OCR
Oxford Cambridge and RSA

Friday 14 June 2019 – Afternoon

A Level Mathematics A

H240/03 Pure Mathematics and Mechanics

PRINTED ANSWER BOOKLET

Time allowed: 2 hours

You must have:
- Question Paper H240/03 (inserted)

You may use:
- a scientific or graphical calculator

Please write clearly in black ink. **Do not write in the barcodes.**

Centre number						Candidate number				

First name(s) ______________________

Last name ______________________

INSTRUCTIONS
- The Question Paper will be found inside the Printed Answer Booklet.
- Use black ink. HB pencil may be used for graphs and diagrams only.
- Answer **all** the questions.
- **Write your answer to each question in the space provided in the Printed Answer Booklet.** Additional paper may be used if necessary but you must clearly show your candidate number, centre number and question number(s).
- You are permitted to use a scientific or graphical calculator in this paper.
- Give non-exact numerical answers correct to 3 significant figures unless a different degree of accuracy is specified in the question.
- The acceleration due to gravity is denoted by $g\,\text{m}\,\text{s}^{-2}$. Unless otherwise instructed, when a numerical value is needed, use $g = 9.8$.

INFORMATION
- **You are reminded of the need for clear presentation in your answers.**
- The Printed Answer Booklet consists of **16** pages. The Question Paper consists of **12** pages.

Section A: Pure Mathematics

1	Sine rule: $\frac{\sin 32°}{13.5} = \frac{\sin(CAB)}{8.3}$ $\sin(CAB) = 0.3258...$ $CAB = 19.0°$.
2(a)(i)	$x^2+y^2-6x+4y+4=0$ $\Rightarrow (x-3)^2-9+(y+2)^2-4+4=0$ $\Rightarrow (x-3)^2+(y+2)^2=9$, $C=(3,-2)$.
2(a)(ii)	$r=\sqrt{9}=3$.
2(b)	$y=kx-3$. $(x-3)^2+(kx-3+2)^2=9$ } Points the intersect. $x^2+(kx-3)^2-6x+4(kx-3)+4=0$. $(1+k^2)x^2+(-6-2k)x+1=0$. b^2-4ac: $(-6-2k)^2-4(1+k^2)$ $=36+24k+4k^2-4-4k^2$ $=32+24k$ For no intersections, $b^2-4ac<0$. So $32+24k<0$ $k<-4/3$. **(answer space continued on next page)**

2(b)	(continued)
3(a)	We shall find the critical values of
	$|x-2| \leq |2x-6| \quad — ①$
	Square both sides
	$(x-2)^2 \leq (2x-6)^2$
	We can lose the absolute value, as they are now always positive. Expand:
	$x^2-4x+4 \leq 4x^2-24x+36.$
	Rearange: $0 \leq 3x^2-20x+32. \quad — ②$
	We know ① holds true if ② holds true, So evaluate the quadratic.
	$3x^2-20x+32=(3x-8)(x-4)$
	Critical points $x=8/3$ and $x=4$.
	Positive quadratic, so ② and therefor ① is true for 
	$x \leq \frac{8}{3}$ and $x \geq 4$.

3(b)	It is a translation in x direction by 4 units. Then it is a stretch by scale factor 0.5 in the x-direction.
4(a)	$y = 3x \sin 2x$ $\frac{dy}{dx} = \frac{d}{dx}(3x)\sin 2x + 3x\frac{d}{dx}(\sin 2x)$ (Product rule) $= 3\sin 2x + 6x\cos 2x = 0$ $\div \cos 2x$ $\frac{3\sin 2x}{\cos 2x} + 6x\frac{\cos 2x}{\cos 2x} = 0$ $\div 3$ $\Rightarrow \tan 2x + 2x = 0$
4(b)	$f(x) = \tan(2x) + 2x = 0.$ $f'(x) = 2\sec^2(2x) + 2$ $x_{n+1} = x_n - \frac{\tan(2x_n) + 2x_n}{2\sec^2(2x_n) + 2}.$ Let $x_0 = 1$ $x_1 = 1.01365728$ $x_2 = 1.01433905$ $\vdots$ $x = 1.0144$ (4 dp)

