MATHS MADE EASY

AQA

Please write clearly in block capitals.

Centre number | | | | | | Candidate number | | | | |

Surname MODEL SOLUTIONS

Forename(s)

Candidate signature

GCSE MATHEMATICS

H

Higher Tier Paper 2 Calculator

Thursday 6 June 2019 Morning Time allowed: 1 hour 30 minutes

Materials

For this paper you must have:
- a calculator
- mathematical instruments.

Instructions

- Use black ink or black ball-point pen. Draw diagrams in pencil.
- Fill in the boxes at the top of this page.
- Answer **all** questions.
- You must answer the questions in the spaces provided. Do not write outside the box around each page or on blank pages.
- Do all rough work in this book. Cross through any work you do not want to be marked.

Information

- The marks for questions are shown in brackets.
- The maximum mark for this paper is 80.
- You may ask for more answer paper, graph paper and tracing paper. These must be tagged securely to this answer book.

Advice

In all calculations, show clearly how you work out your answer.

For Examiner's Use	
Pages	Mark
2–3	
4–5	
6–7	
8–9	
10–11	
12–13	
14–15	
16–17	
18–19	
20–21	
22–23	
24–25	
TOTAL	

Answer **all** questions in the spaces provided

Do not write outside the box

1 Circle the point that lies on the curve $y = x^2 - 4x + 1$

[1 mark]

(–1, 4) (–1, –4) (–1, –2) (–1, 6)

2 The height of a tree is 12 metres, correct to the nearest metre.

Circle the error interval.

[1 mark]

11.5 m ⩽ height < 12.5 m

11.5 m ⩽ height ⩽ 12.5 m

11.5 m < height ⩽ 12.5 m

11.5 m < height < 12.5 m

Do not write outside the box

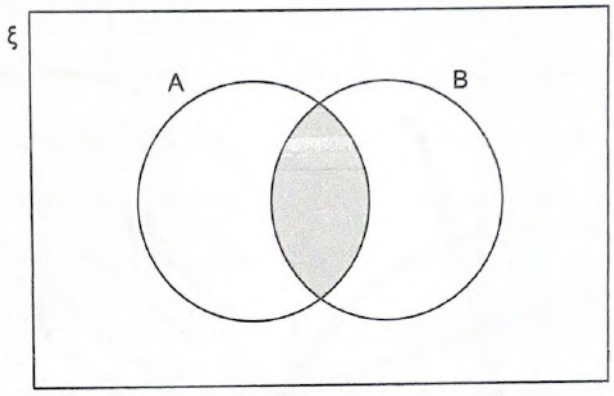
3 $2a$ is five times bigger than b.

Circle the ratio $a : b$

[1 mark]

10 : 1 | 1 : 10 | (5 : 2) | 2 : 5

4



Which of these represents the shaded region?

Circle your answer.

[1 mark]

