

A Level Transition Card Answers

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Proof

P1

Prime numbers between 35 and 50 are:

37, 41, 43, 47

P2

a) If x is one more than a multiple of 3 then we can write $x = 3n + 1$ where n is an integer.
Therefore $x^2 - 3x - 1 = (3n + 1)^2 - 3(3n + 1) - 1$
 $x^2 - 3x - 1 = 9n^2 + 6n + 1 - 9n - 3 - 1$
 $x^2 - 3x - 1 = 9n^2 - 3n - 3$
 $x^2 - 3x - 1 = 3(3n^2 - n - 1)$
Since n is an integer, $3n^2 - n - 1$ must be an integer, so $3(3n^2 - n - 1)$ is a multiple of 3.
Hence, $x^2 - 3x - 1$ is a multiple of 3 when x is one more than a multiple of 3.

b) The square numbers under 100 are:

1, 4, 9, 16, 25, 36, 49, 64, 81

None of these are divisible by 11, so the statement is proved by exhaustion.

P3

Try $x = 3$:

$3^4 - 1 = 81 - 1 = 80$ which is not a multiple of 3.

Therefore, we have found a counterexample, so the statement is disproved.

P4

Suppose there are not infinitely many prime numbers.
So the number of prime numbers is some finite number n .

Then we can list them: p_1, p_2, \dots, p_n

Now define $N = p_1 \times p_2 \times \dots \times p_n$

Consider $N + 1$

All of the prime numbers on the list are NOT factors of $N + 1$

But $N + 1$ must have a prime factorisation or be a prime number itself.

This requires the existence of primes that are not on our list.

Therefore, there are more than n primes, which is a contradiction of our assumption that there are n primes.

This contradicts the original assumption.

Hence, there must be an infinite number of prime numbers.

Algebra & Functions

A1

$$16 = 2^4$$

$$\text{So } 16^{-\frac{3}{4}} = (2^4)^{-\frac{3}{4}}$$

$$16^{-\frac{3}{4}} = 2^{4 \times (-\frac{3}{4})}$$

$$16^{-\frac{3}{4}} = 2^{-3}$$

$$16^{-\frac{3}{4}} = \left(\frac{1}{2}\right)^3$$

$$16^{-\frac{3}{4}} = \frac{1}{2^3}$$

$$16^{-\frac{3}{4}} = \frac{1}{8}$$

A2

$$\sqrt{18} \times \sqrt{2} = \sqrt{18 \times 2}$$

$$\sqrt{18} \times \sqrt{2} = \sqrt{36}$$

$$\sqrt{18} \times \sqrt{2} = 6$$

A3

$$(1 + \sqrt{6})(1 - \sqrt{6}) = 1 + \sqrt{6} - \sqrt{6} - 6$$

$$(1 + \sqrt{6})(1 - \sqrt{6}) = -5$$

A4

$$\frac{7}{8+\sqrt{2}} = \frac{7}{8+\sqrt{2}} \times \frac{8-\sqrt{2}}{8-\sqrt{2}}$$

$$\frac{7}{8+\sqrt{2}} = \frac{7(8-\sqrt{2})}{(8+\sqrt{2})(8-\sqrt{2})}$$

$$\frac{7}{8+\sqrt{2}} = \frac{56-7\sqrt{2}}{64+8\sqrt{2}-8\sqrt{2}-2}$$

$$\frac{7}{8+\sqrt{2}} = \frac{56-7\sqrt{2}}{62}$$

A5

$$\frac{x^2-3x-4}{x^4+2x^3+x^2} = \frac{(x-4)(x+1)}{x^2(x^2+2x+1)}$$

$$\frac{x^2-3x-4}{x^4+2x^3+x^2} = \frac{(x-4)(x+1)}{x^2(x+1)^2}$$

$$\frac{x^2-3x-4}{x^4+2x^3+x^2} = \frac{x-4}{x^2(x+1)}$$

A6

$$\frac{x^2+6x+8}{3x^2} \times \frac{x^3+2x^2}{(x+2)^2} = \frac{(x+2)(x+4)}{3x^2} \times \frac{x^2(x+2)}{(x+2)^2}$$

$$\frac{x^2+6x+8}{3x^2} \times \frac{x^3+2x^2}{(x+2)^2} = \frac{x^2(x+2)^2(x+4)}{3x^2(x+2)^2}$$

