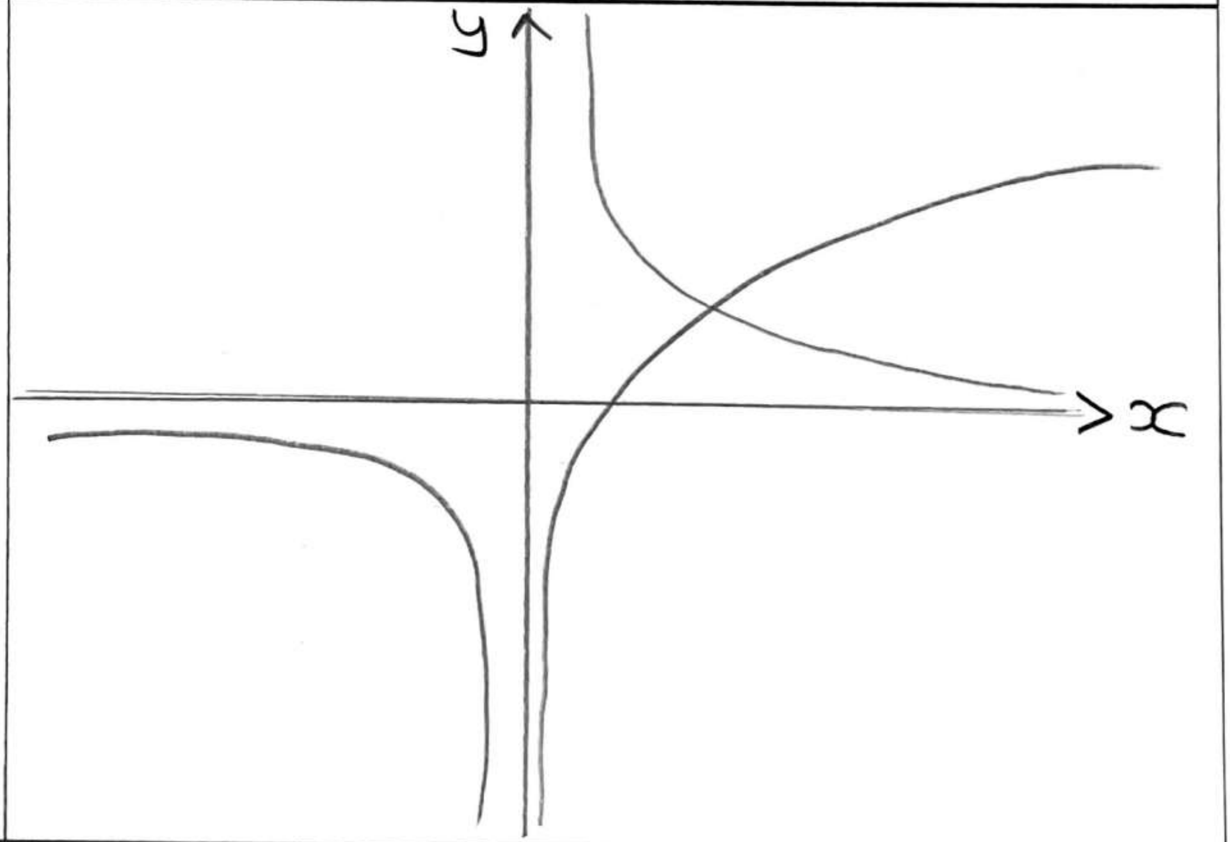
Last name ______________________________

INSTRUCTIONS
- Use black ink. You can use an HB pencil, but only for graphs and diagrams.
- Write your answer to each question in the space provided in the **Printed Answer Booklet**. If you need extra space use the lined pages at the end of the Printed Answer Booklet. The question numbers must be clearly shown.
- Answer **all** the questions.
- Where appropriate, your answer should be supported with working. Marks might be given for using a correct method, even if your answer is wrong.
- Give non-exact numerical answers correct to 3 significant figures unless a different degree of accuracy is specified in the question.
- The acceleration due to gravity is denoted by $g\,\text{m}\,\text{s}^{-2}$. When a numerical value is needed use $g = 9.8$ unless a different value is specified in the question.

INFORMATION
- The total mark for this paper is **75**.
- The marks for each question are shown in brackets **[ ]**.
- This document has **16** pages.

ADVICE
- Read each question carefully before you start your answer.

Section A: Pure Mathematics

1(a)	Cosine rule: $AB^2 = 9.5^2 + 9.5^2 - 2 \times 9.5 \times 9.5 \times \cos(25°)$ $AB^2 = 16.91...$ $AB = 4.11\text{cm}.$
1(b)	Area of circle segment $= \frac{25}{360} \times \pi (9.5)^2 = 19689...$ Area of triangle $= \frac{1}{2} \times 9.5^2 \times \sin(25) = 19.0706..$ Area shaded $= 0.619\text{cm}^2.$
2(a)	

2(b) They intersect only once, and so $y = \ln x = k/x$ only has one real ~~root~~ solution. $\ln x = k/x$ rearranges to

$$x \ln x = k$$

$$x \ln x - k = 0.$$

So $x \ln x - k = 0$ has only one root.