A ∪ B | (A ∩ B)′ | (A ∩ B) | A′ ∪ B′

Turn over for the next question

4

Do not write outside the box

5 Using ruler and compasses, show the region inside the grid that is

less than 4 cm from *A*

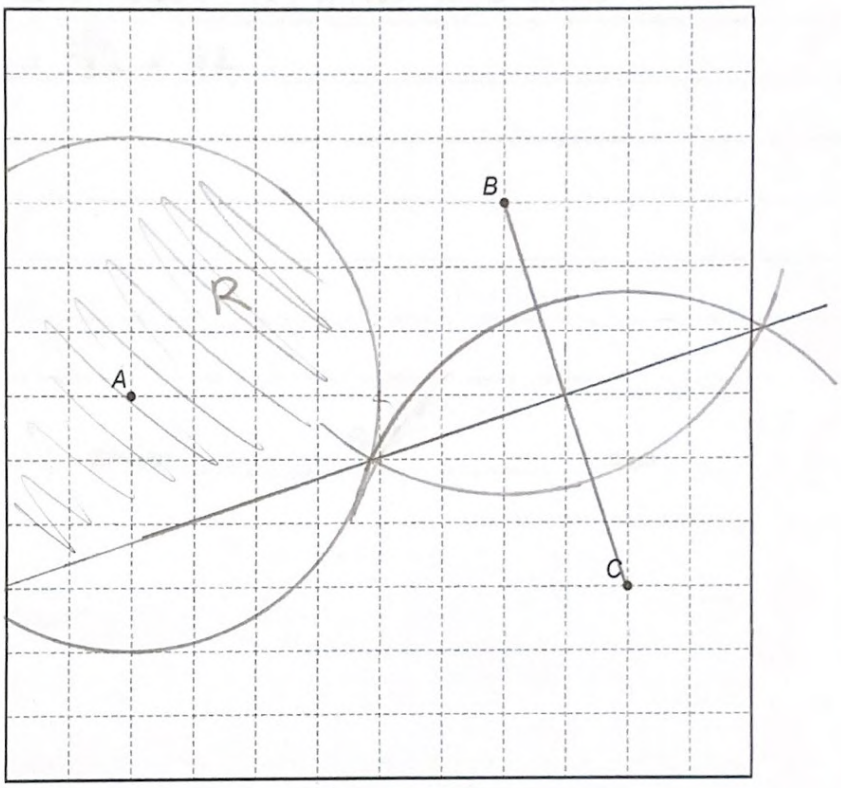
and

nearer to *B* than to *C*.

Label the region R.

Show all your construction lines.

[3 marks]



Do not write outside the box

6 Beth drives 200 miles in 4 hours.

She drives the first 18 miles at an average speed of 36 mph

Work out her average speed for the rest of the journey.

[3 marks]

$S = \frac{d}{t}$

$36 = \frac{18}{t}$ $\Rightarrow t = 0.5$ hrs

So Beth drives 182 miles in 3.5 hrs

$S = \frac{182}{3.5} = 52$

Answer 52 mph

Turn over for the next question

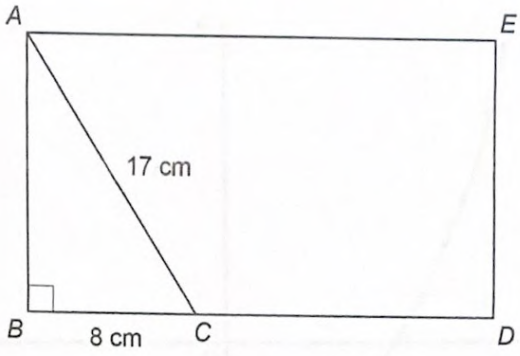
6

Do not write outside the box

7 The diagram shows rectangle *ABDE* and right-angled triangle *ABC*.

AC = 17 cm

BC = 8 cm



Not drawn accurately

BC : *CD* = 1 : 2

Work out the area of rectangle *ABDE*.

[4 marks]