$$\frac{x^2+6x+8}{3x^2} \times \frac{x^3+2x^2}{(x+2)^2} = \frac{x+4}{3}$$

A7

$$\frac{2x+1}{x+5} - \frac{9x}{(x+6)^2} = \frac{(2x+1)(x+6)^2 - 9x(x+5)}{(x+5)(x+6)^2}$$

$$\frac{2x+1}{x+5} - \frac{9x}{(x+6)^2} = \frac{(2x+1)(x^2+12x+36) - 9x^2 - 45x}{(x+5)(x+6)^2}$$

$$\frac{2x+1}{x+5} - \frac{9x}{(x+6)^2} = \frac{2x^3+25x^2+84x+36 - 9x^2 - 45x}{(x+5)(x+6)^2}$$

$$\frac{2x+1}{x+5} - \frac{9x}{(x+6)^2} = \frac{2x^3+16x^2+39x+36}{(x+5)(x+6)^2}$$

A8

Factors are $(x - 3)$, $(x - 5)$ and $(x + 1)$

Hence, $f(x) = (x - 3)(x - 5)(x + 1)$

A9

Using the long division method, $x^3 + 6x^2 + 12x + 8$

A10

$$3x^2 + 12x + 17 = 3(x^2 + 4x) + 17$$

$$3x^2 + 12x + 17 = 3(x + 2)^2 - 12 + 17$$

$$3x^2 + 12x + 17 = 3(x + 2)^2 + 5$$

A11

$$a = 9, b = 122 \text{ and } c = 17885$$

$$x = 38.313$$

$$x = -51.868$$

A12

$$b^2 - 4ac = (-42)^2 - 4 \times 7 \times 63 = 0$$

Hence, the equation has one real root.

