OCR
Oxford Cambridge and RSA

# Wednesday 07 October 2020 – Afternoon

## AS Level Mathematics A

**H230/01** Pure Mathematics and Statistics

Printed Answer Booklet

**Time allowed: 1 hour 30 minutes**

**You must have:**
- Question Paper H230/01 (inside this document)
- a scientific or graphical calculator

Please write clearly in black ink. **Do not write in the barcodes.**

| Centre number | | | | | | Candidate number | | | | |
|---|---|---|---|---|---|---|---|---|---|---|

First name(s) ______________________

Last name ______________________

**INSTRUCTIONS**
- Use black ink. You can use an HB pencil, but only for graphs and diagrams.
- Write your answer to each question in the space provided in the **Printed Answer Booklet**. If you need extra space use the lined pages at the end of the Printed Answer Booklet. The question numbers must be clearly shown.
- Answer **all** the questions.
- Where appropriate, your answer should be supported with working. Marks might be given for using a correct method, even if your answer is wrong.
- Give non-exact numerical answers correct to 3 significant figures unless a different degree of accuracy is specified in the question.
- The acceleration due to gravity is denoted by $g\,\mathrm{m\,s^{-2}}$. When a numerical value is needed use $g = 9.8$ unless a different value is specified in the question.

**INFORMATION**
- The total mark for this paper is **75**.
- The marks for each question are shown in brackets **[ ]**.
- This document has **16** pages.

**ADVICE**
- Read each question carefully before you start your answer.



**Turn over**

## Section A: Pure Mathematics

**1(a)**

$$\frac{d}{dx}\left(x^3\right)+\frac{d}{dx}\left(-3x\right)+\frac{d}{dx}\left(5x^{-2}\right)$$

$$=3x^2-3-\frac{10}{x^3}$$

**1(b)**

$$\int\left(6x^2-\frac{2}{x^3}\right)dx$$

$$=\int 6x^2\,dx-2\int x^{-3}\,dx$$

$$=2x^3-2\times{}^{-2}/x^{+2}+C$$

$$=2x^3+{}^{1}/x^{2}+C$$

2(a)

$$\frac{p-1}{1-2} = \frac{-3-1}{4-2} = -2$$

$$(p-1)/-1 = -2$$

$$p-1 = 2$$

$$p = 3.$$

2(b)

$$\overrightarrow{AB} = \begin{pmatrix} 4 \\ -2 \end{pmatrix}, \overrightarrow{BD} = \begin{pmatrix} q-1 \\ -1 \end{pmatrix}, \overrightarrow{DA} = \begin{pmatrix} -3-q \\ 3 \end{pmatrix}$$

$$DA^2 = AB^2 + BD^2$$

$$(-3-q)^2 + 3^2 = (q-1)^2 + 1 + 16 + 4$$

$$q^2 + 6q + 18 = q^2 - 2q + 22$$

$$8q = 4, \quad q = 0.5.$$

**3(a)**

$$4\sin^2\theta = \left(\frac{\sin^2\theta}{\cos^2\theta}\right)$$

$\times\cos^2\theta$ and $\div 4\sin^2\theta$

$$\cos^2\theta = \sin^2\theta / 4\sin^2\theta = 1/4.$$

$\sqrt{\ }$

$$\cos\theta = 1/2 \quad \text{or} \quad \cos\theta = -1/2.$$

$\cos^{-1}$

$$\theta = 60°, 120°.$$

OR $\sin^2\theta = 0$, $\sin\theta = 0$.

$$\theta = 0°, 180°$$

So $\theta = 0°, 60°, 120°$ or $180°$.