$AB^2 = 17^2 - 8^2$

$\Rightarrow AB = \sqrt{17^2 - 8^2} = 15$

$BC : CD = 1 : 2$, so $CD = 8 \times 2 = 16\text{ cm}$

$\hookrightarrow BD = 8\text{ cm} + 16\text{ cm} = 24\text{ cm}$

Area of $ABDE = 15 \times 24 = 360\text{ cm}^2$

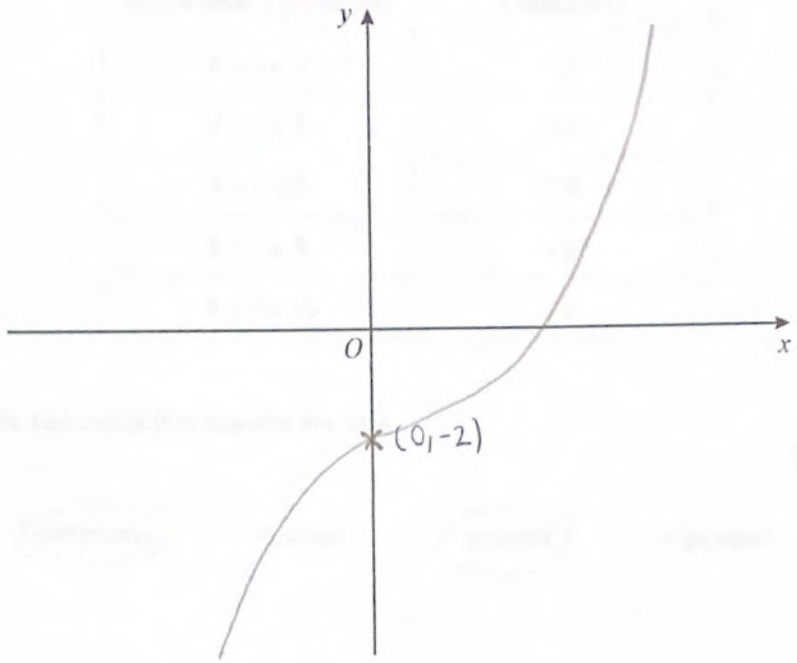
Answer 360 cm^2

Do not write outside the box

8 On the axes, sketch the curve $y = x^3 - 2$

You **must** show the coordinates of the y-intercept.

[2 marks]



Turn over for the next question

6

Do not write outside the box

9 In a sport, injury time is added time played at the end of a match.

The table shows the injury time, t (minutes) played in 380 matches.

Injury time, t (minutes)	Frequency
$0 < t \leqslant 2$	59
$2 < t \leqslant 4$	158
$4 < t \leqslant 6$	106
$6 < t \leqslant 8$	45
$8 < t \leqslant 10$	12

9 (a) Circle the **two** words that describe the data.

[1 mark]

(continuous) discrete (grouped) ungrouped

9 (b) Which class interval contains the median?

You **must** show your working.

[2 marks]

$380 \div 2 = 190$

$59 + 158 = 217$

Answer 2 $< t \leqslant$ 4

Do not write outside the box

9 (c) What percentage of the matches had **more than** 6 minutes of injury time?

[2 marks]

$\frac{45+12}{380} \times 100 = 15$

Answer 15 %

10 x is an integer.

$$-4 < x \leqslant 2$$

and

$$2 \leqslant x + 3 < 9$$

Work out all the possible values of x.

[3 marks]

$-4 < x \leqslant 2 \rightarrow -3, \ldots, 2$

$2 \leqslant x+3 < 9 \Rightarrow -1 \leqslant x < 96 \rightarrow -1, \ldots, 5$

Answer $-1, 0, 1, 2$

8

Do not write outside the box

11 Joe and Kyle share an amount of money in the ratio $7 : n$

Joe gets 35% of the money.

Work out the value of n.

[2 marks]

$\frac{7}{7+n} = 0.35$

$\Rightarrow 7 n = \frac{7}{0.35} - 7 = 13$

Answer 13

12 A biased coin is thrown 250 times.

The relative frequency of Heads is worked out after every 50 throws.

Total number of throws	50	100	150	200	250
Relative frequency	0.4	0.29	0.4	0.32	0.3

Circle the best estimate of the probability of Heads.

[1 mark]

(0.3) 0.32 0.342 0.4

Do not write outside the box

13 The amounts spent on clothes by 40 boys and 40 girls in one month were recorded.

The table shows information about the amounts spent by the boys.

Amount, x (£)	Midpoint	Number of boys	m x f
$0 \leqslant x < 20$	10	22	220
$20 \leqslant x < 40$	30	9	270
$40 \leqslant x < 60$	50	6	300
$60 \leqslant x < 80$	70	3	210
		Total = 40	1000

The mean for the girls was £35

Estimate the mean for the girls as a percentage of the mean for the boys.

