AQA

Please write clearly in block capitals.

Centre number						Candidate number				

Surname MODEL SOLUTIONS

Forename(s)

Candidate signature

I declare this is my own work.

A-level
MATHEMATICS
Paper 1

Wednesday 3 June 2020 Afternoon Time allowed: 2 hours

Materials
- You must have the AQA Formulae for A-level Mathematics booklet.
- You should have a graphical or scientific calculator that meets the requirements of the specification.

Instructions
- Use black ink or black ball-point pen. Pencil should only be used for drawing.
- Fill in the boxes at the top of this page.
- Answer **all** questions.
- You must answer each question in the space provided for that question. If you need extra space for your answer(s), use the lined pages at the end of this book. Write the question number against your answer(s).
- Show all necessary working; otherwise marks for method may be lost.
- Do all rough work in this book. Cross through any work that you do not want to be marked.

Information
- The marks for questions are shown in brackets.
- The maximum mark for this paper is 100.

Advice
- Unless stated otherwise, you may quote formulae, without proof, from the booklet.
- You do not necessarily need to use all the space provided.

For Examiner's Use	
Question	Mark
1	
2	
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TOTAL	

Maths Made Easy

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Answer **all** questions in the spaces provided.

1 The first three terms, in ascending powers of x, of the binomial expansion of $(9+2x)^{\frac{1}{2}}$ are given by

$$(9+2x)^{\frac{1}{2}} \approx a + \frac{x}{3} - \frac{x^2}{54}$$

where a is a constant.

1 (a) State the range of values of x for which this expansion is valid.

Circle your answer.

[1 mark]

$|x| < \frac{2}{9}$ $\quad$ $|x| < \frac{2}{3}$ $\quad$ $|x| < 1$ $\quad$ $|x| < \frac{9}{2}$

1 (b) Find the value of a.

Circle your answer.

[1 mark]

1 $\quad$ 2 $\quad$ 3 $\quad$ 9

Maths Made Easy

2 A student is searching for a solution to the equation $f(x) = 0$

He correctly evaluates

$$f(-1) = -1 \text{ and } f(1) = 1$$

and concludes that there must be a root between -1 and 1 due to the change of sign.

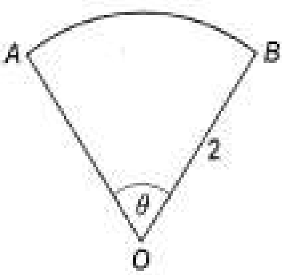
Select the function $f(x)$ for which the conclusion is **incorrect**.

Circle your answer.

[1 mark]

$f(x) = \frac{1}{x}$ $\quad$ $f(x) = x$ $\quad$ $f(x) = x^3$ $\quad$ $f(x) = \frac{2x+1}{x+2}$

3 The diagram shows a sector OAB of a circle with centre O and radius 2



The angle AOB is θ radians and the perimeter of the sector is 6

Find the value of θ

Circle your answer.

[1 mark]

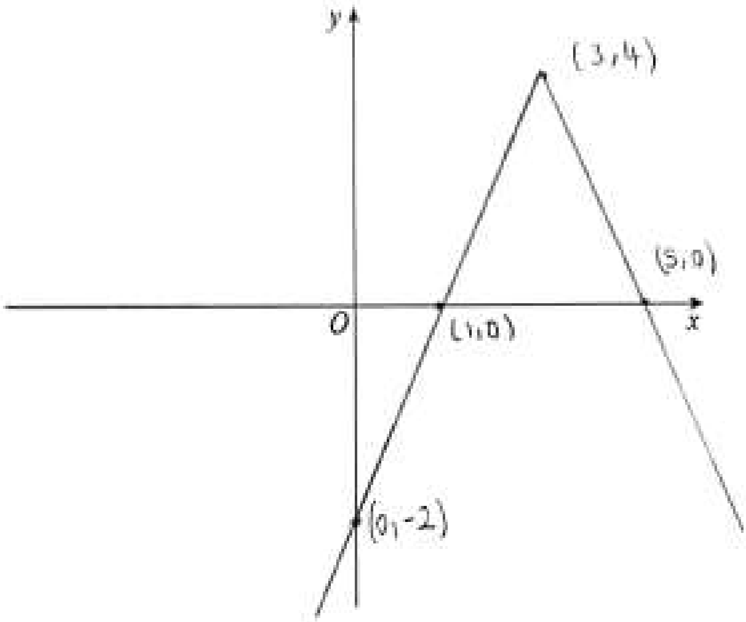
1 $\quad$ $\sqrt{3}$ $\quad$ 2 $\quad$ 3