3

$$y = 4x^{1/2} - 3x + 1$$

$$\frac{dy}{dx} = 2x^{-1/2} - 3$$

Now we have differentiated y, we can find the gradient of the tangent at $x = 4$ which is $\frac{dy}{dx} = \frac{2}{\sqrt{4}} - 3 = -2$

The gradient of the normal at $x = 4$ equals the negative reciprical of the gradient of the tangent. Call the gradient of the normal m, then

$$-2m = 1, \quad m = 1/2.$$

When $x = 4$, $y = 4\sqrt{4} - 3 \times 4 + 1 = -3$

So $y - (-3) = 1/2 (x - 4)$

$$2y + 6 = x - 4$$

$$x - 2y - 10 = 0.$$

4(a)

If $(2x-1)$ is a factor of $f(x)$, then $f(1/2)=0$, by the factor theorem.

$$f(1/2)=6(1/2)^3+k(1/2)^2+57(1/2)-20=0$$

$$\frac{3}{4}+\frac{k}{4}+\frac{57}{2}-20=0$$

$$\Rightarrow \quad \frac{k}{4}=-\frac{37}{4}$$

$$k=-37.$$

4(b)

$(2x-1)$ is a factor,

So $f(x)=(2x-1)(3x^2+cx+20)$

Where $2x^2c-3x^2=-37x^2$

$$c=-17.$$

$$f(x)=(2x-1)(3x^2-17x+20)$$
$$=(2x-1)(3x-5)(x-4).$$

4(c)(i)	Factors are $(e^{-t}-4)(2e^{-1}-1)(3e^{-1}-5)$. $e^{-t} = 1/2, 5/3, 4$ $t = -\ln(1/2), -\ln(5/3), -\ln(4)$.
4(c)(ii)	$\sum t = -(\ln(1/2) + \ln(5/3) + \ln(4))$ $= -\ln(5\times4/3\times2) = -\ln(10/3)$ $= \ln(3/10)$

5(a)	For stationary point $x+b=0$, $x=-3$ $b=3$. $y= a(-3+3)^2+c=2$ $\Rightarrow \quad c=2$
5(b)	The translated curve: $y=a(x+b-4)^2+c$ $b=3, \quad c=2.$ $y=a(x-1)^2+2$ $(3,-18)$ $-18=a(3-1)^2+2=$ $-20=4a, \quad a=-5.$

6 For the tangent $3y + x = 7$ can be rearanged to $y = -\frac{1}{3}x + 7/3$, which gives a gradient of $-1/3$. The perpendicular line to this has a gradient equal to its negative reciprical, we will call this line l and say it passes through $(3,-2)$.

Gradient of l, m is such that $-\frac{1}{3}m = -1$
So $m = 3$. $y = 3x + c$.
Substitute in $(3,-2)$.

$$-2 = 3\times(3) + c, \quad c = -11.$$

Line l: $y = 3x - 11$. The point where L and $3y + x = 7$ meet is on the circumfrence of the circle, as it's a tangent meeting a radius.

$$\left.\begin{array}{l} y = -\frac{1}{3}x + 7/3 \\ y = 3x - 11 \end{array}\right\} \quad \begin{array}{l} -x + 7 = 9x - 33 \\ 40 = 10x \\ x = 4. \end{array}$$

$y = 3\times 4 - 11 = 1.$
Point $(4,1)$ is on the circle, as the circle is centred $(3,-2)$ then it's equation is

