



Oxford Cambridge and RSA

AS Level Mathematics A

H230/02 Pure Mathematics and Mechanics

Wednesday 23 May 2018 – Morning

Time allowed: 1 hour 30 minutes



You must have:

- Printed Answer Booklet

You may use:

- a scientific or graphical calculator

INSTRUCTIONS

- Use black ink. HB pencil may be used for graphs and diagrams only.
- Complete the boxes provided on the Printed Answer Booklet with your name, centre number and candidate number.
- Answer **all** the questions.
- **Write your answer to each question in the space provided in the Printed Answer Booklet.** Additional paper may be used if necessary but you must clearly show your candidate number, centre number and question number(s).
- Do **not** write in the barcodes.
- You are permitted to use a scientific or graphical calculator in this paper.
- Final answers should be given to a degree of accuracy appropriate to the context.
- The acceleration due to gravity is denoted by $g \text{ m s}^{-2}$. Unless otherwise instructed, when a numerical value is needed, use $g = 9.8$.

INFORMATION

- The total number of marks for this paper is **75**.
- The marks for each question are shown in brackets [].
- **You are reminded of the need for clear presentation in your answers.**
- The Printed Answer Booklet consists of **12** pages. The Question Paper consists of **8** pages.

Formulae
AS Level Mathematics A (H230)

Binomial series

$$(a + b)^n = a^n + {}^n C_1 a^{n-1} b + {}^n C_2 a^{n-2} b^2 + \dots + {}^n C_r a^{n-r} b^r + \dots + b^n \quad (n \in \mathbb{N}),$$

$$\text{where } {}^n C_r = {}_n C_r = \binom{n}{r} = \frac{n!}{r!(n-r)!}$$

Differentiation from first principles

$$f'(x) = \lim_{h \rightarrow 0} \frac{f(x+h) - f(x)}{h}$$

Standard deviation

$$\sqrt{\frac{\sum(x-\bar{x})^2}{n}} = \sqrt{\frac{\sum x^2}{n} - \bar{x}^2} \quad \text{or} \quad \sqrt{\frac{\sum f(x-\bar{x})^2}{\sum f}} = \sqrt{\frac{\sum fx^2}{\sum f} - \bar{x}^2}$$

The binomial distribution

If $X \sim B(n, p)$ then $P(X = x) = \binom{n}{x} p^x (1-p)^{n-x}$, Mean of X is np , Variance of X is $np(1-p)$

Kinematics

$$v = u + at$$

$$s = ut + \frac{1}{2}at^2$$

$$s = \frac{1}{2}(u + v)t$$

$$v^2 = u^2 + 2as$$

$$s = vt - \frac{1}{2}at^2$$

Section A: Pure Mathematics

Answer **all** the questions

- 1 In triangle ABC , $AB = 20$ cm and angle $B = 45^\circ$.
- (i) Given that $AC = 16$ cm, find the two possible values for angle C , correct to 1 decimal place. [4]
- (ii) Given instead that the area of the triangle is $75\sqrt{2}$ cm², find BC . [2]
- 2 (i) The curve $y = \frac{2}{3+x}$ is translated by four units in the positive x -direction. State the equation of the curve after it has been translated. [2]
- (ii) Describe fully the single transformation that transforms the curve $y = \frac{2}{3+x}$ to $y = \frac{5}{3+x}$. [2]

- 3 In each of the following cases choose one of the statements

$$P \Rightarrow Q \quad P \Leftarrow Q \quad P \Leftrightarrow Q$$

to describe the relationship between P and Q .

- (i) $P: y = 3x^5 - 4x^2 + 12x$
 $Q: \frac{dy}{dx} = 15x^4 - 8x + 12$ [1]
- (ii) $P: x^5 - 32 = 0$ where x is real
 $Q: x = 2$ [1]
- (iii) $P: \ln y < 0$
 $Q: y < 1$ [1]
- 4 (i) Express $4x^2 - 12x + 11$ in the form $a(x+b)^2 + c$. [3]
- (ii) State the number of real roots of the equation $4x^2 - 12x + 11 = 0$. [1]
- (iii) Explain fully how the value of r is related to the number of real roots of the equation $p(x+q)^2 + r = 0$ where p, q and r are real constants and $p > 0$. [2]

- 5 In this question you must show detailed reasoning.

The line $x + 5y = k$ is a tangent to the curve $x^2 - 4y = 10$. Find the value of the constant k . [5]

- 6 A pan of water is heated until it reaches 100°C . Once the water reaches 100°C , the heat is switched off and the temperature $T^{\circ}\text{C}$ of the water decreases. The temperature of the water is modelled by the equation

$$T = 25 + ae^{-kt},$$

where t denotes the time, in minutes, after the heat is switched off and a and k are positive constants.

- (i) Write down the value of a . [1]
- (ii) Explain what the value of 25 represents in the equation $T = 25 + ae^{-kt}$. [1]

When the heat is switched off, the initial rate of decrease of the temperature of the water is 15°C per minute.