Turn over for the next question

Do not write outside the box

4 (a) Sketch the graph of

$$y = 4 - |2x - 6|$$



[3 marks]

4 (b) Solve the inequality

$$4 - |2x - 6| > 2$$

[2 marks]

$|2x-6| < 2$

$2x-6 < 2 \qquad 2x-6 > -2$

$2x < 8 \qquad 2x > 4$

$x < 4 \qquad x > 2$

$2 < x < 4$.

5 Prove that, for integer values of n such that $0 \leq n < 4$

$$2^{n+2} > 3^n$$

[2 marks]

$n = 0 \quad 2^{n+2} = 4 > 3^0 = 1$

$n = 1 \quad 2^{n+2} = 8 > 3^1 = 3$

$n = 2 \quad 2^{n+2} = 16 > 3^2 = 9$

$n = 3 \quad 2^{n+2} = 32 > 3^3 = 27$

$\therefore \quad 0 \leq n < 4$

Turn over for the next question

Do not write outside the box

Do not write outside the box

6 Four students, Tom, Josh, Floella and Georgia are attempting to complete the indefinite integral

$$\int \frac{1}{x}\,dx \qquad \text{for } x > 0$$

Each of the students' solutions is shown below:

Tom	$\int \frac{1}{x}\,dx = \ln x$
Josh	$\int \frac{1}{x}\,dx = k\ln x$
Floella	$\int \frac{1}{x}\,dx = \ln Ax$
Georgia	$\int \frac{1}{x}\,dx = \ln x + c$

6 (a) (i) Explain what is wrong with Tom's answer.

[1 mark]

He has not included a constant of integration.

6 (a) (ii) Explain what is wrong with Josh's answer.

[1 mark]

The constant should be added rather than multiplied.

6 (b) Explain why Floella and Georgia's answers are equivalent.

[2 marks]

Rules of logs: $\ln Ax = \ln A + \ln x = \ln x + c$, where $c = \ln A$.

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7 Consecutive terms of a sequence are related by

$$u_{n+1} = 3 - (u_n)^2$$

7 (a) In the case that $u_1 = 2$

7 (a) (i) Find u_3

[2 marks]

$u_2 = 3 - 2^2 = -1$

$u_3 = 3 - (-1)^2 = 2$

7 (a) (ii) Find u_{50}

[1 mark]

$u_{50} = -1$ (50 is even)

7 (b) State a different value for u_1 which gives the same value for u_{50} as found in part (a)(ii).

[1 mark]