$$(x-3)^2 + (y+2)^2 = r^2$$

Sub in $(4,1)$

$$(4-3)^2 + (1+2)^2 = r^2 = 10$$

Circle equation

$$(x-3)^2 + (y+2)^2 = 10.$$

7(a) Volume $= \frac{1}{2} \times$ base $\times$ height $\times$ length

$= \frac{1}{2} \times 2x \times x \times y = x^2y.$

Length of $BC = \sqrt{x^2+x^2} = x\sqrt{2} = AB = DE = EF.$

Triangles: $ABC = DEF = \frac{1}{2}x \times 2x = x^2\ cm^2.$

Slants: $ABDE = BCEF = xy\sqrt{2}\ cm^2$

Base: $ACDF = 2xy\ cm^2$

Surface area, $S = 2xy + 2\times(x^2) + (yx\sqrt{2})\times 2 = 600.$

$= 2xy + 2xy\sqrt{2} + 2x^2$

$= y(2x + 2\sqrt{2}x) = 600 - 2x^2$

$$y = \frac{300 - x^2}{x(1+\sqrt{2})}.$$

Volume $= x^2y = x^2\left(\frac{300-x^2}{x(1+\sqrt{2})}\right)$

$= x(300-x^2)\left(\frac{1}{(1+\sqrt{2})}\right)$

$= x(300-x^2)\left(\frac{1-\sqrt{2}}{(1+\sqrt{2})(1-\sqrt{2})}\right)$

$= x(300-x^2)\left(\frac{1-\sqrt{2}}{-1}\right)$

$V = (\sqrt{2}-1)x(300-x^2).$ □

7(b)	$V = kx(300 - x^2)$ $\frac{dV}{dx} = k(300 - 3x^2)$ $k(300 - 3x^2) = 0$ $300 = 3x^2$ $x^2 = 100$ $x = 10$, x cannot be negative. $x = 10cm$.
7(c)	Volume (max) $= (\sqrt{2} - 1) \times 10 \times (300 - 10^2)$ $= 828.4 cm^3$ (1dp). Nearest integer $V = 828 cm^3$.
7(d)	The volume cannot be negative so $300 - x^2 > 0$, So x must be less than $\sqrt{300}$ cm.

Section B: Mechanics

8 $(2p-3q-13)i+(-4+5p-6)j=0i+0j$

$-4+5p-6=0 \Rightarrow p=2$

$2p-3q-13=0$ & $p=2$

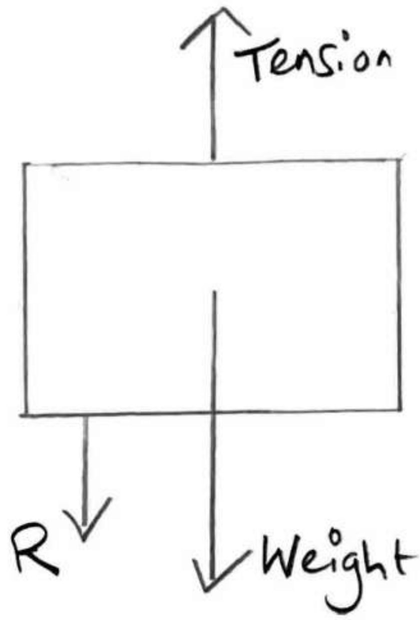
So $2\times 2-3q-13=0 \Rightarrow q=-3$.

9(a) $9500\,N - 55g\,N - 830g\,N = (830+55)a$.

$9500-8673=885a$

$827=885a$.

$a=827/885=0.934\ ms^{-2}$.

9(b)	
9(c)	Reaction R, $T-(55g+R)=55a$ $\left(\text{where } a=\frac{827}{885}\right)$ $9500-539-R=55a$ $R=950-539-55a=8910\text{N}$ (4sf)

10(a)	$v=(2t+1)(-t+3)=-2t^2+5t+3.$ $\frac{dv}{dt}=-4t+5$, when $t=4$, $\frac{dv}{dt}=-11.$ So the deceleration of P is $11ms^{-2}$.
10(b)	$t=3$ seconds.
10(c)	$\int_0^3 -2t^2+5t+3\,dt=\left[-\frac{2t^3}{3}+\frac{5t^2}{2}+3t\right]_0^3$ $=\left(-\frac{2\times3^3}{3}+\frac{5\times3^2}{2}+3^2\right)-0=\frac{27}{2}.$ $\int_3^4 -2t^2+5t+3\,dt=\left[-\frac{2t^3}{3}+\frac{5t^2}{2}+3t\right]_3^4$ $=\left(-\frac{2(4)^3}{3}+\frac{5(4)^2}{2}+3\times4\right)-\frac{27}{2}=-\frac{25}{6}.$ Total distance $=\frac{25}{6}+\frac{27}{2}=\frac{53}{3}$ or $17.7m$.

11(a) For the car $s = 2t^2$ (as ~~accelerati~~ acceleration $= d^2s/dt^2 = 4$).

Motorcycle: $s = 16t - 1.5 \times 16 = 16(t - 1.5)$.
As $V = 16$ and distance before lights $= V \times t$.

Where they're level

$$2t^2 = 16t - 24$$
$$\Rightarrow t^2 - 8t + 12 = 0$$
$$\Rightarrow (t-2)(t-6) = 0.$$

So $t_1 = 2$, $t_2 = 6$.

11(b)	The motorcycle is ahead on the car.
11(c)	Let $f(t) = t^2 - 8t + 12$ be the distance between them. $f'(t) = 2t - 8$, $f'(4) = 0$ so $f(t)$ has a maximum at $t = 4$. $f(4) = 4^2 - 8 \times 4 + 12 = 8$m.