OCR
Oxford Cambridge and RSA

Wednesday 07 October 2020 – Afternoon

A Level Mathematics A

H240/01 Pure Mathematics

Printed Answer Booklet

Time allowed: 2 hours

You must have:
- Question Paper H240/01 (inside this document)
- a scientific or graphical calculator

Please write clearly in black ink. **Do not write in the barcodes.**

Centre number | | | | | | Candidate number | | | | |

First name(s) ______________________

Last name ______________________

INSTRUCTIONS
- Use black ink. You can use an HB pencil but only for graphs and diagrams.
- Write your answer to each question in the space provided in the **Printed Answer Booklet.** If you need extra space use the lined pages at the end of the Printed Answer Booklet. The question numbers must be clearly shown.
- Answer **all** the questions.
- Where appropriate, your answer should be supported with working. Marks might be given for using a correct method, even if your answer is wrong.
- Give non-exact numerical answers correct to 3 significant figures unless a different degree of accuracy is specified in the question.
- The acceleration due to gravity is denoted by $g\,\mathrm{m\,s^{-2}}$. When a numerical value is needed use $g = 9.8$ unless a different value is specified in the question.

INFORMATION
- The total mark for this paper is **100**.
- The marks for each question are shown in brackets **[ ]**.
- This document has **16** pages.

ADVICE
- Read each question carefully before you start your answer.

1(a) For small θ, $\cos\theta \approx 1 - \frac{1}{2}\theta^2$ and $\tan\theta \approx \theta$.

So $2\cos\theta + (1-\tan\theta)^2 \approx 2(1-\frac{1}{2}\theta^2) + (1-\theta)^2$

$= 2 - \theta^2 + 1 - 2\theta + \theta^2 = 3 - 2\theta$.

1(b) For small θ, $\sin\theta \approx \theta$.

$3 - 2\theta \approx 2\cos\theta + (1-\tan\theta)^2 = 28\sin\theta \approx 28\theta$.

$3 - 2\theta = 28\theta$

$30\theta = 3$

$\theta = 0.1$.

2(a)

$\sqrt{12}\, a^{1/2} \times a^{5/2} \times \sqrt{3} = \sqrt{3 \times 12}\, a^{6/2}$

$= \sqrt{36}\, a^3 = 6a^3.$

2(b)

$(64b^3)^{1/3} = \sqrt[3]{64}\, b = 4b.$

$(4b^4)^{-1/2} = \frac{1}{\sqrt{4}} \times \frac{1}{b^2} = \frac{1}{2b^2}.$

$(64b^3)^{1/3} \times (4b^4)^{-1/2} = \frac{4b}{2b^2} = \frac{2}{b}.$

2(c)

$9^{3c} = (3^2)^{3c} = 3^{6c},\ 27^{2c} = (3^3)^{2c} = 3^{6c}.$

$7 \times 3^{6c} - 4 \times 3^{6c} = 3 \times 3^{6c}$

$= 3^{6c+1}.$

3(a)

Area $= 2\pi r^2 + 2\pi rh$.

Volume $= \pi r^2 h = 16000\pi$

$r^2 h = 16000$, $\quad h = 16000/r^2$.

So Area $= 2\pi r^2 + 2\pi r\left(\dfrac{16{,}000}{r^2}\right)$

$= 2\pi r^2 + 32{,}000\pi r^{-1}$. $\square$

3(b)

$\dfrac{dA}{dr} = 4\pi r - 32{,}000\pi r^{-2}$.

maximum or minimum at $dA/dr = 0$.

$4\pi r - 32000\pi r^{-2} = 0$,

$4r = 32000/r^2$

$r^3 = 8000$, $\quad r = 20$.

So when $r = 20$,

Surface Area $= 2400\pi\ \text{cm}^2$.

$d^2A/dr^2 = 4\pi + 64{,}000\pi r^{-3}$

When $r = 20$, $\dfrac{d^2A}{dr^2} = 4\pi + 8\pi > 0$,

Hence it's a minimum.

4 Assume N is the greatest multiple of 5.
So
$N = 5k$, for some integer k.

Then $N + 5 = 5k + 5 = 5(k+1)$.

$k+1$ is an integer and so $N+5$ is a multiple of 5, and $N+5 > N$.
But N is the largest multiple of 5,
thus contradittion.

5(a)

$\overrightarrow{BQ} = \frac{1}{2}\overrightarrow{BA} = \frac{1}{2}(a-b)$.

$\overrightarrow{PQ} = \frac{1}{4}b + \frac{1}{2}(a-b) = \frac{1}{2}a - \frac{1}{4}b$.

5(b)

As they're on the same line $\overrightarrow{PR}$ and $\overrightarrow{PQ}$ must ~~be~~ be scalar multiples of eachother as they have the same direction.

