



Oxford Cambridge and RSA

**Thursday 22 October 2020 – Afternoon**

**A Level Further Mathematics B (MEI)**

**Y435/01 Extra Pure**

**Time allowed: 1 hour 15 minutes**



**You must have:**

- the Printed Answer Booklet
- the Formulae Booklet for Further Mathematics B (MEI)
- a scientific or graphical calculator

**INSTRUCTIONS**

- Use black ink. You can use an HB pencil, but only for graphs and diagrams.
- Write your answer to each question in the space provided in the **Printed Answer Booklet**. If you need extra space use the lined pages at the end of the Printed Answer Booklet. The question numbers must be clearly shown.
- Fill in the boxes on the front of the Printed Answer Booklet.
- Answer **all** the questions.
- Where appropriate, your answer should be supported with working. Marks might be given for using a correct method, even if your answer is wrong.
- Give your final answers to a degree of accuracy that is appropriate to the context.
- Do **not** send this Question Paper for marking. Keep it in the centre or recycle it.

**INFORMATION**

- The total mark for this paper is **60**.
- The marks for each question are shown in brackets [ ].
- This document has **4** pages.

**ADVICE**

- Read each question carefully before you start your answer.

Answer **all** the questions.

1 The matrix  $\mathbf{A}$  is given by  $\mathbf{A} = \begin{pmatrix} 0 & 2 \\ 3 & -1 \end{pmatrix}$ .

Find

- the eigenvalues of  $\mathbf{A}$ ,
- an eigenvector associated with each eigenvalue.

[5]

2 A sequence is defined by the recurrence relation  $t_{n+1} = \frac{t_n}{n+3}$  for  $n \geq 1$ , with  $t_1 = 8$ .

Verify that the particular solution to the recurrence relation is given by  $t_n = \frac{a}{(n+b)!}$  where  $a$  and  $b$  are constants whose values are to be determined.

[5]

3 A sequence is defined by the recurrence relation  $u_{n+2} = 4u_{n+1} - 5u_n$  for  $n \geq 0$ , with  $u_0 = 0$  and  $u_1 = 1$ .

(a) Find an exact real expression for  $u_n$  in terms of  $n$  and  $\theta$ , where  $\tan \theta = \frac{1}{2}$ .

[7]

A sequence is defined by  $v_n = a^{\frac{1}{2}n}u_n$  for  $n \geq 0$ , where  $a$  is a positive constant.

(b) In each of the following cases, describe the behaviour of  $v_n$  as  $n \rightarrow \infty$ .

- $a = 0.1$
- $a = 0.2$
- $a = 1$

[5]

- 4 (a) In each of the following cases, a set  $G$  and a binary operation  $\circ$  are given. The operation  $\circ$  may be assumed to be associative on  $G$ .

Determine which, if any, of the other three group axioms are satisfied by  $(G, \circ)$  and which, if any, are not satisfied.

(i)  $G = \{2n + 1 : n \in \mathbb{Z}\}$  and  $\circ$  is addition. [3]

(ii)  $G = \{a + b\sqrt{2} : a, b \in \mathbb{Z}\}$  and  $\circ$  is multiplication. [3]

(iii)  $G$  is the set of all real numbers and  $\circ$  is multiplication. [3]

- (b) A group  $M$  consists of eight  $2 \times 2$  matrices under the operation of matrix multiplication. Five of the eight elements of  $M$  are as follows.

$$\frac{1}{\sqrt{2}} \begin{pmatrix} 1 & i \\ i & 1 \end{pmatrix} \quad \frac{1}{\sqrt{2}} \begin{pmatrix} -1 & i \\ i & -1 \end{pmatrix} \quad \frac{1}{\sqrt{2}} \begin{pmatrix} 1 & -i \\ -i & 1 \end{pmatrix} \quad \begin{pmatrix} 0 & i \\ i & 0 \end{pmatrix} \quad \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$$

(i) Find the other three elements of  $M$ . [3]

$(N, *)$  is another group of order 8, with identity element  $e$ . You are given that  $N = \langle a, b, c \rangle$  where  $a*a = b*b = c*c = e$ .

(ii) State whether  $M$  and  $N$  are isomorphic to each other, giving a reason for your answer. [1]

5 In this question you must show detailed reasoning.

The matrix  $\mathbf{A}$  is given by  $\begin{pmatrix} \frac{1}{3} & -\frac{2}{3} & -\frac{2}{3} \\ -\frac{2}{3} & \frac{1}{3} & -\frac{2}{3} \\ -\frac{2}{3} & -\frac{2}{3} & \frac{1}{3} \end{pmatrix}$  and the vector  $\mathbf{e}$  is given by  $\begin{pmatrix} 1 \\ 1 \\ 1 \end{pmatrix}$ . You are given that  $\mathbf{e}$  is an

eigenvector of  $\mathbf{A}$  with an associated eigenvalue of  $-1$ .

$\mathbf{f}$  is any vector which is perpendicular to  $\mathbf{e}$ .

(a) Show that  $\mathbf{f}$  is also an eigenvector of  $\mathbf{A}$ . [4]

(b) State the eigenvalue associated with  $\mathbf{f}$ . [1]

You are now given that  $\mathbf{A}$  represents a reflection in 3-D space.

(c) Explain the significance of  $\mathbf{e}$  and  $\mathbf{f}$  in relation to the transformation that  $\mathbf{A}$  represents. [2]

(d) State the cartesian equation of the plane of reflection of the transformation represented by  $\mathbf{A}$ . [1]

6 A surface  $S$  is defined by  $z = f(x, y) = 4x^4 + 4y^4 - 17x^2y^2$ .

(a) (i) Show that there is only one stationary point on  $S$ . [5]

The value of  $z$  at the stationary point is denoted by  $s$ .

(ii) State the value of  $s$ . [1]

(iii) By factorising  $f(x, y)$ , sketch the contour lines of the surface for  $z = s$ . [3]

(iv) Hence explain whether the stationary point is a maximum point, a minimum point or a saddle point. [1]

$C$  is a point on  $S$  with coordinates  $(a, a, f(a, a))$  where  $a$  is a constant and  $a \neq 0$ .  $\Pi$  is the tangent plane to  $S$  at  $C$ .

(b) (i) Find the equation of  $\Pi$  in the form  $\mathbf{r} \cdot \mathbf{n} = p$ . [3]

(ii) The shortest distance from the origin to  $\Pi$  is denoted by  $d$ . Show that  $\frac{d}{a} \rightarrow \frac{3\sqrt{2}}{4}$  as  $a \rightarrow \infty$ . [3]

(iii) Explain whether the origin lies above or below  $\Pi$ . [1]

**END OF QUESTION PAPER**

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