[5 marks]

mean for boys = $\frac{1000}{40}$ = £25

$\frac{35}{25} \times 100 = 140$ %.

Answer 140 %

8

Do not write outside the box

14 Ali and Mel are making 3-digit codes.

The digit 0 is **not** used. 1, 2, 3, 4, 5, 6, 7, 8, 9

Ali only uses odd digits.

Mel only uses even digits.

14 (a) Ali can make x more codes than Mel.

Assume that digits **cannot** be repeated.

Work out the value of x.

[3 marks]

Ali : $5 \times 4 \times 3 = 60$ codes

Mel : $4 \times 3 \times 2 = 24$ codes

$60 - 24 = 36$

Answer 36

14 (b) In fact, digits **can** be repeated.

What does this tell you about the actual value of x?

Tick **one** box.

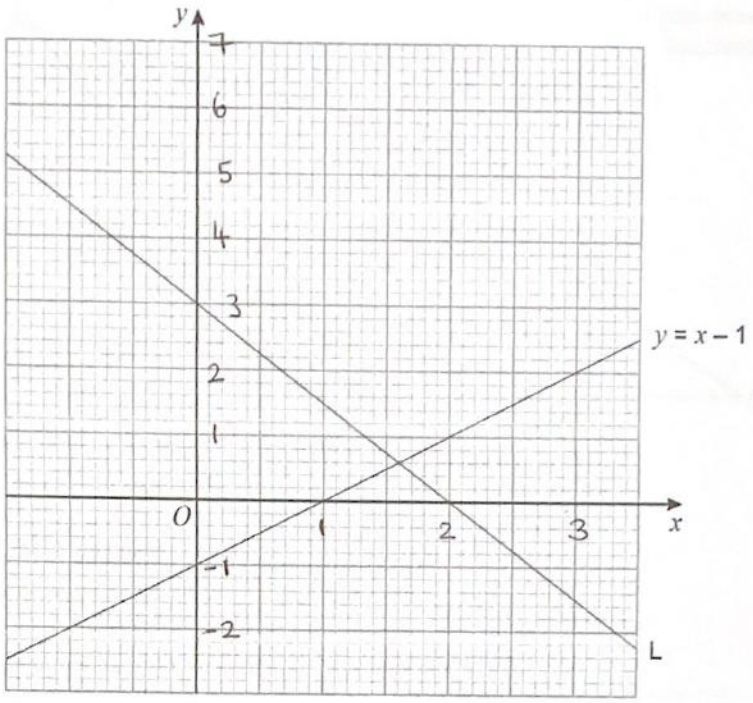
[1 mark]

- [x] It is bigger than my answer to part (a)
- [ ] It is smaller than my answer to part (a)
- [ ] It is the same as my answer to part (a)

Do not write outside the box

15 Here is line L and the graph of $y = x - 1$

The scales of the axes are not shown.



Work out the equation of line L.

[4 marks]

The scale for the x-axis is double the y axis (gradient of $\frac{1}{2}$ = 1)

gradient of L = $2 \times \frac{3-0}{0-4} = -\frac{3}{2}$

L: $y - 0 = -\frac{3}{2}(x - 2)$

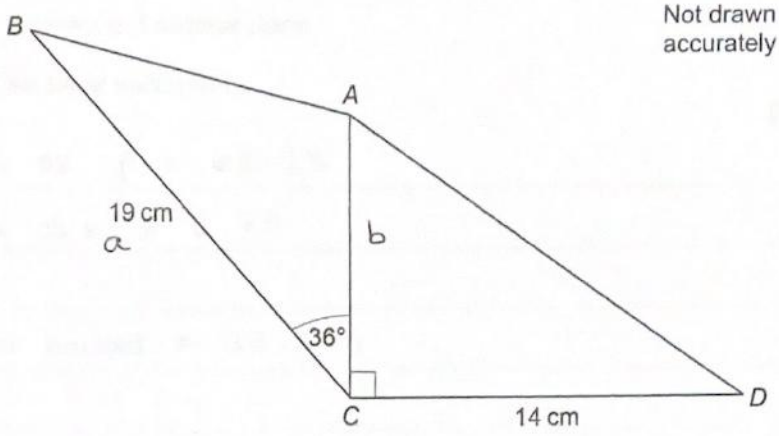
$y = -\frac{3}{2}x + 3$

Answer $y = -\frac{3}{2}x + 3$

8

Do not write outside the box

16 *ABC* and *ACD* are triangles.



