Maths Made Easy

AQA

Please write clearly in block capitals.

Centre number

Candidate number

Surname MODEL SOLUTIONS

Forename(s)

Candidate signature

I declare this is my own work.

AS
MATHEMATICS

Paper 1

Wednesday 13 May 2020 Morning Time allowed: 1 hour 30 minutes

Materials

- You must have the AQA Formulae for A-level Mathematics booklet.
- You should have a graphical or scientific calculator that meets the requirements of the specification.

Instructions

- Use black ink or black ball-point pen. Pencil should only be used for drawing.
- Fill in the boxes at the top of this page.
- Answer **all** questions.
- You must answer each question in the space provided for that question. If you need extra space for your answer(s), use the lined pages at the end of this book. Write the question number against your answer(s).
- Show all necessary working; otherwise marks for method may be lost.
- Do all rough work in this book. Cross through any work that you do not want to be marked.

Information

- The marks for questions are shown in brackets.
- The maximum mark for this paper is 80.

Advice

- Unless stated otherwise, you may quote formulae, without proof, from the booklet.
- You do not necessarily need to use all the space provided.

For Examiner's Use	
Question	Mark
1	
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TOTAL	

Section A

Answer **all** questions in the spaces provided.

1 At the point (1, 0) on the curve $y = \ln x$, which statement below is correct?

Tick (✓) **one** box.

[1 mark]

- The gradient is negative and decreasing ☐
- The gradient is negative and increasing ☐
- The gradient is positive and decreasing ✓
- The gradient is positive and increasing ☐

2 Given that $f(x) = 10$ when $x = 4$, which statement below must be correct?

Tick (✓) **one** box.

[1 mark]

- $f(2x) = 5$ when $x = 4$ ☐
- $f(2x) = 10$ when $x = 2$ ✓
- $f(2x) = 10$ when $x = 8$ ☐
- $f(2x) = 20$ when $x = 4$ ☐

Do not write outside the box

3 Jia has to solve the equation

$$2 - 2\sin^2\theta = \cos\theta$$

where $-180° \leq \theta \leq 180°$

Jia's working is as follows:

$$2 - 2(1 - \cos^2\theta) = \cos\theta$$
$$2 - 2 + 2\cos^2\theta = \cos\theta$$
$$2\cos^2\theta = \cos\theta$$
$$2\cos\theta = 1$$
$$\cos\theta = 0.5$$
$$\theta = 60°$$

Jia's teacher tells her that her solution is incomplete.

3 (a) Explain the **two** errors that Jia has made.

[2 marks]

Jia should not have cancelled by cosθ.
Jia hasn't included a second solution to
cosθ = 0.5.

3 (b) Write down all the values of θ that satisfy the equation

$$2 - 2\sin^2\theta = \cos\theta$$

where $-180° \leq \theta \leq 180°$

[2 marks]

$\theta = -90°, -60°, 60°, 90°$

Turn over ▶

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4 In the binomial expansion of $(\sqrt{3}+\sqrt{2})^4$ there are two irrational terms.

Find the difference between these two terms.

[3 marks]

$(\sqrt{3}+\sqrt{2})^4 = (\sqrt{3})^4 + 4(\sqrt{3})^3\sqrt{2} + 6(\sqrt{3})^2(\sqrt{2})^2 + 4\sqrt{3}(\sqrt{2})^3$
$+ (\sqrt{2})^4$
$= 9 + 4 \times 3\sqrt{3} \times \sqrt{2} + 36 + 4 \times \sqrt{3} \times 2\sqrt{2}$

Irrational terms: $12\sqrt{6}$ and $8\sqrt{6}$

$12\sqrt{6} - 8\sqrt{6} = 4\sqrt{6}$

5 Differentiate from first principles

$$y = 4x^2 + x$$

[4 marks]

$$\frac{dy}{dx} = \lim_{h \to 0} \left(\frac{4(x+h)^2 + (x+h) - 4x^2 + x}{h} \right)$$

$$= \lim_{h \to 0} \left(\frac{4x^2 + 8xh + 4h^2 + x + h - 4x^2 + x}{h} \right)$$

$$= \lim_{h \to 0} \left(\frac{8xh + 4h^2 + h}{h} \right)$$

$$= \lim_{h \to 0} \left(8x + 4h + 1 \right)$$

$$= 8x + 1$$

Do not write outside the box

Do not write outside the box

6 (a) It is given that

$$f(x) = x^3 - x^2 + x - 6$$

Use the factor theorem to show that $(x - 2)$ is a factor of $f(x)$.