A13

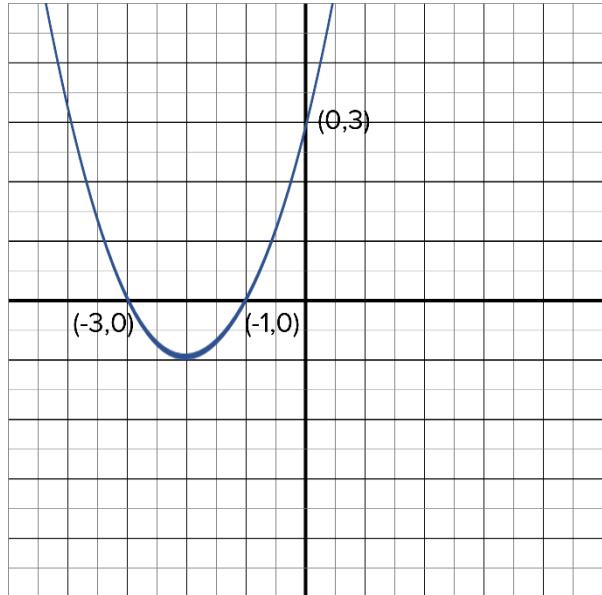
$$2x^2 - 10x + 15 = 2(x^2 - 5x) + 15$$

$$2x^2 - 10x + 15 = 2\left(x - \frac{5}{2}\right)^2 - \frac{25}{2} + 15$$

$$2x^2 - 10x + 15 = 2\left(x - \frac{5}{2}\right)^2 + \frac{5}{2}$$

$$\text{Vertex} = \left(\frac{5}{2}, \frac{5}{2}\right)$$

A14



A15

$$y = 2x + 1$$

$$3(2x + 1) + 2 = 2x^2 - 10x + 29$$

$$6x + 3 + 2 = 2x^2 - 10x + 29$$

$$6x + 5 = 2x^2 - 10x + 29$$

$$2x^2 - 16x + 24 = 0$$

$$x^2 - 8x + 12 = 0$$

$$(x - 6)(x - 2) = 0$$

$$x = 6 \text{ or } x = 2$$

$$y = 2 \times 6 + 1 \text{ or } y = 2 \times 2 + 1$$

$$x = 6, y = 13 \text{ or } x = 2, y = 5$$

A16

$$\emptyset$$

A17

$$x^2 < 8x - 15$$

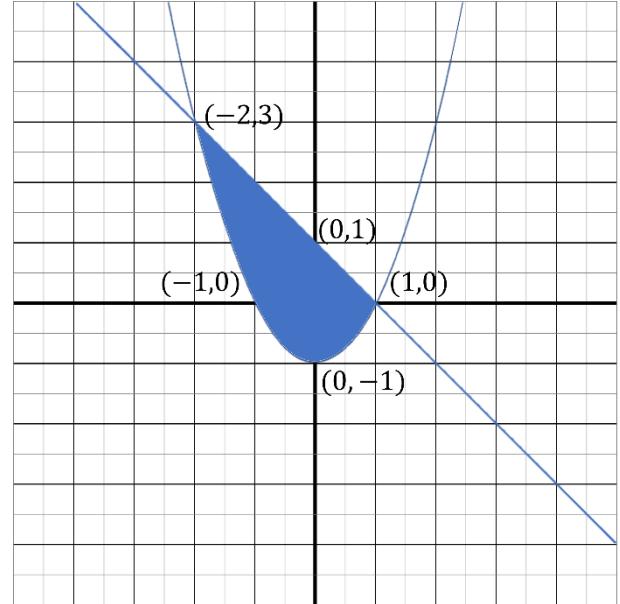
$$x^2 - 8x + 15 < 0$$

$$(x - 3)(x - 5) < 0$$

Critical values: $x = 3, x = 5$

$$3 < x < 5$$

A18



A19

$$x^3 - 2x^2 - x + 2 = (x - 1)(x^2 - x - 2)$$

$$x^3 - 2x^2 - x + 2 = (x - 1)(x - 2)(x + 1)$$

So the other factors are $(x - 2)$ and $(x + 1)$

A20

First, factorise $x^3 - 6x^2 + 11x - 6$. Start by guessing a factor, e.g. $x - 1$
 If $x = 1$, then $x^3 - 6x^2 + 11x - 6 = 1 - 6 + 11 - 6 = 0$, so $x - 1$ is a factor.

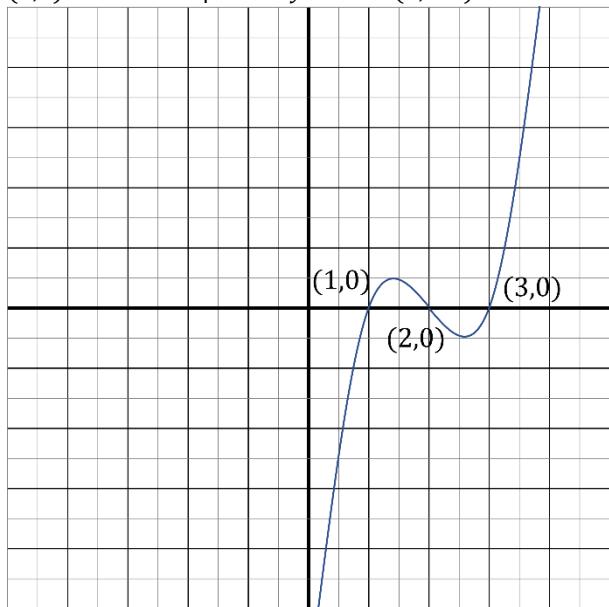
Now, we can use any polynomial division technique to determine:

$$x^3 - 6x^2 + 11x - 6 = (x - 1)(x^2 - 5x + 6)$$

Now factorise the quadratic to find the other two factors.

$$x^3 - 6x^2 + 11x - 6 = (x - 1)(x - 2)(x - 3)$$

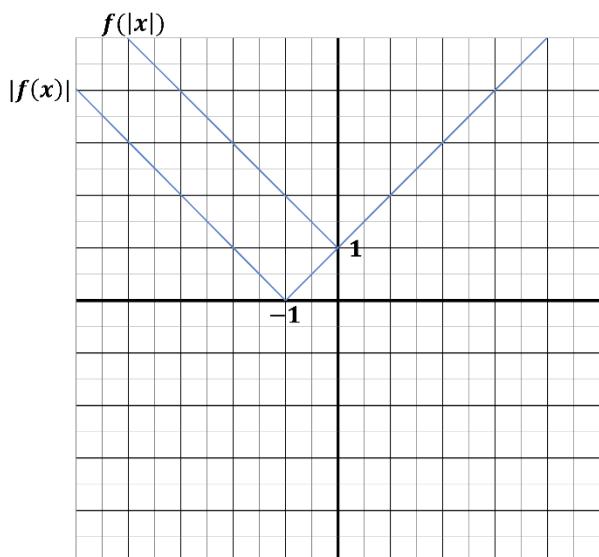
Hence, the graph intercepts the x axis at $(1,0)$, $(2,0)$ and $(3,0)$ and intercepts the y axis at $(0,-6)$.



