Oxford Cambridge and RSA

Thursday 22 October 2020 – Afternoon

A Level Further Mathematics B (MEI)

Y435/01 Extra Pure

Time allowed: 1 hour 15 minutes

8314362605

You must have:
- the Printed Answer Booklet
- the Formulae Booklet for Further Mathematics B (MEI)
- a scientific or graphical calculator

INSTRUCTIONS
- Use black ink. You can use an HB pencil, but only for graphs and diagrams.
- Write your answer to each question in the space provided in the **Printed Answer Booklet**. If you need extra space use the lined pages at the end of the Printed Answer Booklet. The question numbers must be clearly shown.
- Fill in the boxes on the front of the Printed Answer Booklet.
- Answer **all** the questions.
- Where appropriate, your answer should be supported with working. Marks might be given for using a correct method, even if your answer is wrong.
- Give your final answers to a degree of accuracy that is appropriate to the context.
- Do **not** send this Question Paper for marking. Keep it in the centre or recycle it.

INFORMATION
- The total mark for this paper is **60**.
- The marks for each question are shown in brackets **[ ]**.
- This document has **4** pages.

ADVICE
- Read each question carefully before you start your answer.

Answer **all** the questions.

1 The matrix **A** is given by $\mathbf{A} = \begin{pmatrix} 0 & 2 \\ 3 & -1 \end{pmatrix}$.

Find

- the eigenvalues of **A**,
- an eigenvector associated with each eigenvalue. **[5]**

2 A sequence is defined by the recurrence relation $t_{n+1} = \dfrac{t_n}{n+3}$ for $n \geqslant 1$, with $t_1 = 8$.

Verify that the particular solution to the recurrence relation is given by $t_n = \dfrac{a}{(n+b)!}$ where a and b are constants whose values are to be determined. **[5]**

3 A sequence is defined by the recurrence relation $u_{n+2} = 4u_{n+1} - 5u_n$ for $n \geqslant 0$, with $u_0 = 0$ and $u_1 = 1$.

(a) Find an exact real expression for u_n in terms of n and θ, where $\tan\theta = \frac{1}{2}$. **[7]**

A sequence is defined by $v_n = a^{\frac{1}{2}n}u_n$ for $n \geqslant 0$, where a is a positive constant.

(b) In each of the following cases, describe the behaviour of v_n as $n \to \infty$.

- $a = 0.1$
- $a = 0.2$
- $a = 1$ **[5]**

4 **(a)** In each of the following cases, a set G and a binary operation $\circ$ are given. The operation $\circ$ may be assumed to be associative on G.

Determine which, if any, of the other three group axioms are satisfied by $(G, \circ)$ and which, if any, are not satisfied.

(i) $G = \{2n + 1 : n \in \mathbb{Z}\}$ and $\circ$ is addition. **[3]**

(ii) $G = \{a + b\sqrt{2} : a, b \in \mathbb{Z}\}$ and $\circ$ is multiplication. **[3]**

(iii) G is the set of all real numbers and $\circ$ is multiplication. **[3]**

(b) A group M consists of eight 2×2 matrices under the operation of matrix multiplication. Five of the eight elements of M are as follows.

$$\frac{1}{\sqrt{2}}\begin{pmatrix} 1 & \mathrm{i} \\ \mathrm{i} & 1 \end{pmatrix} \quad \frac{1}{\sqrt{2}}\begin{pmatrix} -1 & \mathrm{i} \\ \mathrm{i} & -1 \end{pmatrix} \quad \frac{1}{\sqrt{2}}\begin{pmatrix} 1 & -\mathrm{i} \\ -\mathrm{i} & 1 \end{pmatrix} \quad \begin{pmatrix} 0 & \mathrm{i} \\ \mathrm{i} & 0 \end{pmatrix} \quad \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$$

(i) Find the other three elements of M. **[3]**

$(N, *)$ is another group of order 8, with identity element e. You are given that $N = <a, b, c>$ where $a * a = b * b = c * c = e$.

(ii) State whether M and N are isomorphic to each other, giving a reason for your answer. **[1]**

5 **In this question you must show detailed reasoning.**

The matrix $\mathbf{A}$ is given by $\begin{pmatrix} \frac{1}{3} & -\frac{2}{3} & -\frac{2}{3} \\ -\frac{2}{3} & \frac{1}{3} & -\frac{2}{3} \\ -\frac{2}{3} & -\frac{2}{3} & \frac{1}{3} \end{pmatrix}$ and the vector $\mathbf{e}$ is given by $\begin{pmatrix} 1 \\ 1 \\ 1 \end{pmatrix}$. You are given that $\mathbf{e}$ is an eigenvector of $\mathbf{A}$ with an associated eigenvalue of -1.

$\mathbf{f}$ is any vector which is perpendicular to $\mathbf{e}$.

(a) Show that $\mathbf{f}$ is also an eigenvector of $\mathbf{A}$. **[4]**

(b) State the eigenvalue associated with $\mathbf{f}$. **[1]**

You are now given that $\mathbf{A}$ represents a reflection in 3-D space.

(c) Explain the significance of $\mathbf{e}$ and $\mathbf{f}$ in relation to the transformation that $\mathbf{A}$ represents. **[2]**

(d) State the cartesian equation of the plane of reflection of the transformation represented by $\mathbf{A}$. **[1]**

6 A surface S is defined by $z = \mathrm{f}(x, y) = 4x^4 + 4y^4 - 17x^2y^2$.

(a) **(i)** Show that there is only one stationary point on S. **[5]**

The value of z at the stationary point is denoted by s.

(ii) State the value of s. **[1]**

(iii) By factorising $\mathrm{f}(x, y)$, sketch the contour lines of the surface for $z = s$. **[3]**

(iv) Hence explain whether the stationary point is a maximum point, a minimum point or a saddle point. **[1]**

C is a point on S with coordinates $(a, a, \mathrm{f}(a, a))$ where a is a constant and $a \neq 0$. Π is the tangent plane to S at C.

(b) **(i)** Find the equation of Π in the form $\mathbf{r.n} = p$. **[3]**

(ii) The shortest distance from the origin to Π is denoted by d. Show that $\frac{d}{a} \to \frac{3\sqrt{2}}{4}$ as $a \to \infty$. **[3]**

(iii) Explain whether the origin lies above or below Π. **[1]**

END OF QUESTION PAPER