[2 marks]

if $(x-2)$ is a factor then $x = 2$ will be a solution of $f(x)$.

$f(2) = 2^3 - 2^2 + 2 - 6 = 0$

$\therefore$ $x = 2$ is a solution $\Rightarrow (x-2)$.

6 (b) Find the quadratic factor of $f(x)$.

[1 mark]

$x^2 + x + 3$

6 (c) Hence, show that there is only one real solution to $f(x) = 0$

[3 marks]

let $x^2 + x + 3 = 0$

Discriminant $= 1^2 - 4 \times 1 \times 3 = -11 < 0$

$\therefore$ there ~~is~~ are no real ~~[illegible]~~ Solutions to $x^2 + x + 3$, thus only one solutions

Do not write outside the box

6 (d) Find the exact value of x that solves

$$e^{3x} - e^{2x} + e^{x} - 6 = 0$$

[3 marks]

Let $y = e^x$

then $e^{3x} - e^{2x} + e^{x} - 6 = y^3 - y^2 + y - 6 = 0$

solution : $y = 2$

so $e^x = 2$

$\Rightarrow x = \ln 2$.

Turn over for the next question

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7 Curve C has equation $y = x^2$

C is translated by vector $\begin{bmatrix} 3 \\ 0 \end{bmatrix}$ to give curve C_1

Line L has equation $y = x$

L is stretched by scale factor 2 parallel to the x-axis to give line L_1

Find the exact distance between the two intersection points of C_1 and L_1

[6 marks]

$C_1 : y = (x-3)^2$

$L_1 : y = \frac{1}{2}x$

Let $\frac{1}{2}x = (x-3)^2$

$\Rightarrow \frac{1}{2}x = x^2 - 6x + 9$

$\Rightarrow x^2 - \frac{13}{2}x + 9 = 0$

$x = 2$ or 4.5

$y = 1$ or 2.25

Distance between points $= \sqrt{(4.5-2)^2 + (2.25-1)^2}$

$= \frac{5\sqrt{5}}{4}$

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8 (a) Find the equation of the tangent to the curve $y = e^{4x}$ at the point (a, e^{4a}).

[3 marks]

$\frac{dy}{dx} = 4e^{4x}$

at (a, e^{4a}) gradient $= 4e^{4a}$

Equation of tangent: $y - e^{4a} = 4e^{4a}(x - a)$

$\Rightarrow y = 4xe^{4a} - 4ae^{4a} + e^{4a}$

$\Rightarrow y = e^{4a}(4x - 4a + 1)$

8 (b) Find the value of a for which this tangent passes through the origin.

[2 marks]

let $x = y = 0$

Tangent: $0 = e^{4a}(4(0) - 4a + 1)$

~~$\Rightarrow 0 = e^{4a} - 4ae^{4a}$~~

$\Rightarrow e^{4a}(1 - 4a) = 0$

$\Rightarrow a = 1/4$

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8 (c) Hence, find the set of values of m for which the equation

$$e^{4x} = mx$$

has no real solutions.

[3 marks]

$e^{4x} > 0$, so $m \geqslant 0$

For $a = 1/4$, contact point $(1/4, e)$

Gradient from $(0,0)$ to $(1/4, e)$ is $= \frac{e - 0}{1/4 - 0} = 4e$

$0 \leqslant m \leqslant 4e$

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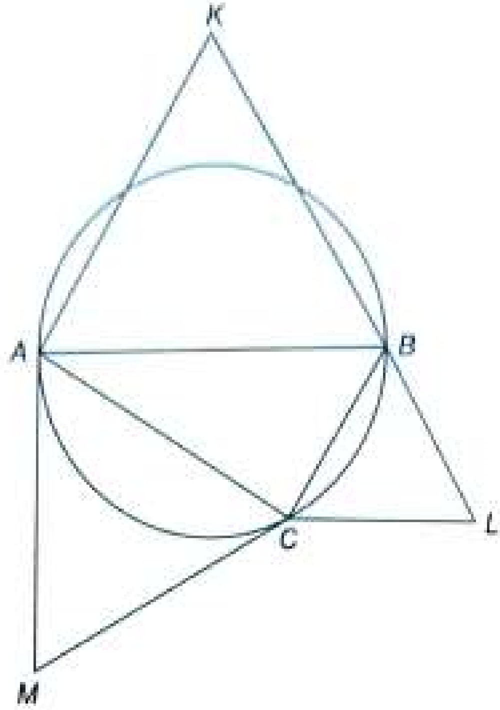
9 The diagram below shows a circle and four triangles.



