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Candidate surname	Other names	
Centre Number		Candidate Number
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Pearson Edexcel Level 3 GCE		
Wednesday 13 May 2020		
Morning (Time: 2 hours)	Paper Reference 8MA0/01	
Mathematics Advanced Subsidiary Paper 1: Pure Mathematics		
You must have: Mathematical Formulae and Statistical Tables (Green), calculator		Total Marks

Candidates may use any calculator allowed by the regulations of the Joint Council for Qualifications. Calculators must not have the facility for symbolic algebra manipulation, differentiation and integration, or have retrievable mathematical formulae stored in them.

Instructions

- Use **black** ink or ball-point pen.
- If pencil is used for diagrams/sketches/graphs it must be dark (HB or B).
- **Fill in the boxes** at the top of this page with your name, centre number and candidate number.
- Answer **all** questions and ensure that your answers to parts of questions are clearly labelled.
- Answer the questions in the spaces provided
– *there may be more space than you need.*
- You should show sufficient working to make your methods clear.
Answers without working may not gain full credit.
- Inexact answers should be given to three significant figures unless otherwise stated.

Information

- A booklet 'Mathematical Formulae and Statistical Tables' is provided.
- There are 14 questions in this question paper. The total mark for this paper is 100.
- The marks for **each** question are shown in brackets
– *use this as a guide as to how much time to spend on each question.*

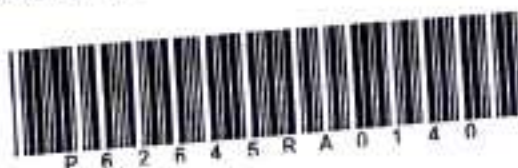
Advice

- Read each question carefully before you start to answer it.
- Try to answer every question.
- Check your answers if you have time at the end.

Turn over ►

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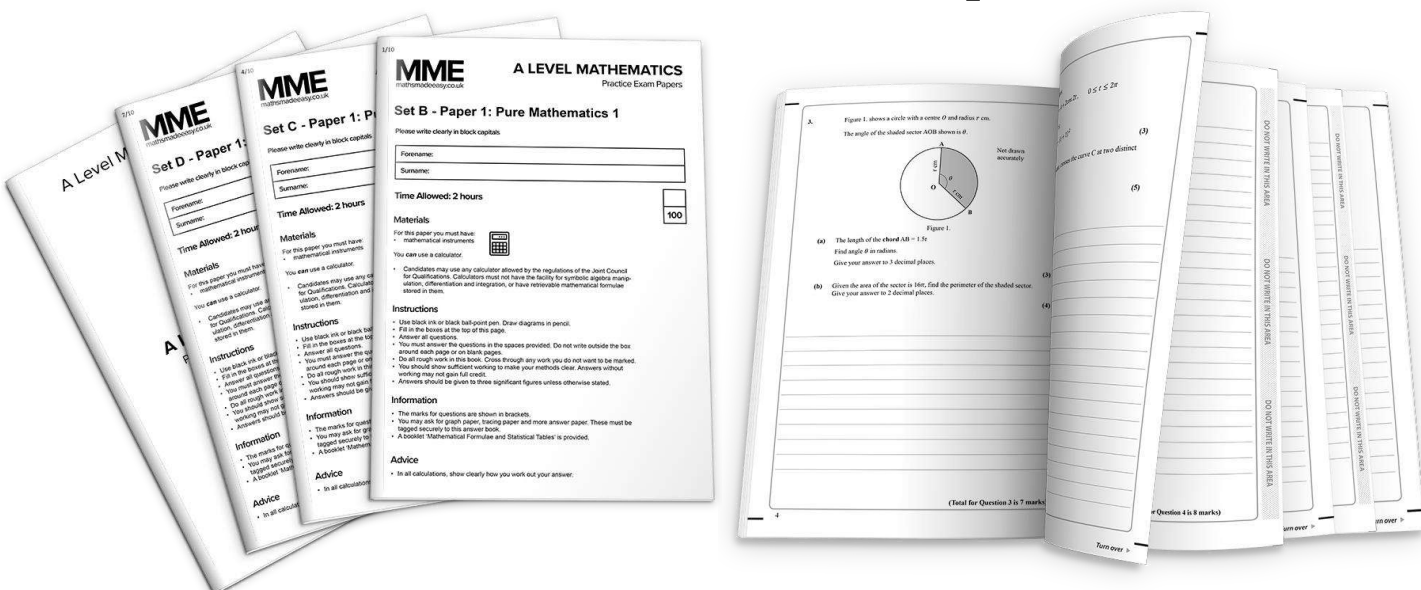

