OCR
Oxford Cambridge and RSA

Wednesday 14 October 2020 – Afternoon

A Level Mathematics A

H240/02 Pure Mathematics and Statistics

Printed Answer Booklet

Time allowed: 2 hours

You must have:
- Question Paper H240/02 (inside this document)
- a scientific or graphical calculator

Please write clearly in black ink. **Do not write in the barcodes.**

Centre number | | | | | | Candidate number | | | | |

First name(s) ______________________

Last name ______________________

INSTRUCTIONS
- Use black ink. You can use an HB pencil, but only for graphs and diagrams.
- Write your answer to each question in the space provided in the **Printed Answer Booklet.** If you need extra space use the lined pages at the end of the Printed Answer Booklet. The question numbers must be clearly shown.
- Answer **all** the questions.
- Where appropriate, your answer should be supported with working. Marks might be given for using a correct method, even if your answer is wrong.
- Give non-exact numerical answers correct to 3 significant figures unless a different degree of accuracy is specified in the question.
- The acceleration due to gravity is denoted by $g\,\text{m s}^{-2}$. When a numerical value is needed use $g = 9.8$ unless a different value is specified in the question.

INFORMATION
- The total mark for this paper is **100**.
- The marks for each question are shown in brackets **[ ]**.
- This document has **20** pages.

ADVICE
- Read each question carefully before you start your answer.

Section A: Pure Mathematics

1(a)(i)	let $u = 2x+3$. Chain rule $\frac{dy}{dx} = \frac{dy}{du} \times \frac{du}{dx}$ $y = u^7$ $\frac{dy}{dx} = 7u^6$, $\frac{du}{dx} = 2$. $\frac{dy}{dx} = 14u^6$. $\frac{d}{dx}\left((2x+3)^6\right) = 14(2x+3)^6$.
1(a)(ii)	Product rule $d/dx\,(uv) = u\,dv/dx + v\,du/dx$. Let $u = x^3$ $\quad v = \ln(x)$ $du/dx = 3x^2$ $\quad dv/dx = 1/x$ $\frac{d}{dx}\left(x^3\ln(x)\right) = 3x^2\ln(x) + \frac{x^3}{x}$ $= 3x^2\ln(x) + x^2 = x^2(3\ln(x)+1)$
1(b)	Let $u = 5x$ $\quad \frac{du}{dx} = 5$, $dx = \frac{1}{5}du$ $\int \cos 5x\,dx = \frac{1}{5}\int \cos u\,du = \frac{\sin(5x)}{5} + C$.
1(c)	$y = \int \frac{dy}{dx}dx + c = 3x^2 - 5x + c$. Passes $(1,3)$. $\quad 3 = 3\times1^2 - 5\times1 + c$, $c = 5$. $y = 3x^2 - 5x + c$.

2 $$\frac{2x^3+x^2-7x-6}{x^2-x-2} = \frac{(2x+3)(x^2-x-2)}{(x+1)(x-2)}$$

$$= \frac{(2x+3)(x+1)(x-2)}{(x+1)(x-2)}$$

$$= 2x+3.$$

3(a)(i)

$1 + (-2)(-x) + \frac{(-2)(-3)}{2!}(-x)^2 + \frac{(-2)(-3)(-4)}{3!}(-x)^3$

$= 1 + 2x + 3x^2 + 4x^3 \ldots$

3(a)(ii)

As $n < 0$ in the binomial expansion $(1+x)^n$

$\cancel{n(n-1)\ldots(n-r+1) = n!}$

$(n+1)x^n.$

3(b)	$\dfrac{1}{1-x}$
3(c)	$2+3x+4x^2+5x^3+\dots$ $=1+x+x^2+x^3+\dots+1+2x+3x^2+4x^3+\dots$ $=\dfrac{1}{1-x}+\dfrac{1}{(1-x)^2}=\dfrac{1-x+1}{(1-x)^2}=\dfrac{2-x}{(1-x)^2}.$

4 Equation is equal to $3\sin^4\phi + \sin^2\phi - 4 = 0$, Which factorises to $(3\sin^2\phi + 4)(\sin^2\phi - 1) = 0$.

This implies either $\sin^2\phi = -4/3$ or $\sin^2\phi = 1$. $\sin^2\phi \neq -4/3$ as it's squared. So

$\sin^2\phi = 1$, $\sin\phi = 1$ or $\sin\phi = -1$.