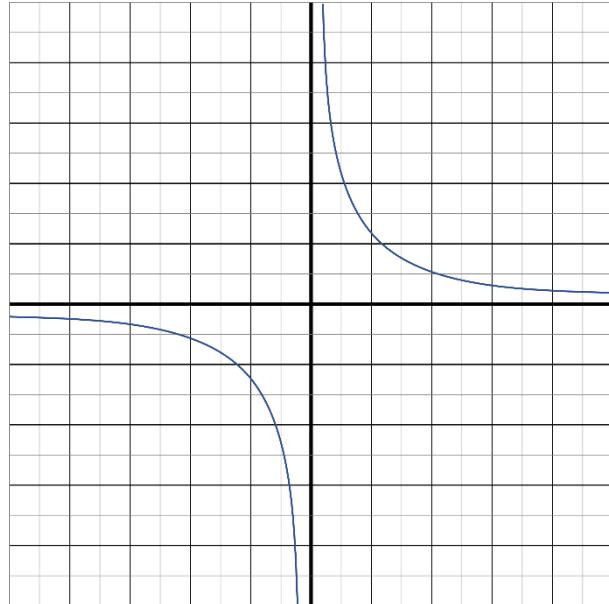
A21

$$\begin{aligned}f(|-3|) &= f(3) \\f(|-3|) &= 3^3 - 3 \times 3 \\f(|-3|) &= 27 - 9 \\f(|-3|) &= 18\end{aligned}$$