$\lambda\overrightarrow{PQ} = \overrightarrow{PR} = \lambda\frac{1}{4}(2a-b)$.

So let $k = \lambda/4$ then $\overrightarrow{PR} = k(2a-b)$ for constant k.

5(c)

$\overrightarrow{OP} + \overrightarrow{PR} = \overrightarrow{OR} = \frac{3}{4}b + k(2a-b)$.

$\overrightarrow{AO} + \overrightarrow{OR} = \overrightarrow{AR} = -a + \frac{3}{4}b + k(2a-b)$

$= (2k-1)a + (\frac{3}{4}-k)b$.

R is an extension of OA and so $\overrightarrow{AR}$ is a multiple of a.

So $\frac{3}{4} - k = 0$, $k = \frac{3}{4}$.

So $\overrightarrow{AR} = \frac{1}{2}a$, $\overrightarrow{OA} = a$.

$\overrightarrow{OA} : \overrightarrow{AR} = 2:1$.

6(a)	$\log_{10} S = \log_{10}(ab^t)$ $= \log_{10}(a) + \log_{10}(b^t)$ $= \log_{10}(a) + \log_{10}(b) \times t.$ Where $m = \log_{10}(b)$, $c = \log_{10}(a)$ (constants) This takes linear form. $\log_{10}(S) = m \times t + c.$
6(b)	gradient $= m$, vertical axis intercept $= c$, $\log_{10} a = 0.583$ $\Rightarrow a = 10^{0.583} = 3.83$ (3sf). $= 3.8$ (2sf). $\log_{10} b = 0.146 \Rightarrow b = 10^{0.146} = 1.4$ (2sf).
6(c)	Find t where $3.8 \times 1.4^t = 200$ $1.4^t = 52.63\ldots$ $t \times \ln(1.4) = \ln(52.63\ldots)$ $t = 11.8.$ 12 years after starting so 2027.
6(d)	There is a finite market, sales will not increase forever.

7(a)	Anna revision $= 30 + 9\times 15 = 165$ minutes. Ben revision $= 30 \times 1.1^9 = 71$ minutes $165 - 71 = 94$ minutes.
7(b)	Anna work on day X : $30 + 15\times(X-1)$. Ben work on day x : $30\times 1.1^{x-1}$. $30\times 1.1^{x-1} > 15X + 15$. $1.1^{x-1} > 0.5X + 0.5$. $X-1 \log_{1.1}(1.1) > \log_{1.1}(0.5X+0.5)$ $X-1 > \log_{1.1}(0.5X+0.5)$ $X > \log_{1.1}(0.5X+0.5)+1$.
7(c)	$x_1 = 10$. $x_2 = 18.9$, $x_3 = 25.1$, $x_4 = 28$ $x_5 = 29.0$, $x_6 = 29.4$, $x_7 = 29.6$, $x_8 = 29.6$. $X = 30$
7(d)(i)	She will run out of hours in the day.
7(d)(ii)	Increasing by 10% will need decimals of minutes, and so will lose accuracy.

8(a)	$\frac{d}{dx}\left(e^{2x}\right)=2e^{x}, \frac{d}{dx}\left(2+3x^{2}\right)=6x$ $\frac{d}{dx}\left(\left(2+3x^{2}\right)e^{2x}\right)=6xe^{2x}+2e^{2x}\left(2+3x^{2}\right)$ (by product rule). $=e^{2x}\left(6x^{2}+6x+4\right).$
8(b)	$e^{2x}>0$ for all x. $6x^{2}+6x+4=6\left(x+\frac{1}{2}\right)^{2}+\frac{5}{2},$ $6\left(x+\frac{1}{2}\right)^{2}\geqslant 0$, so $6x^{2}+6x+4\geqslant \frac{5}{2}>0.$ So the gradient $e^{2x}\left(6x^{2}+6x+4\right)>0$ for all real x, as such $e^{2x}\left(2+3x^{2}\right)$ is always increasing.

9(a)	When $x=0$ $y=\|-3\|=3$, $(0,3)$. When $y=0$ $\|2x-3\|=0$, $x=3/2$ $(0,3)$ and $(1.5,0)$.
9(b)(i)	$a<2$, (if it's gradient were greater they'd only meet once). At x intersection. $1.5a+2=0$, $a=-4/3$. So $a>-4/3$ or else it would miss. $-\frac{4}{3}<a<2$.
9(b)(ii)	$2x-3=ax+2$. $(2-a)x=5$, $x=5/(2-a)$. Also when $3-2x=ax+2$ $1=(a+2)x$, $x=1/(2+a)$.