4(c)	$h = \pi/8$ ($\frac{1}{4}$ of $\pi/2$). $\frac{1}{2}h\left[0 + 2\left(3\times\frac{\pi}{8}\times\sin\left(\frac{\pi}{4}\right) + 3\left(\frac{\pi}{4}\right)\sin\left(\frac{\pi}{2}\right) + 3\left(\frac{3\pi}{8}\right)\sin\left(\frac{3\pi}{4}\right)\right) + 0\right]$ $= \frac{1}{16}\pi\left(\frac{3}{8}\pi\sqrt{2} + \frac{3}{2}\pi + \frac{9}{8}\pi\sqrt{2}\right)$ $= \frac{3}{32}\pi^2(\sqrt{2}+1)$.
4(d)(i)	$\int 3x\sin 2x\,dx = \frac{3}{4}(\sin(2x) - 2x\cos(2x)) + c$ $\frac{3}{4}\left[\sin(2x) - 2x\cos(2x)\right]_0^{\pi/2} = 3\pi/4$.
4(d)(ii)	$\frac{3}{32}\pi^2(\sqrt{2}+1) \approx 2.23$ $3\pi/4 \approx 2.36$. Trapezium rule is an underestimate.
4(d)(iii)	Not all strips of the trapeziums are under the curve and not all are over.

5(a)

RHS $= (\cot\theta + \operatorname{cosec}\theta)^2$

Expand brackets RHS: $\cot^2\theta + 2\cot\theta\operatorname{cosec}\theta + \operatorname{cosec}^2\theta$.

$$RHS = \frac{\cos^2\theta}{\sin^2\theta} + 2\left(\frac{\cos\theta}{\sin\theta}\right)\left(\frac{1}{\sin\theta}\right) + \frac{1}{\sin^2\theta}$$

All have common divisors $\sin^2\theta$.

$$RHS = \frac{\cos^2\theta + 2\cos\theta + 1}{\sin^2\theta}.$$

$$\sin^2\theta = (1-\cos^2\theta) = (1-\cos\theta)(1+\cos\theta)$$

$$RHS = \frac{\cos^2\theta + 2\cos\theta + 1}{(1-\cos\theta)(1+\cos\theta)}$$

Factorize denominator

$$RHS = \frac{(\cos\theta+1)(\cos\theta+1)}{(1-\cos\theta)(\cos\theta+1)} = \frac{1+\cos\theta}{1-\cos\theta}$$

$= LHS$

5(b) If $3(\cos\theta + \operatorname{cosec}\theta)^2 = 2\sec\theta$

By part a this implies

$$3 \times \frac{1+\cos\theta}{1-\cos\theta} = \frac{2}{\cos\theta}$$

$\Rightarrow 3(1+\cos\theta)(\cos\theta) = 2(1-\cos\theta)$

$\Rightarrow 3\cos^2\theta + 3\cos\theta = 2 - 2\cos\theta$

$\Rightarrow 3\cos^2\theta + 5\cos\theta - 2 = 0.$

$\Rightarrow (3\cos\theta - 1)(\cos\theta + 2).$

$\cos\theta$ cannot $= -2$, as $-1 \leq \cos\theta \leq 1$ for all θ.

So $\cos\theta = 1/3$.

For $0 < \theta < 2\pi$ there are two

$\theta = 1.23$, or ~~3~~ 5.05 (to 2dp).

6 $\dfrac{2x-1}{(2x+3)(x+1)} = \dfrac{A}{2x+3} + \dfrac{B}{x+1} + \dfrac{C}{(x+1)^2}$

$\Rightarrow 2x-1 \equiv A(x+1)^2 + B(2x+3)(x+1) + C(2x+3)$

If $x=-1$ then $-3=C$
If $x=-3/2$ then $A=-16$.
If $x=0$ then $B=8$.

$y = \dfrac{-16}{2x+3} + \dfrac{8}{x+1} - \dfrac{3}{(x+1)^2}$

$\int y\,dx = -16\int \dfrac{1}{2x+3}\,dx + 8\int \dfrac{1}{x+1}\,dx - 3\int \dfrac{1}{(x+1)^2}\,dx$

$= -16 \times \frac{1}{2}\ln(2x+3) + 8\ln(x+1) - 3/(x+1) + c$

y intercept $(0, -1/3)$
x intercept $(1/2, 0)$.

~~Area shaded~~ $\int_0^{1/2} y\,dx = \left[-8\ln(2x+3) + 8\ln(x+1) - \dfrac{3}{(x+1)}\right]_0^{1/2}$

$= (-8\ln 4 + 8\ln(3/2) + 2) - (-8\ln 3 + 3)$.