$u_1 = -2$

Turn over for the next question

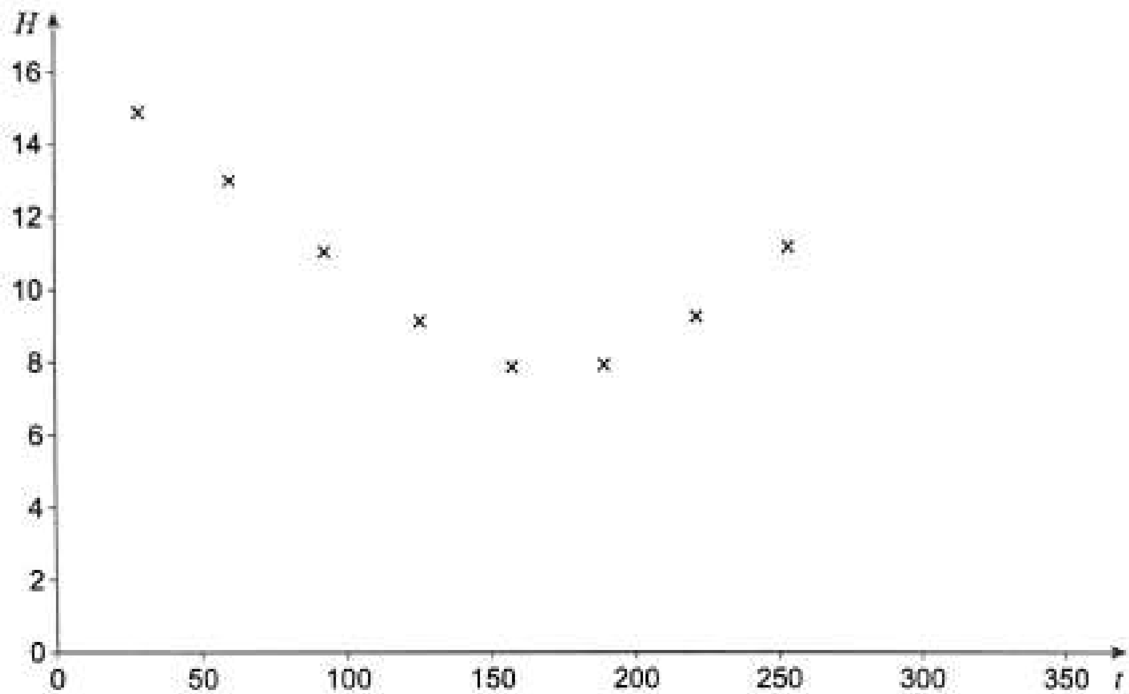
Do not write outside the box

8 Mike, an amateur astronomer who lives in the South of England, wants to know how the number of hours of darkness changes through the year.

On various days between February and September he records the length of time, H hours, of darkness along with t, the number of days after 1 January.

His results are shown in **Figure 1** below.

Figure 1



Mike models this data using the equation

$$H = 3.87\sin\left(\frac{2\pi(t + 101.75)}{365}\right) + 11.7$$

8 (a) Find the minimum number of hours of darkness predicted by Mike's model. Give your answer to the nearest minute.

[2 marks]

min value where $\sin\left(\frac{2\pi(t+101.75)}{365}\right) = -1$

$\therefore \cancel{\frac{2\pi(t+101.75)}{365} = \frac{3\pi}{2}}$

$\cancel{\Rightarrow t = \frac{3\times 365}{4} - 101.75 =}$

$H = -3.87 + 11.7 = 7.83 = 7$ hours 50 mins

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8 (b) Find the maximum number of consecutive days where the number of hours of darkness predicted by Mike's model exceeds 14

[3 marks]