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Predicted Papers



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1. A curve has equation

$$y = 2x^3 - 4x + 5$$

Find the equation of the tangent to the curve at the point $P(2, 13)$.

Write your answer in the form $y = mx + c$, where m and c are integers to be found.

Solutions relying on calculator technology are not acceptable.

(5)

$$y = 2x^3 - 4x + 5$$

$$\frac{dy}{dx} = 6x^2 - 4$$

$$x = 2$$

$$\frac{dy}{dx} = 6 \times 2^2 - 4$$

$$= 6 \times 4 - 4$$

$$= 24 - 4$$

$$= 20$$

$$y = 20x + c$$

$$13 = 20 \times 2 + c$$

$$13 = 40 + c$$

$$c = -27$$

$$y = 20x - 27$$

2. [In this question the unit vectors \mathbf{i} and \mathbf{j} are due east and due north respectively.]

A coastguard station O monitors the movements of a small boat.

At 10:00 the boat is at the point $(4\mathbf{i} - 2\mathbf{j})$ km relative to O .

At 12:45 the boat is at the point $(-3\mathbf{i} - 5\mathbf{j})$ km relative to O .

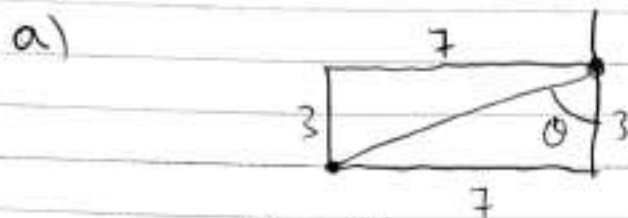
The motion of the boat is modelled as that of a particle moving in a straight line at constant speed.

- (a) Calculate the bearing on which the boat is moving, giving your answer in degrees to one decimal place.

(3)

- (b) Calculate the speed of the boat, giving your answer in km h^{-1}

(3)



$$\tan \theta = \frac{7}{3}$$

$$\theta = \tan^{-1}\left(\frac{7}{3}\right)$$

$$\text{bearing} = 180 + \tan^{-1}\left(\frac{7}{3}\right)$$

$$\text{bearing} = 180 + 67$$

$$\text{bearing} = 246.8^\circ$$

b)

$$\sqrt{3^2 + 7^2} = \sqrt{58}$$

$$\text{Travelled } \sqrt{58} \text{ km in } 2.75 \text{ hr.}$$

$$\text{speed} = \frac{\sqrt{58}}{2.75} = 2.77 \text{ km h}^{-1}$$

3.

In this question you must show all stages of your working.

Solutions relying on calculator technology are not acceptable.

(i) Solve the equation

$$x\sqrt{2} - \sqrt{18} = x$$

writing the answer as a surd in simplest form.

(3)

(ii) Solve the equation

$$4^{3x-2} = \frac{1}{2\sqrt{2}}$$

(3)

$$\begin{aligned}
 \text{i)} \quad x\sqrt{2} - \sqrt{18} &= x \\
 x(\sqrt{2} - 1) &= \sqrt{18} \\
 x &= \frac{\sqrt{18}}{\sqrt{2} - 1} \\
 x &= \frac{\sqrt{18}(\sqrt{2} + 1)}{(\sqrt{2} - 1)(\sqrt{2} + 1)} \\
 x &= \frac{\sqrt{36} + \sqrt{18}}{2 - 1} \\
 x &= \frac{6 + 3\sqrt{2}}{1} \\
 x &= 6 + 3\sqrt{2}
 \end{aligned}$$