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The area of *ACD* is 80.5 cm^2

Work out the area of *ABC*.

Give your answer to 3 significant figures.

[4 marks]

$0.5 \times AC \times 14 = 80.5$

$\Rightarrow 7AC = 80.5 \div 7 = 11.5$ cm

Area of a triangle = $\frac{1}{2}ab\sin C$

Area of ABC = $0.5 \times 19 \times 11.5 \times \sin 36$

$= 64.2$

Answer 64.2 cm^2

Do not write outside the box

17 $m = \frac{p-2b}{2}$

$p = 68.3$ correct to 1 decimal place.

$b = 8.7$ correct to 1 decimal place.

Work out the lower bound for m.

[3 marks]

min of p = 68.25

max of b = 8.75

lower bound = 25.375

Answer 25.375

Turn over for the next question

7

Do not write outside the box

18 In a bag there are blue discs, green discs and white discs.

There are four times as many blue discs as green discs.

number of blue discs : number of white discs = 3 : 5

One disc is selected at random.

Work out the probability that the disc is either blue or white.

[3 marks]

let b = no. of blue discs

g = no. of green discs

w = no. of white discs

~~b = 4g (x3)~~ b 3b = 12g

~~3b =~~ 3b = 5w

b g b w

4 : 1 , 3 : 5

⇒ 12 : 3 , 12 : 20

12n + 3n + 20n = 35n

b g w

32/35

g
12

blue or white.

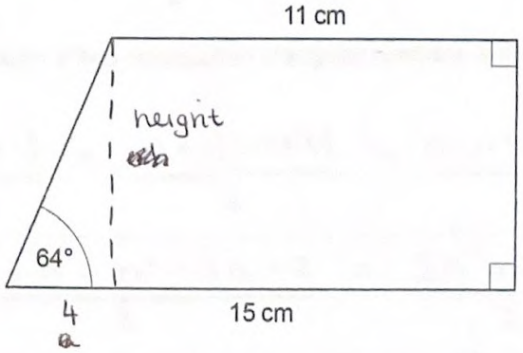
32/35

Answer _____

Do not write outside the box

19 Work out the area of the trapezium.

Not drawn accurately



[4 marks]

$\tan 64 = \frac{h}{4} \Rightarrow h = 8.201\ldots$

$\text{Area} = \frac{1}{2} \times (15+11) \times h = 106.6$

Answer 106.6 cm^2

Turn over for the next question

7

Do not write outside the box

20 Expressions for consecutive triangular numbers are

$$\frac{n(n+1)}{2} \quad \text{and} \quad \frac{(n+1)(n+2)}{2}$$

Prove that the sum of two consecutive triangular numbers is always a square number.

[4 marks]

$$\frac{n(n+1)}{2} + \frac{(n+1)(n+2)}{2} = \frac{n(n+1)+(n+1)(n+2)}{2}$$

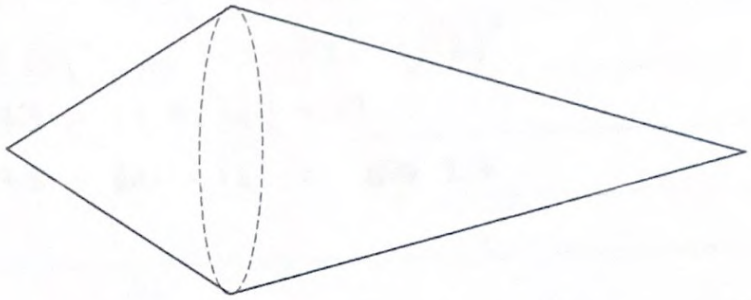
$$= \frac{n^2+n+n^2+3n+2}{2} = \frac{2n^2+4n+2}{2}$$

$$= n^2+2n+1 = (n+1)^2 \text{ (Square number).}$$

Do not write outside the box

21 A solid shape is made by joining two cones.

Each cone has the same radius.