$$3.87 \sin\left(\frac{2\pi(t+101.75)}{365}\right) + 11.7 = 14$$

$$\Rightarrow \sin\left(\frac{2\pi(t+101.75)}{365}\right) = \frac{230}{387}$$

$t = -64.78,\ 43.78,\ 300.22,\ 408.78$

$408 - 300 = 108$

Question 8 continues on the next page

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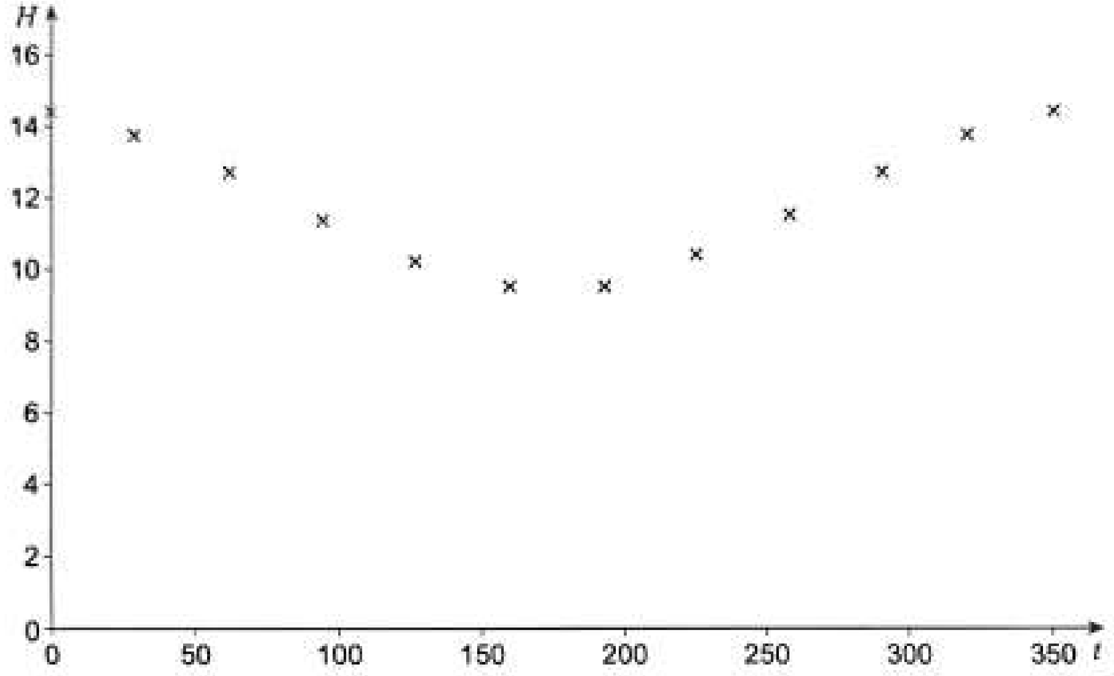
8 (c) Mike's friend Sofia, who lives in Spain, also records the number of hours of darkness on various days throughout the year.

Her results are shown in **Figure 2** below.

Figure 2



Sofia attempts to model her data by refining Mike's model.

She decides to increase the 3.87 value, leaving everything else unchanged.

Explain whether Sofia's refinement is appropriate.

[2 marks]

Increasing the 3.87 value would increase the range of the graph, which doesn't match up to her graph.

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9 Chloe is attempting to write $\frac{2x^2+x}{(x+1)(x+2)^2}$ as partial fractions, with constant numerators.

Her incorrect attempt is shown below.

Step 1 $\frac{2x^2+x}{(x+1)(x+2)^2} \equiv \frac{A}{x+1} + \frac{B}{(x+2)^2}$

Step 2 $2x^2 + x \equiv A(x+2)^2 + B(x+1)$

Step 3 Let $x = -1 \Rightarrow A = 1$
Let $x = -2 \Rightarrow B = -6$

Answer $\frac{2x^2+x}{(x+1)(x+2)^2} \equiv \frac{1}{x+1} - \frac{6}{(x+2)^2}$

9 (a) (i) By using a counter example, show that the answer obtained by Chloe cannot be correct.

[2 marks]

Let $x = 2$.

LHS $= \frac{2(2)^2+x}{3 \times 4^2} = 5/24$

RHS $= \frac{1}{3} - \frac{6}{4^2} = -1/24$

LHS $\neq$ RHS $\therefore$ she cannot be correct.

9 (a) (ii) Explain her mistake in Step 1.

[1 mark]

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9 (b) Write $\dfrac{2x^2 + x}{(x+1)(x+2)^2}$ as partial fractions, with constant numerators.