$$\begin{aligned}
 \text{ii)} \quad 4^{3x-2} &= \frac{1}{2\sqrt{2}} \\
 4^{3x-2} &= \frac{1}{2^1 \times 2^{\frac{1}{2}}} \\
 4^{3x-2} &= \frac{1}{2^{\frac{3}{2}}} \\
 4^{3x-2} &= 2^{-\frac{3}{2}} \\
 4^{3x-2} &= (4^{\frac{1}{2}})^{-\frac{3}{2}} \\
 4^{3x-2} &= 4^{-\frac{3}{2}} \\
 3x-2 &= -\frac{3}{2} \\
 3x &= \frac{5}{2} \\
 x &= \frac{5}{12}
 \end{aligned}$$

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4. In 1997 the average CO₂ emissions of new cars in the UK was 190 g/km.

In 2005 the average CO₂ emissions of new cars in the UK had fallen to 169 g/km.

Given A g/km is the average CO₂ emissions of new cars in the UK n years after 1997 and using a linear model,

(a) form an equation linking A with n .

(3)

In 2016 the average CO₂ emissions of new cars in the UK was 120 g/km.

(b) Comment on the suitability of your model in light of this information.

(3)

a) Linear equation: $A = mn + c$.

$c = 190$ ($n = 0$ in 1997)

$$169 = 8m + 190$$

$$8m = -21$$

$$m = -\frac{21}{8}$$

$$A = -\frac{21}{8}n + 190$$

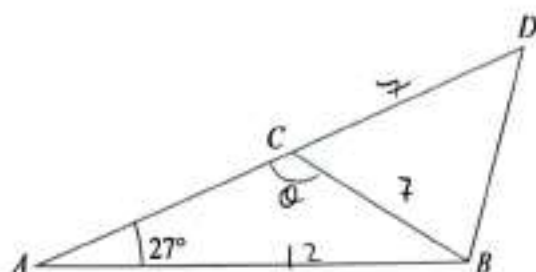
b) $n = 19$

$$A = -\frac{21}{8} \times 19 + 190$$

$$A = 140.125$$

It is predicting a much higher value so is not suitable.

5.



Not to scale

Figure 1

Figure 1 shows the design for a structure used to support a roof.

The structure consists of four steel beams, AB , BD , BC and AD .

Given $AB = 12$ m, $BC = BD = 7$ m and angle $BAC = 27^\circ$

(a) find, to one decimal place, the size of angle ACB .

(3)

The steel beams can only be bought in whole metre lengths.

(b) Find the minimum length of steel that needs to be bought to make the complete structure.

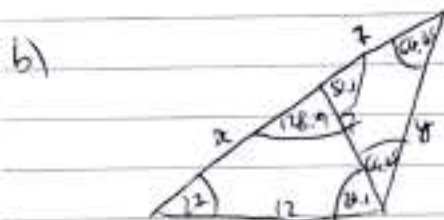
(3)

$$\begin{aligned} \frac{\sin(27^\circ)}{7} &= \frac{\sin(\theta)}{12} \\ \sin(\theta) &= \frac{12 \sin(27^\circ)}{7} \\ \theta &= \sin^{-1}\left(\frac{12 \sin(27^\circ)}{7}\right) \\ \theta &= 51.1^\circ \end{aligned}$$

But the angle is clearly larger.

$$\theta = 180 - 51.1$$

$$\theta = 128.9^\circ$$



$$\begin{aligned} x^2 &= 7^2 + 12^2 - 2 \times 7 \times 12 \times \cos(24.1) \\ x &= \sqrt{49 + 144 - 168 \cos(24.1)} \\ x &= 6.30 \text{ m} \end{aligned}$$

$$\begin{aligned} y^2 &= 7^2 + 7^2 - 2 \times 7 \times 7 \times \cos(51.1) \\ y &= \sqrt{49 + 49 - 98 \cos(51.1)} \\ y &= 7 \sqrt{2(1 - \cos(51.1))} \\ y &= 6.04 \text{ m} \end{aligned}$$

$$\text{Total} = 12 + 7 + 7 + 6.30 + 6.04 = 42 \text{ m}$$

6. (a) Find the first 4 terms, in ascending powers of x , in the binomial expansion of

$$(1 + kx)^{10}$$

where k is a non-zero constant. Write each coefficient as simply as possible.