One cone has slant height = 2 × radius

The other cone has slant height = 3 × radius

The total surface area of the shape is 57.8π cm^2

Curved surface area of a cone = πrl where r is the radius and l is the slant height

Work out the radius.

[3 marks]

1st cone curved S.a = $\pi \times r \times 2r = 2\pi r^2$

2nd cone curved S.a = $\pi \times r \times 3r = 3\pi r^2$

↳ $5\pi r^2 = 57.8\pi$

⇒ $5r^2 = 57.8$

⇒ $r^2 = 11.56$

⇒ $r = 3.4$

Answer 3.4 cm

7

Turn over ▶

Do not write outside the box

22 Show that $(5\sqrt{3} - \sqrt{12})^2$ simplifies to an integer.

[3 marks]

$(5\sqrt{3} - \sqrt{12})^2 = (5\sqrt{3} - \sqrt{12})(5\sqrt{3} - \sqrt{12})$

$= (5\sqrt{3})^2 - 2(5\sqrt{3} \times \sqrt{12}) + (\sqrt{12})^2$

$= 75 - 2(5\sqrt{36}) + 12$

$= 75 - 60 + 12 = 27$

23 A and B are similar cuboids.

surface area of A : surface area of B = 16 : 25

Work out volume of A : volume of B

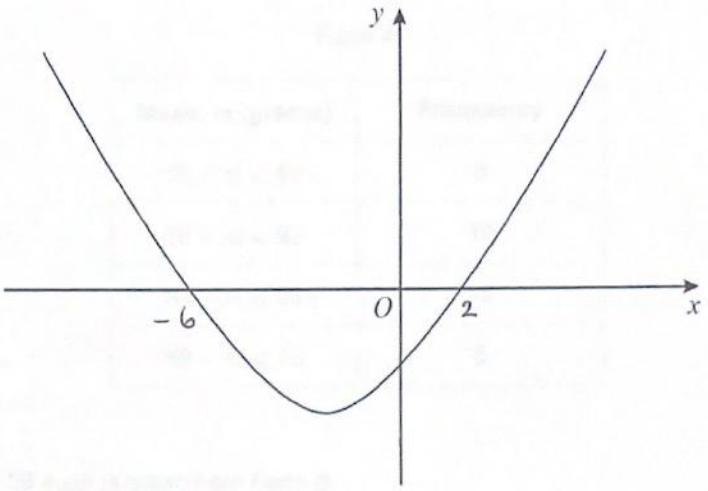
Circle your answer.

[1 mark]

4 : 5 16 : 25 **64 : 125** 256 : 625

Do not write outside the box

24 Here is a sketch of the curve $y = x^2 + 4x - 12$



Work out the values of x for which $x^2 + 4x - 12 < 0$

Give your answer as an inequality.