- (iii) Calculate the value of k . [3]
- (iv) Find the time taken for the temperature of the water to drop from 100°C to 45°C . [3]
- (v) A second pan of water is heated, but the heat is turned off when the water is at a temperature of less than 100°C . Suggest how the equation for the temperature as the water cools would be modified by this. [1]
- 7 (i) Show that the equation

$$2 \sin x \tan x = \cos x + 5$$

can be expressed in the form

$$3 \cos^2 x + 5 \cos x - 2 = 0. \quad [3]$$

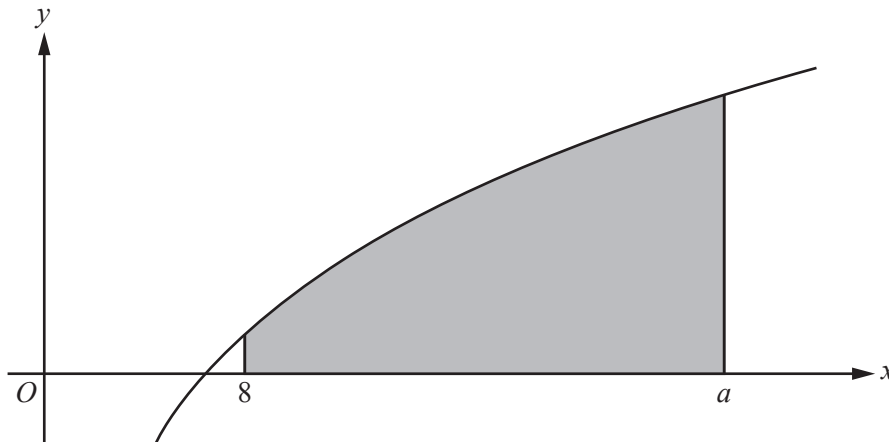
- (ii) Hence solve the equation

$$2 \sin 2\theta \tan 2\theta = \cos 2\theta + 5,$$

giving all values of θ between 0° and 180° , correct to 1 decimal place. [5]

8 In this question you must show detailed reasoning.

The diagram shows part of the graph of $y = 2x^{\frac{1}{3}} - \frac{7}{x^{\frac{1}{3}}}$. The shaded region is enclosed by the curve, the x -axis and the lines $x = 8$ and $x = a$, where $a > 8$.



Given that the area of the shaded region is 45 square units, find the value of a .

[9]

Section B: Mechanics
Answer **all** the questions

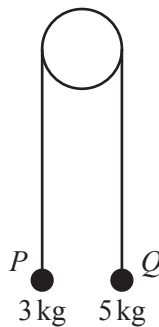
- 9 In this question the horizontal unit vectors \mathbf{i} and \mathbf{j} are in the directions east and north respectively.

A model ship of mass 2 kg is moving so that its acceleration vector $\mathbf{a} \text{ m s}^{-2}$ at time t seconds is given by $\mathbf{a} = 3(2t - 5)\mathbf{i} + 4\mathbf{j}$. When $t = T$, the magnitude of the horizontal force acting on the ship is 10 N.

Find the possible values of T .

[4]

- 10 Particles P and Q , of masses 3 kg and 5 kg respectively, are attached to the ends of a light inextensible string. The string passes over a smooth fixed pulley. The system is held at rest with the string taut. The hanging parts of the string are vertical and P and Q are above a horizontal plane (see diagram).



- (i) Find the tension in the string immediately after the particles are released.

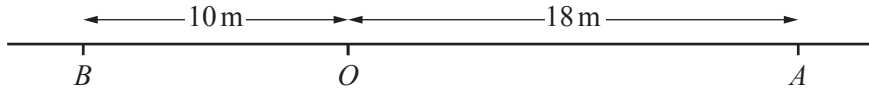
[4]

After descending 2.5 m, Q strikes the plane and is immediately brought to rest. It is given that P does not reach the pulley in the subsequent motion.

- (ii) Find the distance travelled by P between the instant when Q strikes the plane and the instant when the string becomes taut again.

[4]

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A particle P is moving along a straight line with constant acceleration. Initially the particle is at O . After 9 s, P is at a point A , where $OA = 18$ m (see diagram) and the velocity of P at A is 8 m s^{-1} in the direction \overrightarrow{OA} .

(i) (a) Show that the initial speed of P is 4 m s^{-1} . [2]

(b) Find the acceleration of P . [2]

B is a point on the line such that $OB = 10$ m, as shown in the diagram.

(ii) Show that P is never at point B . [4]

A second particle Q moves along the same straight line, but has variable acceleration. Initially Q is at O , and the displacement of Q from O at time t seconds is given by

$$x = at^3 + bt^2 + ct,$$

where a , b and c are constants.

It is given that

- the velocity and acceleration of Q at the point O are the same as those of P at O ,
- Q reaches the point A when $t = 6$.

(iii) Find the velocity of Q at A . [5]

END OF QUESTION PAPER

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