(3)

Given that in the expansion of $(1 + kx)^{10}$ the coefficient of x^3 is 3 times the coefficient of x ,

- (b) find the possible values of k .

(3)

$$\begin{aligned} \text{a) } (1+kx)^{10} &= 1 + \binom{10}{1} kx + \binom{10}{2} k^2 x^2 + \binom{10}{3} k^3 x^3 \\ &= 1 + 10kx + 45k^2 x^2 + \frac{10!}{2!3!} k^3 x^3 \\ &= 1 + 10kx + 45k^2 x^2 + \frac{10 \times 9 \times 8}{6} k^3 x^3 \\ &= 1 + 10kx + 45k^2 x^2 + 120k^3 x^3. \end{aligned}$$

$$\begin{aligned} \text{b) } 120k^3 &= 3 \times 10k \\ 120k^3 &= 30k \\ k^2 &= \frac{1}{4} \\ k &= \pm \frac{1}{2}. \end{aligned}$$

7. Given that k is a positive constant and $\int_1^k \left(\frac{5}{2\sqrt{x}} + 3 \right) dx = 4$

(a) show that $3k + 5\sqrt{k} - 12 = 0$

(4)

(b) Hence, using algebra, find any values of k such that

$$\int_1^k \left(\frac{5}{2\sqrt{x}} + 3 \right) dx = 4$$

(4)

a) $\int_1^k \left(\frac{5}{2\sqrt{x}} + 3 \right) dx = 4$

$$\int_1^k \frac{5}{2} x^{-\frac{1}{2}} + 3 dx = 4$$

$$\left[5x^{\frac{1}{2}} + 3x \right]_1^k = 4$$

$$5k^{\frac{1}{2}} + 3k - 5 - 3 = 4$$

$$5k^{\frac{1}{2}} + 3k - 8 = 4$$

$$3k + 5k^{\frac{1}{2}} - 12 = 0$$

$$(3k^2 - 4)(k^2 + 3) = 0$$

$$\sqrt{k} = \frac{4}{3}$$

$$k = \frac{16}{9}$$

$$\sqrt{k} = -3$$

Not valid.

8. The temperature, $\theta^\circ\text{C}$, of a cup of tea t minutes after it was placed on a table in a room, is modelled by the equation

$$\theta = 18 + 65e^{-\frac{t}{8}} \quad t \geq 0$$

Find, according to the model,

- the temperature of the cup of tea when it was placed on the table, (1)
- the value of t , to one decimal place, when the temperature of the cup of tea was 35°C . (3)
- Explain why, according to this model, the temperature of the cup of tea could not fall to 15°C . (1)

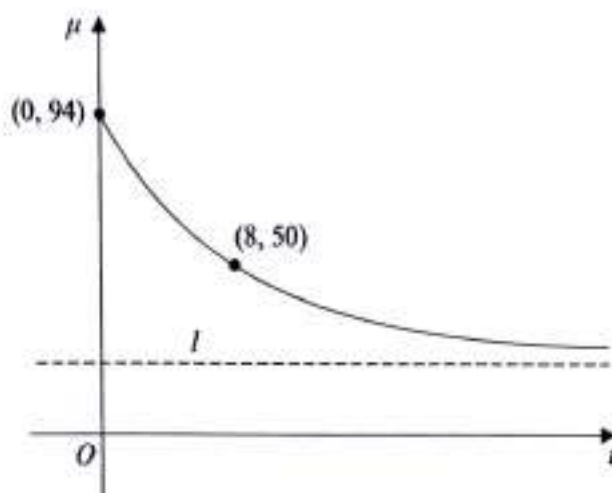


Figure 2

The temperature, $\mu^\circ\text{C}$, of a second cup of tea t minutes after it was placed on a table in a different room, is modelled by the equation

$$\mu = A + Be^{-\frac{t}{8}} \quad t \geq 0$$

where A and B are constants.