[4 marks]

$$\frac{2x^2+x}{(x+1)(x+2)^2} \equiv \frac{A}{x+1} + \frac{B}{x+2} + \frac{C}{(x+2)^2}$$

$\Rightarrow 2x^2 + x \equiv A(x+2)^2 + B(x+1)(x+2) + C(x+1)$

let $x = -1 \Rightarrow A = 1$

$x = -2 \Rightarrow 6 = C$

x^2 terms $\Rightarrow 2 = A + B$

$\Rightarrow B = 1$

$$\therefore \frac{2x^2+x}{(x+1)(x+2)^2} = \frac{1}{x+1} + \frac{1}{x+2} + \frac{6}{(x+2)^2}$$

Turn over ▶

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10 (a) An arithmetic series is given by

$$\sum_{r=5}^{20}(4r+1)$$

10 (a) (i) Write down the first term of the series.

[1 mark]

21

10 (a) (ii) Write down the common difference of the series.

[1 mark]

4

10 (a) (iii) Find the number of terms of the series.

[1 mark]

16

Do not write outside the box

10 (b) A **different** arithmetic series is given by

$$\sum_{r=10}^{100} (br + c)$$

where b and c are constants.

The sum of this series is 7735

10 (b) (i) Show that $55b + c = 85$

[4 marks]

$n = 91$

$a = 10b + c$

$d = b$

$L = 100b + c$

$\frac{91}{2}(2(10b+c) + 90b) = 7735$

$\Rightarrow 91(55b+c) = 7735$

$\Rightarrow 55b + c = 85$

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10 (b) (ii) The 40th term of the series is 4 times the 2nd term.

Find the values of b and c.

[4 marks]

$4(11b+c) = 49b + c$

$\Rightarrow 44b + 4c = 49b + c$

$\Rightarrow 5b = 3c$

$\Rightarrow c = \frac{5}{3}b$

$\Rightarrow 55b + \frac{5}{3}b = 85$

$\Rightarrow 170b = 255$

$\Rightarrow b = 1.5$

$\Rightarrow c = 2.5$

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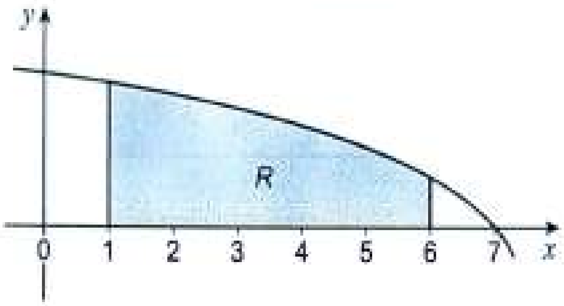
11 The region R enclosed by the lines $x = 1$, $x = 6$, $y = 0$ and the curve

$$y = \ln(8 - x)$$

is shown shaded in **Figure 3** below.

Figure 3



All distances are measured in centimetres.

11 (a) Use a single trapezium to find an approximate value of the area of the shaded region, giving your answer in cm^2 to two decimal places.

[2 marks]

$y = \ln(8-1) = 1.94\ldots$

$y = \ln(8-6) = 0.69\ldots$

$\text{Area} = \frac{5}{2}(\ln 7 + \ln 2)$

$= 6.5976\ldots$

$= 6.60 \text{ cm}^2$ (2 dp)

Question 11 continues on the next page

Turn over ▶

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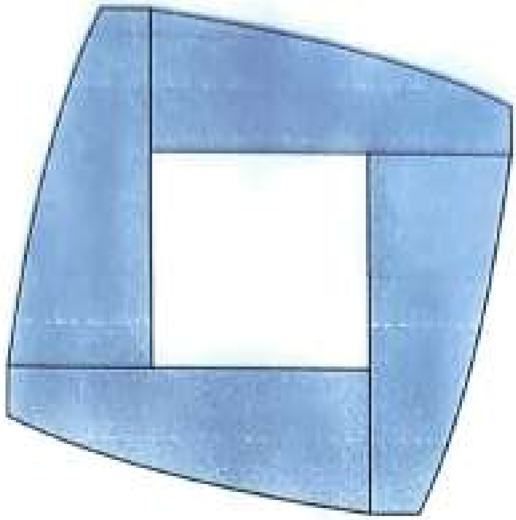
11 (b) Shape B is made from four copies of region R as shown in **Figure 4** below.