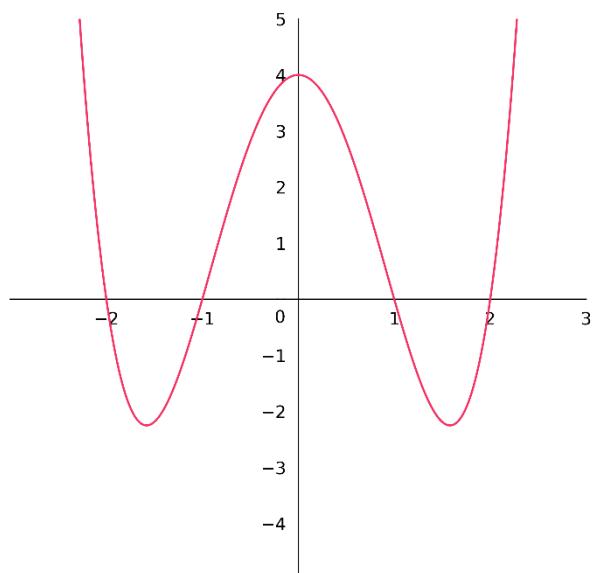
A22



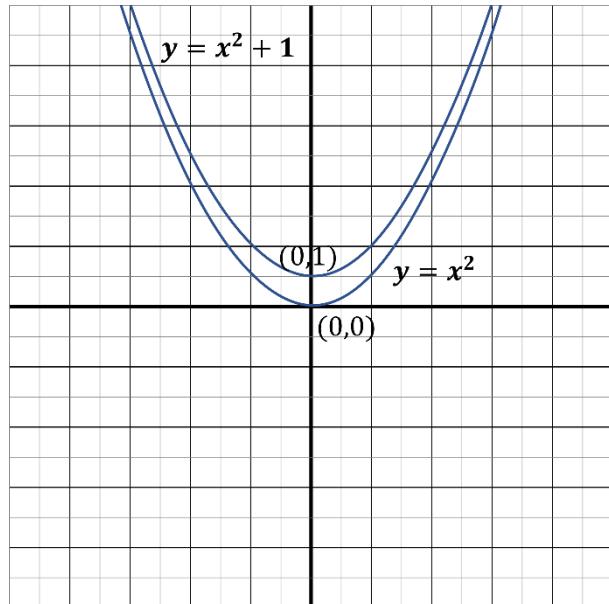
A23



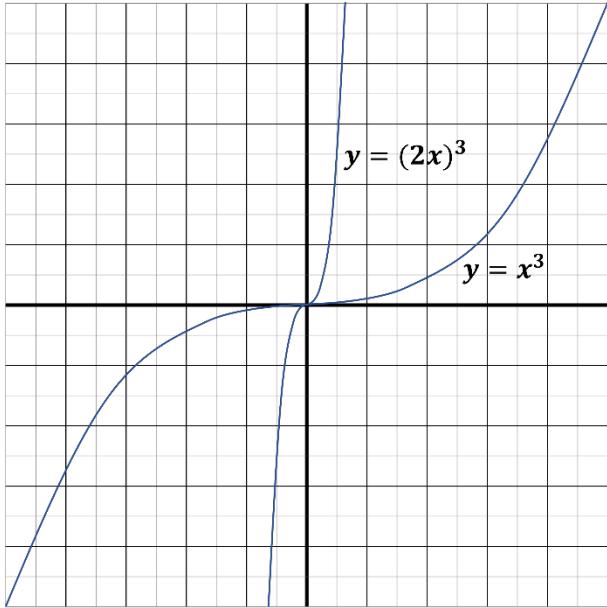
A24



A25



A26



A27

$$y \propto x^2$$

$$y = kx^2$$

$$75 = k \times 5^2$$

$$75 = 25k$$

$$k = 3$$

A28

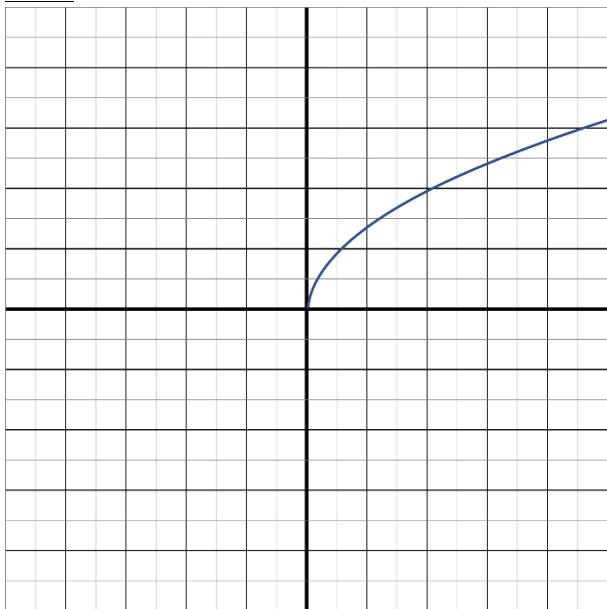
$$y \propto \frac{1}{x}$$

$$y = \frac{k}{x}$$

$$0.4 = \frac{k}{5}$$

$$k = 2$$

A29



A30

$$fg(x) = f(g(x))$$

$$fg(x) = f(x^2 - 1)$$

$$fg(x) = 3(x^2 - 1)^2 - 4$$

$$fg(x) = 3(x^4 - 2x^2 + 1) - 4$$

$$fg(x) = 3x^4 - 6x^2 + 3 - 4$$

$$fg(x) = 3x^4 - 6x^2 - 1$$

A31

$$f(x) = \sin(2x - 8)$$

$$y = \sin(2x - 8)$$

$$\sin^{-1}(y) = 2x - 8$$

$$\sin^{-1}(y) + 8 = 2x$$

$$x = \frac{\sin^{-1}(y)+8}{2}$$

Hence, $f^{-1}(x) = \frac{\sin^{-1}(x)+8}{2}$

Coordinate Geometry

C1

$$m = \frac{9 - 3}{4 - 2} = \frac{6}{2} = 3$$

$$y = 3x + c$$

$$3 = 3 \times 2 + c$$

$$3 = 6 + c$$

$$c = -3$$

$$y = 3x - 3$$

C2

$$\text{Midpoint} = \left(\frac{-8+8}{2}, \frac{10-2}{2} \right) = \left(\frac{0}{2}, \frac{8}{2} \right) = (0, 4)$$

$$\text{Length} = \sqrt{(-8 - 8)^2 + (10 - (-2))^2}$$

$$\text{Length} = \sqrt{16^2 + 12^2}$$

$$\text{Length} = \sqrt{256 + 144}$$

$$\text{Length} = \sqrt{400}$$

$$\text{Length} = 20$$

C3

$$\text{First line: } m = \frac{2-0}{4-0} = \frac{2}{4} = \frac{1}{2}$$

$$\text{Second line: } m = \frac{0-2}{1-0} = \frac{-2}{1} = -2$$

These are the negative reciprocals of each other, so the lines are perpendicular.