Figure 2 shows a sketch of μ against t with two data points that lie on the curve.

The line l , also shown on Figure 2, is the asymptote to the curve.

Using the equation of this model and the information given in Figure 2

- find an equation for the asymptote l . (4)

a) $t = 0$
 $\theta = 18 + 65e^{-\frac{0}{8}}$
 $\theta = 83^\circ$

Question 8 continued

$$\begin{aligned}
 b) \quad 35 &= 18 + 65e^{-t/8} \\
 65e^{-t/8} &= 17 \\
 e^{-t/8} &= \frac{17}{65} \\
 -\frac{t}{8} &= \ln\left(\frac{17}{65}\right) \\
 t &= -8 \ln\left(\frac{17}{65}\right) \\
 t &= 10.7 \text{ min.}
 \end{aligned}$$

c) Room temperature is 18°C , so as
 $t \rightarrow \infty$, $\theta \rightarrow 18$.

$$\begin{aligned}
 d) \quad t=0: \\
 A+B &= 94 \\
 t=8: \\
 50 &= A + Be^{-1} \\
 B &= 94 - A \\
 50 &= A + (94 - A)e^{-1} \\
 50 &= A(1 - e^{-1}) + 94e^{-1} \\
 50 - 94e^{-1} &= A(1 - e^{-1}) \\
 A &= \frac{50 - 94e^{-1}}{1 - e^{-1}} \\
 A &= \frac{50e - 94}{e - 1}
 \end{aligned}$$

This is the asymptote.

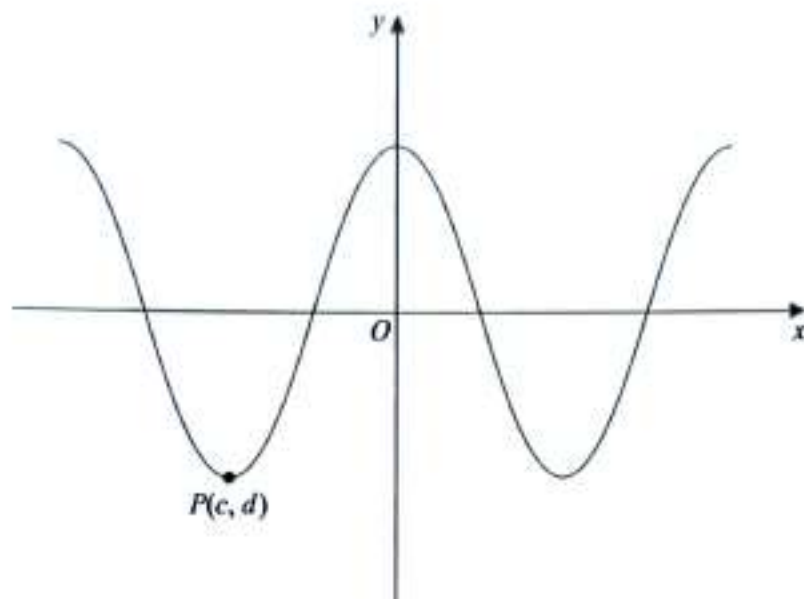


Figure 3

Figure 3 shows part of the curve with equation $y = 3 \cos x^\circ$.

The point $P(c, d)$ is a minimum point on the curve with c being the smallest negative value of x at which a minimum occurs.

- (a) State the value of c and the value of d .

(1)

- (b) State the coordinates of the point to which P is mapped by the transformation which transforms the curve with equation $y = 3 \cos x^\circ$ to the curve with equation

(i) $y = 3 \cos \left(\frac{x^\circ}{4} \right)$

(ii) $y = 3 \cos(x - 36)^\circ$

(2)

- (c) Solve, for $450^\circ \leq \theta < 720^\circ$,

$$3 \cos \theta = 8 \tan \theta$$

giving your solution to one decimal place.

In part (c) you must show all stages of your working.

Solutions relying entirely on calculator technology are not acceptable.