Figure 4



Shape B is cut from metal of thickness 2 mm

The metal has a density of 10.5 g/cm^3

Use the trapezium rule with **six** ordinates to calculate an approximate value of the mass of Shape B.

Give your answer to the nearest gram.

[5 marks]

Area of R $= \frac{1}{2} \times (\ln 7 + \ln 2 + 2(\ln 6 + \ln 5 + \ln 4 + \ln 3)) = 7.2056\ldots \text{ cm}^2$

Volume of Shape B $= 4 \times 7.2056\ldots \times 0.2$
$= 5.764\ldots \text{ cm}^3$

Mass of Shape B $= 5.7645\ldots \times 10.5 = 60.52\ldots$
$= 61\text{g}.$

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11 (c) Without further calculation, give one reason why the mass found in part **(b)** may be:

11 (c) (i) an underestimate.

[1 mark]

The curve in figure 3 is concave.

11 (c) (ii) an overestimate.

[1 mark]

The numbers have been rounded.

Turn over for the next question

Do not write outside the box

12 A curve C has equation

$$x^3 \sin y + \cos y = Ax$$

where A is a constant.

C passes through the point $P\left(\sqrt{3}, \frac{\pi}{6}\right)$

12 (a) Show that $A = 2$

[2 marks]

$(\sqrt{3})^3 \sin \frac{\pi}{6} + \cos \frac{\pi}{6} = A\sqrt{3}$

$\Rightarrow 3\cancel{\sqrt{3}} \cdot \frac{1}{2} + \frac{\sqrt{3}}{2} = A\cancel{\sqrt{3}}$

$\Rightarrow A = 2$

12 (b) (i) Show that $\dfrac{\mathrm{d}y}{\mathrm{d}x} = \dfrac{2 - 3x^2 \sin y}{x^3 \cos y - \sin y}$

[5 marks]

Implicit differentiation:

$3x^2 \sin y + x^3 \cos y \frac{dy}{dx} - \sin y \, {}^{dy}/_{dx} = 2$

$\Rightarrow {}^{dy}/_{dx} (x^3 \cos y - \sin y) = 2 - 3x^2 \sin y$

$\Rightarrow {}^{dy}/_{dx} = \dfrac{2 - 3x^2 \sin y}{x^3 \cos y - \sin y}$

Maths Made Easy

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12 (b) (ii) Hence, find the gradient of the curve at P.

[2 marks]

$\frac{dy}{dx}$ at $P = \dfrac{2 - 3 \times 3 \sin \frac{\pi}{6}}{(\sqrt{3})^3 \cos \frac{\pi}{6} - \sin \frac{\pi}{6}}$

$= -\frac{5}{8}$

12 (b) (iii) The tangent to C at P intersects the x-axis at Q.

Find the exact x-coordinate of Q.

[4 marks]

gradient of tangent $= \frac{8}{5}$

Equation of tangent $\Rightarrow y - \frac{\pi}{6} = \frac{8}{5}(x - \sqrt{3})$

let $y = 0 \Rightarrow x = \sqrt{3} + \frac{4\pi}{15}$

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13 The function f is defined by

$$f(x) = \frac{2x+3}{x-2} \qquad x \in \mathbb{R}, x \neq 2$$

13 (a) (i) Find f^{-1}

[3 marks]

let $y = \frac{2x+3}{x-2}$

$\Rightarrow yx - 2y = 2x + 3$

$\Rightarrow xy - 2x = 3 + 2y$

$\Rightarrow x(y-2) = 3 + 2y$

$\Rightarrow x = \frac{3+2y}{y-2}$

so $f^{-1} = \frac{3+2x}{x-2}$

13 (a) (ii) Write down an expression for $\mathrm{ff}(x)$.

[1 mark]

$ff(x) = x$

Do not write outside the box

13 (b) The function g is defined by

$$g(x) = \frac{2x^2 - 5x}{2} \qquad x \in \mathbb{R}, 0 \le x \le 4$$

13 (b) (i) Find the range of g.