Therefore $\phi = \frac{\pi}{2}$ or $\frac{3\pi}{2}$, no other solutions within range.

5(a) If both $\frac{n^2-1}{2}$ and $\frac{n^2+1}{2}$ are integers then n^2 must be odd.

=> So must n. If n is a positive integer then $n = 2k+1$ (for k integer), if n is an integer

$\frac{n^2-1}{2} \geq 1$ so $n^2 \geq 3$, so ~~n~~

~~$\sqrt{n} \geq 3$~~ $n \geq \sqrt{3}$ or $n \leq \sqrt{3}$.

$n = \sqrt{k}$ where k is an integer greater than 1.

5(b) $(n)^2 + \left(\frac{n^2-1}{2}\right)^2 = n^2 + \frac{n^4-2n^2+1}{4}$

$= \frac{4n^2+n^4-2n^2+1}{4}$

$= \frac{n^4+2n^2+1}{4} = \left(\frac{n^2+1}{2}\right)^2$

6 LHS $= \sqrt{2}\left(\cos(2\theta)\cos 45^\circ - \sin 2\theta \sin 45^\circ\right)$

$= \sqrt{2}\left(\cos(2\theta)\times\frac{1}{\sqrt{2}} - \sin(2\theta)\frac{1}{\sqrt{2}}\right)$

$= \cos(2\theta) - \sin(2\theta)$

LHS $= \cos^2\theta - \sin^2\theta - 2\sin\theta\cos\theta$

$= \cos^2\theta - 2\sin\theta\cos\theta - \sin^2\theta =$ RHS

7(a)	Distance A to B.
7(b)	Midpoint of AB.
7(c)(i)	$\frac{1}{2}(a+b)$
7(c)(ii)	$\frac{1}{2}\lvert a-b\rvert$
7(d)	centre $= \frac{1}{2}\left(\begin{pmatrix}2\\-1\end{pmatrix}+\begin{pmatrix}4\\5\end{pmatrix}\right) = \begin{pmatrix}3\\2\end{pmatrix}$, $(3,2)$. radius $= \frac{1}{2}\left\lvert\begin{pmatrix}2\\-1\end{pmatrix}-\begin{pmatrix}4\\5\end{pmatrix}\right\rvert = \frac{1}{2}\left\lvert\begin{matrix}-2\\-6\end{matrix}\right\rvert$ $= \frac{1}{2} \times 2\sqrt{10} = \sqrt{10}$. Cartesian: $(x-3)^2+(y-2)^2=10$.

8(a)

$$\frac{dP}{100-P} = dt,$$

$|100-P| = Ae^{-t}$, when $t=0, P=2000$

So $A = 1900$.

$$|100-P| = 1900e^{-t}$$

$$P-100 = 1900e^{-t}$$

$$P = 1900e^{-t} + 100.$$

8(b) Decreases and tends towards 100, exponentially.

Section B: Statistics

9(a)	$40000 \times 0.002 = 80.$
9(b)	Number of carsper £.
9(c)	Show four seperate classes within £50,000 to £90,000.
9(d)	$\frac{1}{4} \times 80 = 20.$

10 $p = P$(A ~~g~~random customer is satisfied).

Pierre claims $p = 0.9$, label this H_0.
Yvette suspects $p < 0.9$, label this H_1.

$X \sim Bin(15, 0.9)$ and $X \leq 11$

$P(X \leq 11) = 0.0556$ which is above the 5% significance level.

There is not enough evidence to reject H_0, and so insufficent evidence Yvette's suspicion is correct.