(5)

Question 9 continued

$$a) \quad c = -180^\circ$$

$$d = -3$$

$$b) \quad i) \quad c = -720^\circ$$

$$d = -3$$

$$ii) \quad c = -144^\circ$$

$$d = -3$$

$$c) \quad 3 \cos \theta = 8 \tan \theta$$

$$3 \cos \theta = 8 \frac{\sin \theta}{\cos \theta}$$

$$3 \cos^2 \theta = 8 \sin \theta$$

$$3(1 - \sin^2 \theta) = 8 \sin \theta$$

$$3 - 3 \sin^2 \theta = 8 \sin \theta$$

$$3 \sin^2 \theta + 8 \sin \theta - 3 = 0$$

$$(3 \sin \theta - 1)(\sin \theta + 3) = 0$$

$$\sin \theta = \frac{1}{3}$$

$$\sin \theta = -3 \quad \text{not valid}$$

$$\theta = 520.5^\circ$$

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10.

$$g(x) = 2x^3 + x^2 - 41x - 70$$

(a) Use the factor theorem to show that $g(x)$ is divisible by $(x - 5)$.

(2)

(b) Hence, showing all your working, write $g(x)$ as a product of three linear factors.

(4)

The finite region R is bounded by the curve with equation $y = g(x)$ and the x -axis, and lies below the x -axis.(c) Find, using algebraic integration, the exact value of the area of R .

(4)

$$\begin{aligned} a) \quad g(5) &= 2 \times 5^3 + 5^2 - 41 \times 5 - 70 \\ &= 2 \times 125 + 25 - 205 - 70 \\ &= 250 + 25 - 205 - 70 \\ &= 275 - 275 \\ &= 0 \end{aligned}$$

 $\Rightarrow (x-5)$ is a factor of $g(x)$

$$\begin{aligned} b) \quad g(x) &= (x-5)(2x^2 + 11x + 14) \\ g(x) &= (x-5)(2x+7)(x+2) \end{aligned}$$

c) Roots are $x = 5$, $x = -\frac{7}{2}$, $x = -2$.Under curve between -2 and 5 .

$$R = \int_{-2}^5 (2x^3 + x^2 - 41x - 70) dx$$

$$R = \left[\frac{1}{2}x^4 + \frac{1}{3}x^3 - \frac{41}{2}x^2 - 70x \right]_{-2}^5$$

$$R = \frac{1}{2} \times 5^4 + \frac{1}{3} \times 5^3 - \frac{41}{2} \times 5^2 - 70 \times 5 - \left(\frac{1}{2} \times 2^4 - \frac{1}{3} \times 2^3 + \frac{41}{2} \times 2^2 + 70 \times 2 \right)$$

$$R = \frac{625}{2} + \frac{125}{3} - \frac{1025}{2} - 350 - 8 - \frac{8}{3} + 82 + 140$$

$$R = 57\frac{2}{3}$$

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11. (i) A circle C_1 has equation

$$x^2 + y^2 + 18x - 2y + 30 = 0$$

The line l is the tangent to C_1 at the point $P(-5, 7)$.

Find an equation of l in the form $ax + by + c = 0$, where a , b and c are integers to be found.

(5)

(ii) A different circle C_2 has equation

$$x^2 + y^2 - 8x + 12y + k = 0$$

where k is a constant.

Given that C_2 lies entirely in the 4th quadrant, find the range of possible values for k .

(4)

$$11) i) \quad x^2 + y^2 + 18x - 2y + 30 = 0$$

$$(x+9)^2 + (y-1)^2 - 81 - 1 + 30 = 0$$

Centre is $(-9, 1)$.

Gradient of radius is $\frac{1-7}{-9-(-5)} = \frac{-6}{-4} = \frac{3}{2}$.