[3 marks]

$g(4) = 6$

$g'(x) = \frac{1}{2}(4x - 5)x = 0$

$x = -1.5625$

range : $[-1.5625, 6]$

13 (b) (ii) Determine whether g has an inverse.

Fully justify your answer.

[2 marks]

$g(0) = 0 = g(2.5)$

$\therefore g(x)$ is a many to one function so it does not have an inverse.

13 (c) Show that

$$gf(x) = \frac{48 + 29x - 2x^2}{2x^2 - 8x + 8}$$

[4 marks]

$$gf(x) = \frac{2\left(\frac{2x+3}{x-2}\right)^2 - 5\left(\frac{2x+3}{x-2}\right)}{2}$$

$$= \frac{2\left(\frac{4x^2+12x+9}{x^2-4x+4}\right) - \left(\frac{10x+15}{x-2}\right)}{2}$$

$$= \frac{\frac{2(4x^2+12x+9)}{(x-2)^2} - \frac{(10x+15)(x-2)}{(x-2)^2}}{2}$$

$$= \frac{1}{2} \times \frac{8x^2+24x+18-10x^2+20x-15x+30}{(x-2)^2}$$

$$= \frac{48+29x-2x^2}{2(x-2)^2}$$

$$= \frac{48+29x-2x^2}{2x^2-8x+8}$$

Do not write outside the box

13 (d) It can be shown that fg is given by

$$\text{fg}(x) = \frac{4x^2 - 10x + 6}{2x^2 - 5x - 4}$$

with domain $\{x \in \mathbb{R} : 0 \leq x \leq 4, x \neq a\}$

Find the value of a.

Fully justify your answer.

[2 marks]

$2x^2 - 5x - 4 = 0$

$x = \dfrac{5 \pm \sqrt{57}}{4}$

$\cancel{x}\ a = \dfrac{5 + \sqrt{57}}{4}$ as $0 \leqslant x \leqslant 4$.

Turn over for the next question

Do not write outside the box

14 The function f is defined by

$$f(x) = 3^x\sqrt{x} - 1 \qquad \text{where } x \geq 0$$

14 (a) $f(x) = 0$ has a single solution at the point $x = \alpha$

By considering a suitable change of sign, show that α lies between 0 and 1

[2 marks]

$f(0) = 3^0\sqrt{0} - 1 = -1$

$f(1) = 3^1\sqrt{1} - 1 = 2$

So $f(0) < 0$ and $f(1)$, it shows that there is a solution in the interval $(0, 1)$.

14 (b) (i) Show that

$$f'(x) = \frac{3^x(1 + x\ln 9)}{2\sqrt{x}}$$

[3 marks]

$f'(x) = 3^x \cdot \frac{1}{2}x^{-1/2} + 3^x \ln 3 \cdot x^{1/2}$

$= 3^x\left(\frac{1}{2\sqrt{x}} + \sqrt{x}\ln 3\right)$

$= 3^x\left(\frac{1}{2\sqrt{x}} + \frac{2\sqrt{x}\cdot\sqrt{x}\ln 3}{2\sqrt{x}}\right)$

$= 3^x\left(\frac{1}{2\sqrt{x}} + \frac{2x\ln 3}{2\sqrt{x}}\right)$

$= 3^x\left(\frac{1 + x\ln 9}{2\sqrt{x}}\right)$

$= \frac{3^x(1 + x\ln 9)}{2\sqrt{x}}$

Do not write outside the box

14 (b) (ii) Use the Newton–Raphson method with $x_1 = 1$ to find x_3, an approximation for α.

Give your answer to five decimal places.