[3 marks]

$x^2 + 4x - 12 = (x+6)(x-2)$

Answer $-6 < x < 2$

7

Do not write outside the box

25 A sample of 50 eggs is taken from Farm A.

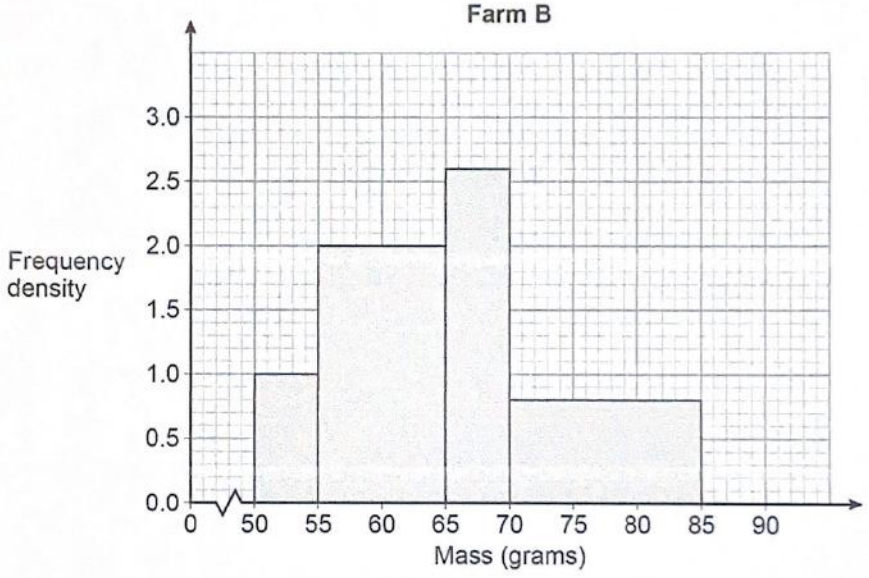
The table shows information about the masses of the eggs from Farm A.

Farm A

Mass, m (grams)	Frequency
$53 < m \leqslant 58$	8
$58 < m \leqslant 63$	19
$63 < m \leqslant 68$	15
$68 < m \leqslant 73$	8

A sample of 50 eggs is taken from Farm B.

The histogram shows information about the masses of the eggs from Farm B.



Do not write outside the box

For medium eggs, $53\text{ g} < \text{mass} \leqslant 63\text{ g}$

The Farm A sample has more medium eggs than the Farm B sample.

Using the table and the histogram, estimate how many more.

You **must** show your working.

[4 marks]

Farm B: $50 - 55 = 5$ eggs so $53 - 55 = 2$ eggs

$55 - 65 = 10 \times 2 = 20$ eggs ~~so $55 - 58 = \frac{3}{10} \times 20 = 6$ eggs~~

so $55 - 63 = \frac{8}{10} \times 20 = 16$ eggs

Farm B, medium eggs = 18 eggs

Farm A, medium eggs = 8 + 19 = 27 eggs

Answer 9 more eggs.

Turn over for the next question

4

Do not write outside the box

26 $(x + 5)(x + 2)(x + a) \equiv x^3 + bx^2 + cx - 30$

Work out the values of the integers a, b and c.

[3 marks]

$(x+5)(x+2)(x+a) = (x^2 + 7x + 10)(x+a)$

$= x^3 + ax^2 + 7x^2 + 7ax + 10x + 10a$

$= x^3 + (7+a)x^2 + (7a+10)x + 10a$

$10a = -30$

$\Rightarrow 7a = -3$

$7(-3) + 10 = c = -11$

$7 - 3 = b = 4$

$a = -3$

$b = 4$

$c = -11$

Do not write outside the box

27 $f(x) = \frac{2x}{5} - 1$

Work out the value of $f^{-1}(3) + f(-0.5)$

[5 marks]

$f^{-1}(x)$: let $y = \frac{2x}{5} - 1$

$\Rightarrow y + 1 = \frac{2x}{5}$

$\Rightarrow 5y + 5 = 2x$

$\Rightarrow x = \frac{5y + 5}{2}$

So $f^{-1}(x) = \frac{5x+5}{2}$

$f^{-1}(3) = \frac{5(3)+5}{2} = 10$

$f(-0.5) = \frac{2(-0.5)}{5} - 1 = -\frac{6}{5}$

$10 - {}^{6}/_{5} = 8.8$

Answer 8.8

END OF QUESTIONS

8