Gradient of tangent is $-\frac{2}{3}$.

$$y = -\frac{2}{3}x + c$$

$$7 = -\frac{2}{3}(-5) + c$$

$$7 = \frac{10}{3} + c$$

$$c = \frac{11}{3}$$

$$y = -\frac{2}{3}x + \frac{11}{3}$$

$$3y = -2x + 11$$

$$2x + 3y - 11 = 0$$

$$ii) \quad x^2 + y^2 - 8x + 12y + k = 0$$

$$(x-4)^2 + (y+6)^2 - 16 - 36 + k = 0$$

$$(x-4)^2 + (y+6)^2 - 52 + k = 0$$

$$(x-4)^2 + (y+6)^2 = 52 - k$$

Need radius < 4 .

$$52 - k < 16$$

$$k > 36$$

$$52 - k > 0$$

$$k < 52$$

$$36 < k < 52$$

12. An advertising agency is monitoring the number of views of an online advert.

The equation

$$\log_{10} V = 0.072t + 2.379 \quad 1 \leq t \leq 30, t \in \mathbb{N}$$

is used to model the total number of views of the advert, V , in the first t days after the advert went live.

(a) Show that $V = ab^t$ where a and b are constants to be found.

Give the value of a to the nearest whole number and give the value of b to 3 significant figures.

(4)

(b) Interpret, with reference to the model, the value of ab .

(1)

Using this model, calculate

(c) the total number of views of the advert in the first 20 days after the advert went live.
Give your answer to 2 significant figures.

(2)

a) $\log_{10} V = 0.072t + 2.379$
 $V = 10^{0.072t + 2.379}$
 $V = 10^{2.379} \cdot 10^{0.072t}$
 $a = 10^{2.379} = 239$
 $b = 10^{0.072} = 1.18$
 $V = ab^t$

b) $V = ab$ when $t = 1$.
 ab is the number of views after 1 day.

c) $V = 10^{2.379} \times 10^{0.072 \times 20}$
 $V = 6600$

13. (a) Prove that for all positive values of a and b

$$\frac{4a}{b} + \frac{b}{a} \geq 4 \quad (4)$$

(b) Prove, by counter example, that this is not true for all values of a and b .

(1)

a) ~~$\frac{4a}{b} + \frac{b}{a} \geq 4$~~ $(2a-b)^2 \geq 0$
 $4a^2 + 4ab + b^2 \geq 0$
 $\frac{4a^2}{ab} + \frac{4ab}{ab} + \frac{b^2}{ab} \geq 0$
 $\frac{4a}{b} + 4 + \frac{b}{a} \geq 0$
 $\frac{4a}{b} + \frac{b}{a} \geq 0$

(note: division by a, b justified by $a > 0, b > 0$).

b) $a = -1, b = 1$
 $\frac{4(-1)}{1} + \frac{1}{-1} =$
 $-4 - 1 =$
 $-5 < 4$

14. A curve has equation $y = g(x)$.

Given that

- $g(x)$ is a cubic expression in which the coefficient of x^3 is equal to the coefficient of x
- the curve with equation $y = g(x)$ passes through the origin
- the curve with equation $y = g(x)$ has a stationary point at $(2, 9)$

(a) find $g(x)$,

(7)

(b) prove that the stationary point at $(2, 9)$ is a maximum.

(2)

$$a) \quad g(x) = Ax^3 + Bx^2 + Cx + D$$

$$A = C$$

$$D = 0$$

$$g(x) = Ax^3 + Bx^2 + Ax$$

$$\frac{dg}{dx} = 3Ax^2 + 2Bx + A$$

$$9 = 8A + 4B + 2A$$

$$9 = 10A + 4B$$

$$12A + 4B + A = 0$$

$$13A + 4B = 0$$

$$3A = -9$$

$$A = -3$$

$$10(-3) + 4B = 9$$

$$-30 + 4B = 9$$

$$4B = 39$$

$$B = \frac{39}{4}$$

$$g(x) = -3x^3 + \frac{39}{4}x^2 - 3x$$

$$b) \quad g'(x) = -9x^2 + \frac{39}{2}x - 3$$

$$g''(x) = -18x + \frac{39}{2}$$

$$g''(2) = -18 \times 2 + \frac{39}{2}$$

$$g''(2) = -36 + \frac{39}{2}$$

$$g''(2) = -\frac{33}{2} < 0$$

Hence, $(2, 9)$ is a maximum.