[2 marks]

$$x_{n+1} = x_n - \frac{f(x_n)}{f'(x_n)}$$

$$\Rightarrow x_2 = 1 - \frac{f(1)}{f'(x_n)} = 0.5829716$$

$$\Rightarrow x_3 = 1 - \frac{f(x_2)}{f'(x_2)} = 0.42465 \ (5\,d.p)$$

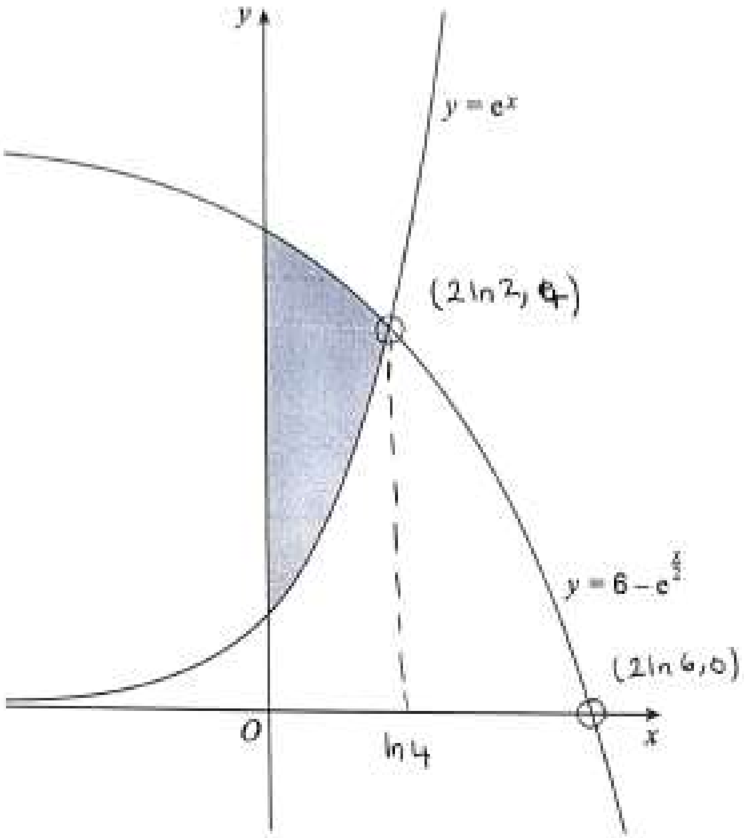
14 (b) (iii) Explain why the Newton–Raphson method fails to find α with $x_1 = 0$

[2 marks]

It would not converge as all values of x_n would be 0.

Do not write outside the box

15 The region enclosed between the curves $y = e^x$, $y = 6 - e^{\frac{x}{2}}$ and the line $x = 0$ is shown shaded in the diagram below.



Show that the exact area of the shaded region is

$$6\ln 4 - 5$$

Fully justify your answer.

[10 marks]

Interception of $y = 6 - e^{\frac{x}{2}}$ and x-axis.

$6 - e^{\frac{x}{2}} = 0$

$\Rightarrow e^{x/2} = 6 \Rightarrow \frac{x}{2} = \ln 6 \Rightarrow x = 2\ln 6$

Find where the line $y = e^x$ and $y = 6 - e^{x/2}$ intersect:

$\Rightarrow e^x = 6 - e^{x/2} \Rightarrow e^x + e^{x/2} - 6 = 0$

$\Rightarrow (e^{\frac{x}{2}} + 3)(e^{x/2} - 2) = 0$

$\Rightarrow e^{x/2} \neq -3$, so $e^{x/2} = 2 \Rightarrow x = 2\ln 2 \; (= \ln 4)$

Do not write outside the box

$$\text{Area} = \int_0^{\ln 4} 6 - e^{\frac{x}{2}} - e^{x}\, dx$$

$$= \left[6x - 2e^{\frac{x}{2}} - e^{x}\right]_0^{\ln 4}$$

$$= \left(6\ln 4 - 2e^{\frac{\ln 4}{2}} - e^{\ln 4}\right) - \left(-2e^{0} - e^{0}\right)$$

$$= (6\ln 4 - 4 - 4) - (-2 - 1)$$

$$= \cancel{6\ln 4 \ldots} \; 6\ln 4 - 5$$

END OF QUESTIONS