C4

The angle at the circumference (top) = $140 \div 2 = 70^\circ$
The reflex angle at the centre = $360 - 140 = 220^\circ$
Angles in a quadrilateral add up to 360° , so:
 $70 + 220 + x + x = 360$
 $290 + 2x = 360$
Hence, $x = 35^\circ$

C5

$$(x - 2)^2 + (y - 3)^2 = 5^2$$

$$x^2 - 4x + 4 + y^2 - 6y + 9 = 25$$

$$x^2 + y^2 - 4x - 6y + 13 - 25 = 0$$

$$x^2 + y^2 - 4x - 6y - 12 = 0$$

C6

Centre is $(-4,3)$

$$\text{Gradient of radius} = \frac{6-3}{-8-(-4)} = -\frac{3}{4}$$

$$\text{Gradient of tangent} = \frac{4}{3}$$

$$y = \frac{4}{3}x + c$$

$$6 = \frac{4}{3} \times (-8) + c$$

$$6 = -\frac{32}{3} + c$$

$$c = \frac{50}{3}$$

$$y = \frac{4}{3}x + \frac{50}{3}$$

Sequences and Series

S1

$$u_2 = 6 \times 3 + 2 = 20$$

$$u_3 = 6 \times 20 + 2 = 122$$

$$u_4 = 6 \times 122 + 2 = 734$$

$$u_5 = 6 \times 734 + 2 = 4406$$

S2

$$a = 5$$

$$d = 6$$

$$S_n = \frac{n}{2}(2a + (n-1)d)$$

$$S_{100} = \frac{100}{2}(2 \times 5 + (100-1) \times 6)$$

$$S_{100} = 30200$$

S3

$$a = 648$$

$$r = \frac{1}{3}$$

$$S_8 = \frac{a(1-r^n)}{1-r}$$

$$S_8 = \frac{648\left(1-\frac{1}{3}^8\right)}{1-\frac{1}{3}}$$

$$S_8 = 971.85$$

Trigonometry and Vectors

T1

$$\frac{\sin(x)}{4} = \frac{\sin(50^\circ)}{5}$$

$$\sin(x) = \frac{4\sin(50^\circ)}{5}$$

$$x = \sin^{-1}\left(\frac{4\sin(50^\circ)}{5}\right)$$

$$x = 37.8^\circ$$

T2

$$4^2 = 3^2 + 5^2 - 2 \times 3 \times 5 \times \cos(x)$$

$$16 = 9 + 25 - 30\cos(x)$$

$$16 = 34 - 30\cos(x)$$

$$30\cos(x) = 18$$

$$\cos(x) = \frac{3}{5}$$

$$x = 53.1^\circ$$

T3

$$\text{Area} = \frac{1}{2}ab \sin(C)$$

$$\text{Area} = \frac{1}{2} \times 3 \times 5 \times \sin(60)$$

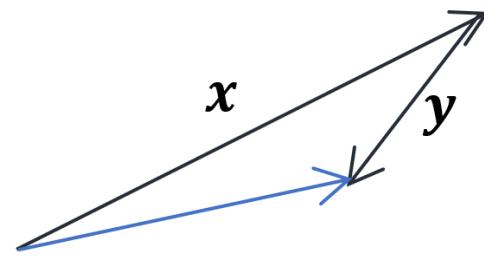
$$\text{Area} = \frac{15}{2} \times \frac{\sqrt{3}}{2}$$

$$\text{Area} = \frac{15\sqrt{3}}{4}$$

T4

Size / magnitude and direction

T5



T6

$$(4\mathbf{i} + 3\mathbf{j}) + (2\mathbf{i} - \mathbf{j}) = (4+2)\mathbf{i} + (3-1)\mathbf{j} = 6\mathbf{i} + 2\mathbf{j}$$

Differentiation and Integration

D1

$$\frac{d}{dx}(x^7) = 7x^6$$

D2

$$\frac{d}{dx}\left(x^{-3} + x^{\frac{1}{2}}\right) = -3x^{-4} + \frac{3}{2}x^{\frac{1}{2}}$$

D3

$$\int x^{\frac{1}{2}} dx = \frac{2}{3}x^{\frac{3}{2}} + c$$

Probability

PR1

There are 13 hearts out of 52 cards.

Hence, the probability is $\frac{13}{52}$ (or $\frac{1}{4}$)

PR2

$P(A \cup B) = P(A) + P(B)$ because A and B are mutually exclusive.

$$0.9 = 0.4 + P(B)$$

$$P(B) = 0.5$$

PR3

$$1 - 0.91 = 0.09$$

PR4

	A	A'	Total
B	0.1	0.3	0.4
B'	0.2	0.4	0.6
Total	0.3	0.7	1

PR5

$$P(A') = 1 - 0.3 = 0.7$$

$$P(A|B) = \frac{0.4 \times 0.3}{0.4 \times 0.3 + 0.6 \times 0.7}$$

$$P(A|B) = \frac{2}{9}$$

PR6