**3(b)**

As $\sin^2\theta = 1 - \cos^2\theta$

$$\Rightarrow \frac{\sin^2\theta - 1 + \cos\theta}{1-\cos\theta} = \frac{1 - 1 - \cos^2\theta + \cos\theta}{1-\cos\theta}$$

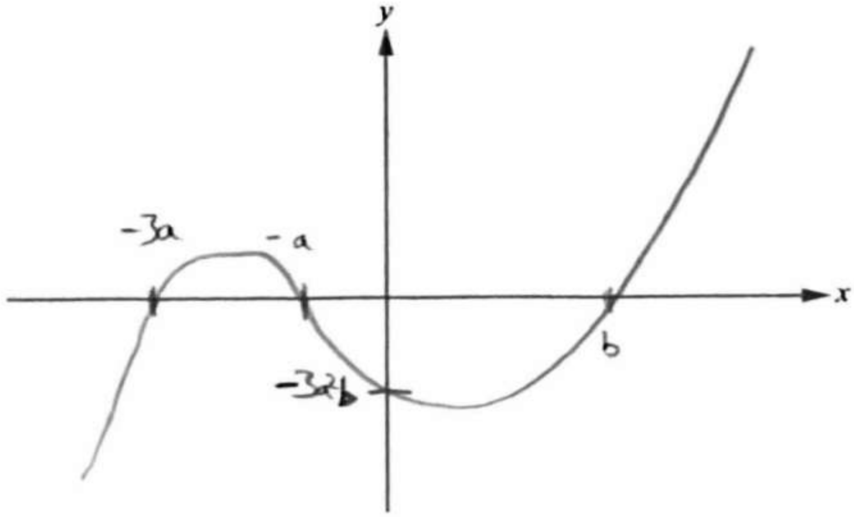
$$= \frac{\cos\theta - \cos^2\theta}{1-\cos\theta} = \frac{\cos\theta(1-\cos\theta)}{(1-\cos\theta)}$$

Which cancles $= \cos\theta$ □

| | |
|---|---|
| 4(a) | $(1+x)^n = 1 + nx + \frac{n(n-1)}{2!}x^2 + \frac{n(n-1)(n-2)}{3!}x^3 \ldots$<br>$(1+x)^4 = 1 + 4x + \frac{4\times3}{2}x^2 + \frac{4\times3\times2}{3!}x^3 + x^4$<br>$(1+x)^4 = 1 + 4x + 6x^2 + 4x^3 + x^4.$ |
| 4(b) | $(1.002)^4 = (1+0.002)^4, \quad x = 0.002.$<br>$(1.002)^4 = 1 + 0.008 + 6\times4\times10^{-6} + 8\times10^{-9}\times4$<br>$+ 1.6\times10^{-11}.$<br>$= 1.008024032016.$<br>$(1002)^4 = (1.002 \times 1000)^4$<br>$= (1.002)\times1\times10^{12}$<br>$(1002)^4 = 1008024032016.$ |

5(a)

5(b)




5(c)

$a=1, \ b=4.$

So area under $\int_{-3}^{-1} (x+a)(x+3a)(x-b)\,dx$

And area above $\int_{-1}^{4} (x+a)(x+3a)(x-b)\,dx$

$(x+1)(x+3)(x-4) = (x^2-x-12)(x+1)$

$= x^3 - x^2 - 12x + x^2 - x - 12$

$= x^3 - 13x - 12$

$\int_{-3}^{-1} x^3 - 13x - 12\,dx = \left[\frac{x^4}{4} - \frac{13x^2}{2} - 12x\right]_{-3}^{-1}$

$= 8.$

$\int_{-1}^{4} x^3 - 13x - 12\,dx = \left[\frac{x^4}{4} - \frac{13x^2}{2} - 12x\right]_{-1}^{4}$