10(a)

$$0.25\times\left(\sin 0+\sin\left(\tfrac{1}{2}\sqrt{0.25}\right)+\sin\left(\tfrac{1}{2}\sqrt{0.5}\right)+\sin\left(\tfrac{1}{2}\sqrt{0.75}\right)\right)$$

$= 0.2533\ldots$ lower bound.

10(b)(i)

$t^2 = x-1$, $\frac{dx}{dt} = 2t$, $dx = 2t\,dt$.

$$\int \sin\left(\tfrac{1}{2}\sqrt{x-1}\right)dx = \int \sin\left(\tfrac{1}{2}t\right)\times 2t\,dt.$$

$$= \cancel{2t\int \sin\left(\tfrac{1}{2}t\right)dt}.$$

$$= \int 2t\sin\left(\tfrac{1}{2}t\right)dt.$$

10(b)(ii)	$\int \sin\left(\frac{1}{2}\sqrt{x-1}\right)dx = \int 2t\sin\left(\frac{t}{2}\right)dt$. $\underset{=}{\text{by parts}}\ -2t\times 2\cos\left(\frac{t}{2}\right) + \int 4\cos\left(\frac{t}{2}\right)dt + c$ $= -4t\cos(t/2) + 8\sin(t/2) + c$. $\int_1^2 \sin\left(\frac{1}{2}\sqrt{x-1}\right)dt = \int_0^1 2t\sin(t/2)\,dt$. $= \left[-4t\cos(t/2) + 8\sin(t/2)\right]_0^1$ $= (-4\cos(1/2) + 8\sin(1/2)) - (-0\times\cos(0) + 8\sin(0))$ $= 8\sin(1/2) - 4\cos(1/2)$ $\square$
11(a)(i)	At point A, $y = mx+2$, and $x^2+y^2-10x-14y=-64$. So $x^2+(mx+2)^2-10x-14(mx+2)+64=0$. $x^2+m^2x^2+4mx+4+10x-14mx+36=0$. $x^2(m^2+1)-10(m+1)x+40=0$.

11(a)(ii)	$b^2 - 4ac = 0$ (as it touches the circle once).
	$(-10(m+1))^2 - 4 \times (m^2+1) \times 40 = 0$
	$100(m+1)^2 - 160(m^2+1) = 0$.
	$60m^2 - 200m + 60 = 0$
	$3m^2 - 10m + 3 = 0$
	$(m-3)(3m-1) = 0$
	$m = 3$ or $m = 1/3$, $m > 3$, so $m \neq 1/3$.
	$y = 3x + 2$
11(b)	Equation of a circle: $(x-a)^2 + (y-b)^2 = r^2$
	So $x^2 - 2ax + a^2 + y^2 - 2by + b^2 - r^2 = 0$.
	Comparing coefficients: for this circle.
	$-2a = -10, a = 5$, $-2b = -14 \Rightarrow b = 7$.
	So $a^2 + b^2 - r^2 = 64 \Rightarrow r^2 = 5^2 + 7^2 - 64 = 10$.
	The circle has radius $\sqrt{10}$, call the centre of the circle point C. Make a right angled triangle ACP. The angle outside the circle $\theta = 1/2$ APB.
	$\tan(\theta) = \sqrt{10}/PA$.
	$PA = 2\sqrt{10}$
	$\tan(\theta) = 1/2$.
	$\tan(2\theta) = \tan(APB) = \frac{1}{1 - 1/4}$.

Turn over

12 $\frac{dy}{dx} = y\frac{(20x-35)}{2x^3-3x^2-11x+6}$

$\int y\,dy = \int \frac{(20x-35)}{2x^3-3x^2-11x+6}\,dx.$

$2x^3-3x^2-11x+6 = (x-3)(2x^2+3x-2)$
$= (x-3)(x+2)(2x-1).$

$\frac{20x-35}{(x-3)(x+2)(2x-1)} = \frac{A}{x+2} + \frac{B}{x-3} + \frac{C}{2x-1}$

$A(x-3)(2x-1)+B(x+2)(2x-1)+C(x+2)(x-3) = 20x-35$
$2Ax^2-7Ax+3A+B\cdot 2x^2+3Bx-2B+Cx^2-Cx-6C$
$2A+2B-C=0$
$-7A+3B-C=20$ $\quad A=-3,\ B=1,\ C=4$
$3A-2B-6C=-35$

$\frac{20x-35}{2x^3-3x^2-11x+6} = \frac{-3}{x+2} + \frac{1}{x-3} + \frac{4}{2x-1}$

$\int \frac{20x-35}{2x^3-3x^2-11x+6}\,dx = -3\ln(x+2)+\ln(x-3)+4\ln(2x-1)$
$+\ln(A).$

$\int \frac{1}{y}\,dy = \ln(|y|).$
So
$y = \frac{A(x-3)(2x-1)^2}{(x+2)^3}$

(answer space continued on next page)