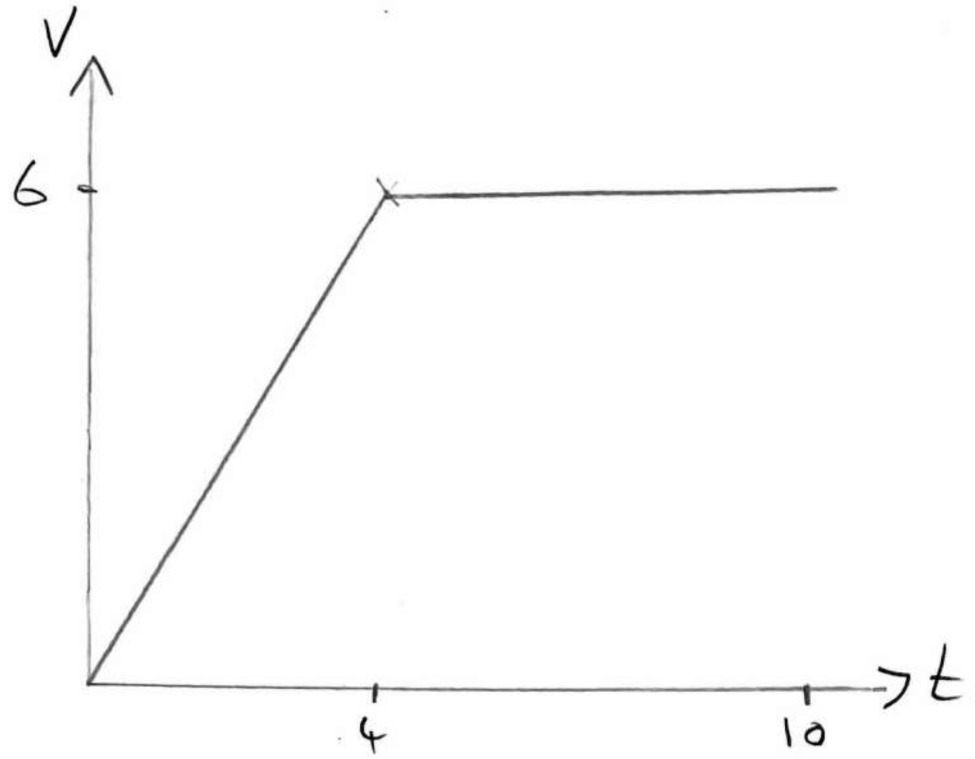
$= 8\ln(3) + 8\ln(3/2) - 8\ln(4) - 1$

$= 8\ln\left(\frac{9}{8}\right) - 1$. Area $= 1 + 8\ln\left(\frac{8}{9}\right)$

Area $= -1 \times \int_0^{1/2} y\,dx$.

(answer space continued on next page)

Section B: Mechanics

7(a)	
7(b)	Is the area under the graph: $\frac{1}{2} \times 4 \times 6 + 6 \times 6 = 48$ m.
8(a)	$S = 2.4 \times 5/3 d$ $S = 4d$.
8(b)	$S = ut + \frac{1}{2} at^2$ $t = 2.4$, $a = -9.8$ $s = 0$. $0 = u \times 2.4 - 9.8 \times 2.4^2 \times \frac{1}{2}$ $u = 9.8 \times 2.4 \times \frac{1}{2}$ $u = 11.76 ms^{-1}$.

8(c)	$d = \left(\frac{5}{3}d\right)t$, so $t = 3/5 = 0.6$, to reach the wall. $s = ut + \frac{1}{2}at^2$. height $= U \times t - \frac{1}{2}gt^2$ $= 11.76 \times 0.6 - 0.5 \times 9.8 \times 0.6^2$ height $= 5.292$ m.
8(d)	V = vertical velocity. $v = U - gt = 5.88\ \text{ms}^{-1}$. $\frac{5}{3}d$ = horizontal velocity. Speed $= \sqrt{\left(\frac{5}{3}d\right)^2 + (5.88)^2} = 16$ $\frac{25}{9}d^2 + 34.5744 = 256$ $d^2 = \frac{9}{25}(256 - 34.744) = 79.71\ldots$ $d = 8.93$ (3sf)

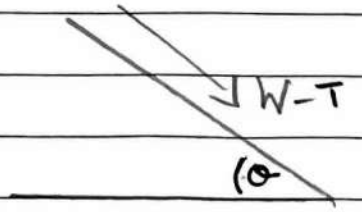
9(a)

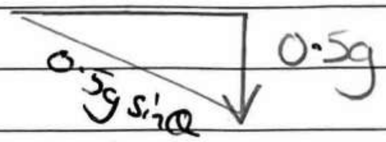
$0.3 = \frac{1}{2} \times a \times 0.4^2$, $a = 3.75$.