11(a)	mean $= \dfrac{100\times162+175\times318+225\times355+300\times165}{1000}$
	$= 201.225.$
	$sd = \sqrt{\dfrac{100^2\times162+175^2\times318+225^2\times355+300^2\times165}{100} - 201.225^2}$
	$sd = 60.73$ (4sf).
11(b)	$P(150 < X < 210) = 0.364$ (3SF).
11(c)	$P(X < 160) = 0.25249.$
	$x_1 = \Phi^{-1}(0.6 + 0.25249)$
	$x_1 = 262.83$ (5sf).

11(d)

112 and 288 are with two standard deviations of the mean.

$P(X<112)=0.0708>0.025$.

11(e)

Reduce the value of σ.

Say $288-200=3\sigma$

$\sigma=29.3$ or about 30.

12(a)	$H_0: \mu = 45.7$, where μ is the mean of all the new journeys and null $H_1: \mu < 45.7$.
12(b)	$N(45.7, \frac{5.6^2}{36})$ and probability $= 0.025$. $P(\bar{X} < a) = 0.025$ So $a = 43.7$ (3sf). So the rejection region is $\bar{X} < 43.7$.

13(a)(i)

$P(AA) = 0.4^2 = 0.16$, $P(BAA) = 0.6 \times 0.4^2$
$= 0.096$.

$P(\text{Andy wins}) = P(AA + BAA) = 0.16 + 0.096$
$= 0.256$.

13(a)(ii)

Winless either ABA or BAB.

$P(BAB) = 0.6^2 \times 0.4 = 0.144$
$P(ABA) = 0.4^2 \times 0.6 = 0.096$

$P(ABA \text{ or } BAB) = 0.144 + 0.096 = 0.24$.

13(b)

Game 1 Game 2 Game 3 . . . .
$0.256 + 0.24 \times 0.256 + 0.24^2 \times 0.256$.

Binomial $0.256 \times (1 + 0.24 + 0.24^2 + \ldots)$

$P(\text{Andy}) = \dfrac{0.256}{1 - 0.24} = \dfrac{32}{95} = 0.337$ (3sf).

14(a)(i)	The existing number of pupils is already catered for. The increase shows how many new pupils there will be.
14(a)(ii)	Wigan, it has the larger increase in number.
14(a)(iii)	Children born in a LA will go to school in that LA.
14(b)	Manchester and Salford for the highest percent and absolute increases.

15(a)

$$\frac{15}{64} \times \frac{2^2}{2!} = \frac{15}{64} \times \frac{4}{2} = \frac{15}{32}.$$

15(b)

There is only one combination of X_1, X_2 and X_3 that adds to 9 with out 2, $X_1 = X_2 = X_3 = 3$.

Could be made with 2,2,5 and 2,3,4.

$P(X_1 + X_2 + X_3 = 9) =$

$$3 \times \left(\frac{15}{32}\right)^2 \times \frac{5}{80} + 6 \times \frac{15}{32} \times \frac{5}{16} \times \frac{5}{32} + \left(\frac{5}{16}\right)^3$$

$= 0.209045$

$P(X_1 + X_2 + X_3 = 9 \text{ \& non} = 2)$:

$P(\text{Each } X = 3) = \left(\frac{5}{16}\right)^3 = 0.030518.$

Condition on $X_1 + X_2 + X_3 = 9$:

$$P(X_1 + X_2 + X_3 = 9 \text{ and no } 2) = \frac{0.030518}{0.209045} = 0.030518.$$

So propability one X is a two, given they add to 9 =

$1 - 0.030518 = 0.854$ (3 s f).

15(c)

P(two 2's in 9 values) $= {}^9C_2 \times \left(1-\frac{15}{32}\right)^7 \times \left(\frac{15}{32}\right)^2$

P(two 2's in 9) $\times$ P(X = 2) $= {}^9C_2 \times \left(1-\frac{15}{32}\right)^7 \times \left(\frac{15}{32}\right)^3$

$= 0.0443$ (3sf).