AB is a diameter of the circle. C is a point on the circumference of the circle.

Triangles ABK, BCL and CAM are equilateral.

Prove that the area of triangle ABK is equal to the sum of the areas of triangle BCL and triangle CAM.

[5 marks]

$\angle ACB = 90^\circ$ (angles in a semi-circle)

$\therefore$ $AB^2 = AC^2 + CB^2$ (using pythagoras' theorem)

Area of $ABK = \frac{1}{2}AB^2 \sin 60$ (Equilateral triangle)

Area of $BCL = \frac{1}{2}(AB^2 - AC^2)\sin 60$

Area of $CAM = \frac{1}{2}AC^2 \sin 60$

$BCL + CAM = \left(\frac{1}{2}AB^2 - \frac{1}{2}AC^2\right)\sin 60 + \frac{1}{2}AC^2 \sin 60$

$= \frac{1}{2}AB^2 \sin 60 = ABK$ area.

Do not write outside the box

10 Raj is investigating how the price, P pounds, of a brilliant-cut diamond ring is related to the weight, C carats, of the diamond.

He believes that they are connected by a formula

$$P = aC^n$$

where a and n are constants.

10 (a) Express $\ln P$ in terms of $\ln C$.

[2 marks]

$\ln P = \ln(aC^n)$

$= \ln a + \ln C^n$

$= \ln a + n \ln C$

10 (b) Raj researches the price of three brilliant-cut diamond rings on a website with the following results.

C	0.60	1.15	1.50
P	495	1200	1720

$\ln C$ -0.51 0.14 0.41

$\ln P$ 6.20 7.09 7.45

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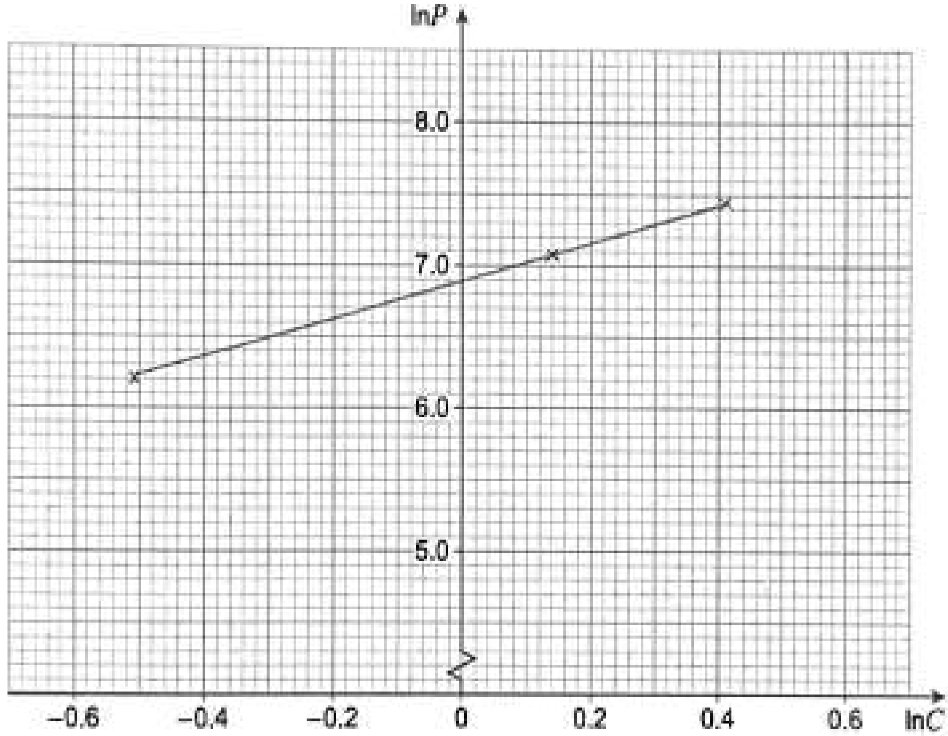
10 (b) (i) Plot $\ln P$ against $\ln C$ for the three rings on the grid below.