↳ by $s = ut + \frac{1}{2}at^2$ $(u = 0)$.

$\tan\theta = 3/4$. $\overset{O}{S}H\overset{A}{C}H\overset{O}{T}A$.

$\sin\theta = 3/5$.





$0.5\sin\theta \times g - T = 0.5a$.

$2.94 - T = 1.875$

$T = 1.065\text{N}$

9(b)

10(a)	$a = (2pt-3)\underline{i} + 8\underline{j}$ (dv/dt).
	$\lvert F\rvert^2 = 400 = m((2pt-3)^2+8^2)$ (when $t=0.5$)
	$t=0.5$, $m=2$
	$400 = (2p-6)^2+16^2$
	$4p^2-24p-108=0$.
	$p^2-6p-9=0$
	$(p-9)(p+3)=0$.
	$p=-3$ $(p\neq 9$ as $p<0)$.
10(b)	$v=(pt^2-3t)i+(8t+q)j$
	$s=\int v\,dt = (\frac{1}{3}pt^3-\frac{3}{2}t^2)\underline{i}+(4t^2+qt)j+c$.
	at $t=0$, $s=2\underline{i}-3\underline{j} \Rightarrow c=2\underline{i}-3\underline{j}$
	$s=(\frac{1}{3}pt^3-\frac{3}{2}t^2+2)\underline{i}+(4t^2+qt-3)\underline{j}$
	$s=(-t^3-\frac{3}{2}t^2+2)\underline{i}+(4t^2+qt-3)\underline{j}$

10(c) At $t=1$, $s = -\frac{1}{2}\underline{i} + (1+q)\underline{j}$

As it's on O to $2\underline{i} - 8\underline{j}$ then

$k(-\frac{1}{2}\underline{i} + (1+q)\underline{j}) = 2\underline{i} - 8\underline{j}$

$\Rightarrow k = -4$, so $-4(1+q) = -8$

$-4 - 4q = -8$

$-4q = -4$

$q = 1$.

11(a) Distance from base of ladder (A) to wall is $h/\sin(30) = l$.

Moment about A is force $\times$ distance

Contact force R_w.

So $R_w l = 2mgd\cos 30° + mg\,a\cos 30$

$R_w = \frac{1}{2h}\left(2mgd\frac{\sqrt{3}}{2} + mga\frac{\sqrt{3}}{2}\right)$

$R_w = \frac{mg(a+2d)\sqrt{3}}{4h}$

11(b)

$R_w \cos 30 + R_A = 2mg + mg$

where R_A is the contact force at A.

$F_A = R_w \sin 30$

(where F_A is the frictional contact force at A).

$\sin 30 R_w = \frac{\sqrt{3}}{8}(3mg - R_w \cos 30)$

Sub in R_w:

$$\frac{mg(a+2d)\sqrt{3}}{8h} = \frac{\sqrt{3}}{8}\left(3mg - \frac{3mg(a+2d)}{8h}\right)$$

$$\frac{\sqrt{3}mg}{8} \times \frac{(a+2d)}{h} = \frac{\sqrt{3}mg}{8}\left(3 - \frac{3(a+2d)}{8h}\right)$$

$$\frac{8(a+2d)}{8h} = 3 - \frac{3(a+2d)}{8h}$$

$$\frac{11(a+2d)}{8} = 3h$$

$$h = \frac{11}{24}(a+2d).$$

(answer space continued on next page)

11(b)	**(continued)** $h \leq 2a\sin 30 = a.$ $\frac{11}{24}(a+2d) \leq a$ $d \leq \frac{13}{22}a.$ greatest possible value for d is $\frac{13}{22}a$.
11(c)	Include a frictional component for contact between the ladder and the wall.
11(d)	