$= 93.75.$

Total area $= 8 + 93.75 = 101.75.$

| | |
|---|---|
| 6(a) | Factorise into $(x-2)(x+3)>0$<br>This is a positive quadratic with $x$ axis ~~intercept~~ intersections at $x=2$ and $x=-3$.<br>So $x^2+x-6>0$ when $x>2$ or $x<-3$.<br>$\{x : x<-3\} \cup \{x : x>2\}$ |
| 6(b) | $x^3-7x^{3/2}-8=(x^{3/2}+1)(x^{3/2}-8)=0$.<br>$x^{3/2}=\sqrt{x^3}\neq -1$, so $x^{3/2}+1=0$ has no real solutions.<br>$x^{3/2}=8$ so $x^3=64$, so $x=4$. |
| 6(c) | Take the natural log of both sides<br>$\ln((3^x)^2)=\ln(3\times 2^x)$<br>$\Rightarrow 2\ln(3^x)=\ln(3)+\ln(2^x)$ (By logarithm laws)<br>$\Rightarrow 2x\ln(3)=\ln(3)+x\ln(2)$ $\quad -x\ln(2)$<br>$2x\ln(3)-x\ln(2)=\ln(3)$.<br>$x(2\ln(3)-\ln(2))=\ln(3)$<br>$\Rightarrow x(\ln(3^2/2))=\ln(2)$ $\quad \div\ln(9/2)$<br>$x=\ln(3)/\ln(9/2)$ |

7 $x+2y=4 \Rightarrow x=4-2y$.

Substituting into the curve

$3(4-2y)y+(4-2y)^2+14=0$

$12y-6y^2+4y^2-16y+4+14=0$

$-2y^2-4y+30=0=2y^2+4y-30$

$=y^2+2y-15=0$.

$(y-3)(y+5)=0$.

The line and curve intersect when $y=3$ or $y=-5$.

$x=4-2\times3=-2$

$x=4-2\times(-5)=14$.

Points of intersection $(14,-5)$ & $(-2,3)$.

## Section B: Statistics

**8(a)**

$0 \le L \le 2$ has $1.6/6$ of the height of $3 \le L \le 4$, so the number of worms is

$$\frac{1.6}{6} \times 30 \times 2 = 16.$$

**8(b)**

Frequency of $5 \to 9 = \frac{2.4}{6} \times 30 \times 4 = 48.$

Frequency of $5 \to 6 = 1/4 \times 48 = 12.$

Frequency of $4 \to 5 = 4.8/6 \times 30 = 24.$

Estimated frequency $4.5 \to 5.5 = \frac{1}{2}(24 + 12)$

$= 18.$

| | |
|---|---|
| 9(a) | These LAs may not have as large a decrease as LAs in London. |
| 9(b) | Brighton and Hove, Cambridge, Oxford and Exeter as they all have relatively high increases in cycling and walking. |
| 9(c) | No, because the data shows percentage change, so an LA with a larger population could have a larger increase with a smaller percentage change. |
| 9(d) | Working from home and train, as both are entirely positive and the largest two increases in percentage (4.3% for work from home and 4.1% for train). |
| 9(e) | Not the case, as the 'driving' figures decrease it does not correlate with the "home" figures. |

| | |
|---|---|
| 10(a) | Let $p = P(\text{Packet contains a gift}) = 0.25$.<br>We will then use the binomial distribution to calculate the probability of there being one or less packets with gifts inside among 20 samples.<br>With $B(20, 0.25)$ distribution and $X = 1$ we can look up<br>$P(X \leq 1) = 0.0243$. ($<2.5\%$ significance level)<br><br>Which would suggest that $p < 0.25$ and Less than One quarter of g. packets with free gifts. |
| 10(b) | The free gifts may not be distributed randomly. |

**11(a)**

| $x$ | 1 | 2 | 3 | 4 |
|---|---|---|---|---|
| P | $2/5$ | $3/10$ | $1/5$ | $1/10$ |

**11(b)**

There are four ways this can happen, rolling 1, 1, 3, rolling 1, 1, 4, rolling 1, 2, 4 or rolling 2, 1, 4.

P(Rolling 1, 1, 3) $= (2/5)^2 \times 1/5 = 0.032$

P(Rolling 1, 1, 4) $= (2/5)^2 \times 1/10 = 0.016$

P(Rolling 1, 2, 4) $= 2/5 \times 3/10 \times 1/10 = 0.012$

P(Rolling 2, 1, 4) $= 3/10 \times 2/5 \times 1/10 = 0.012$.

P(Any one of these happening) $= 0.032 + 0.016 + 0.012 + 0.012 = 0.072$.

(answer space continued on next page)