[2 marks]



10 (b) (ii) Explain which feature of the plot suggests that Raj's belief may be correct.

[1 mark]

The plot is a straight line.

Question 10 continues on the next page

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10 (b) (iii) Using the graph on page 15, estimate the value of a and the value of n.

[4 marks]

$\ln a$ = 'y' intercept

$\therefore \ln a = 6.9$

$\Rightarrow a = 992$

n is the gradient $= \dfrac{7.09 - 6.2}{0.14 + 0.51} = 1.37$

10 (c) Explain the significance of a in this context.

[1 mark]

a is the price for a 1 carat diamond.

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10 (d) Raj wants to buy a ring with a brilliant-cut diamond of weight 2 carats.

Estimate the price of such a ring.

[2 marks]

$992 \times 2^{1.37} = £2564.03$

Turn over for the next question

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Section B

Answer **all** questions in the spaces provided.

11 A go-kart and driver, of combined mass 55 kg, move forward in a straight line with a constant acceleration of $0.2\ \text{m s}^{-2}$

The total driving force is 14 N

Find the total resistance force acting on the go-kart and driver.

Circle your answer.

[1 mark]

0 N   3 N   11 N   14 N

12 One of the following is an expression for the distance between the points represented by position vectors $5\mathbf{i} - 3\mathbf{j}$ and $18\mathbf{i} + 7\mathbf{j}$

Identify the correct expression.

Tick (✓) **one** box.

[1 mark]

$\sqrt{13^2 + 4^2}$ ☐

$\sqrt{13^2 + 10^2}$ ☑

$\sqrt{23^2 + 4^2}$ ☐

$\sqrt{23^2 + 10^2}$ ☐

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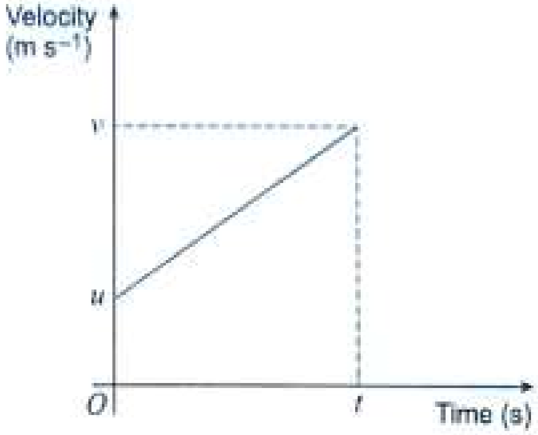
13 An object is moving in a straight line, with constant acceleration $a\,\mathrm{m\,s^{-2}}$, over a time period of t seconds.

It has an initial velocity u and final velocity v as shown in the graph below.



Use the graph to show that

$$v = u + at$$

[3 marks]

acceleration = the rate of change of velocity per unit of time. Therefore, it is the gradient of this line.

$a = \frac{v - u}{t}$

$\Rightarrow \quad at = v - u$

$\Rightarrow \quad v = u + at$

Turn over for the next question

Do not write outside the box

14 A particle of mass 0.1 kg is initially stationary.

A single force **F** acts on this particle in a direction parallel to the vector $7\mathbf{i} + 24\mathbf{j}$

As a result, the particle accelerates in a straight line, reaching a speed of $4\,\text{m s}^{-1}$ after travelling a distance of 3.2 m

Find **F**.

[5 marks]

$s = 3.2,\ u = 0,\ v = 4,\ a = ?$

$v^2 = u^2 + 2as$

$16 = 2 \times a \times 3.2$

$a = \frac{16}{6.4} = 2.5\ \text{ms}^{-2}$

$F = ma = 0.1 \times 2.5 = 0.25\ \text{N}$

magnitude of $7\underline{i} + 24\underline{j} = \sqrt{7^2 + 24^2} = 25$

$F = \frac{0.25}{25}(7\underline{i} + 24\underline{j}) = \frac{1}{100}(7\underline{i} + 24\underline{j})$

Do not write outside the box

15 A particle, P, is moving in a straight line with acceleration $a\ \mathrm{m\,s^{-2}}$ at time t seconds, where

$$a = 4 - 3t^2$$

15 (a) Initially P is stationary.

Find an expression for the velocity of P in terms of t.

[2 marks]

$$v = \int a \, dt$$

$$= 4t - t^3 + c$$

15 (b) When $t = 2$, the displacement of P from a fixed point, O, is 39 metres.

Find the time at which P passes through O, giving your answer to three significant figures.

Fully justify your answer.

[5 marks]

$S = \int v \, dt$

$= 2t^2 - \frac{1}{4}t^4 + k$

$t = 2,\ S = 39 \Rightarrow 39 = 2(2)^2 - \frac{1}{4}(2)^4 + k$

$\Rightarrow 39 = 8 - 4 + k$

$\Rightarrow k = 35$

So we have $S = 2t^2 - \frac{1}{4}t^4 + 35$

To find the time at which P passes through O,

let $S = 0$.

$\Rightarrow 2t^2 - \frac{1}{4}t^4 + 35 = 0$

$\Rightarrow t^4 - 8t^2 - 140 = 0$

let $x = t^2 \Rightarrow x^2 - 8x - 140 = 0$

obtain positive solution of $x = 16.489\ldots$

$\therefore t = 4.06$ seconds

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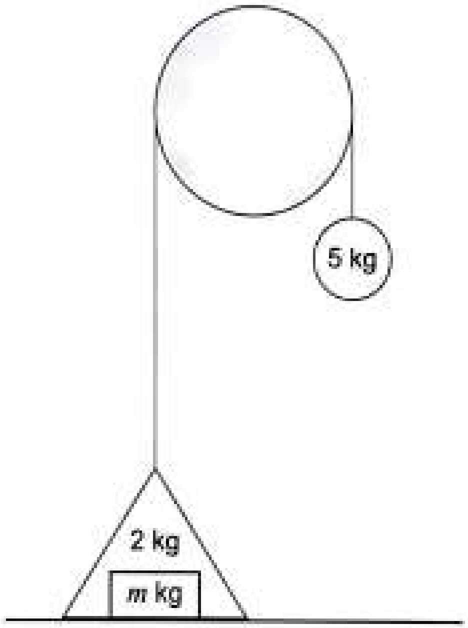
16 A simple lifting mechanism comprises a light inextensible wire which is passed over a smooth fixed pulley.

One end of the wire is attached to a rigid triangular container of mass 2 kg, which rests on horizontal ground.

A load of m kg is placed in the container.

The other end of the wire is attached to a particle of mass 5 kg, which hangs vertically downwards.

The mechanism is initially held at rest as shown in the diagram below.



The mechanism is released from rest, and the container begins to move upwards with acceleration a m s^{-2}

The wire remains taut throughout the motion.

Do not write outside the box

16 (a) Show that

$$a = \left(\frac{3 - m}{m + 7}\right) g$$

[4 marks]

$T - (m+2)g = (m+2)a$ (container)

$5g - T = 5a$ (particle)

$\Rightarrow 5g - 5a - (m+2)g = (m+2)a$

$\Rightarrow 5g - (m+2)g = (5 + 2 + m)a$

$\Rightarrow 3g - mg = (7+m)a$

$\Rightarrow (3 - m)g = (7+m)a$

$\Rightarrow a = \left(\frac{3 - m}{m+7}\right)g$

16 (b) State the range of possible values of m.

[1 mark]

$0 < m < 3$

Question 16 continues on the next page

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16 (c) **In this question use $g = 9.8\text{ m s}^{-2}$**

The load reaches a height of 2 metres above the ground 1 second after it is released.

Find the mass of the load.

[4 marks]

$s = 2,\ u = 0,\ t = 1,\ a = ?$

$s = ut + \frac{1}{2}at^2$

$\Rightarrow 2 = \frac{1}{2}a$

$\Rightarrow a = 4\text{ m s}^{-2}$

$\Rightarrow 4 = \left(\frac{3-m}{m+7}\right)g$

$\Rightarrow m = \frac{3g - 28}{4 + g} = 0.10\text{ Kg}$

Do not write outside the box

Do not write outside the box

16 (d) Ignoring air resistance, describe **one** assumption you have made in your model.

[1 mark]

The top of the container does not reach the pulley.

END OF QUESTIONS