

Please write clearly in	n block capitals.
Centre number	Candidate number
Surname	
Forename(s)	
Candidate signature	I declare this is my own work.

A-level FURTHER MATHEMATICS

Paper 1

Friday 22 May 2020

Morning

Time allowed: 2 hours

Materials

- You must have the AQA formulae and statistical tables booklet for A-level Mathematics and A-level Further Mathematics.
- You should have a scientific calculator that meets the requirements of the specification. (You may use a graphical calculator.)

Instructions

- Use black ink or black ball-point pen. Pencil should only be used for drawing.
- Fill in the boxes at the top of this page.
- Answer all questions.
- You must answer each question in the space provided for that question.
 If you require extra space for your answer(s), use the lined pages at the end of this book. Write the question number against your answer(s).
- Do **not** write outside the box around each page.
- Show all necessary working; otherwise marks for method may be lost.
- Do all rough work in this book. Cross through any work you do not want to be marked.

Information

- The marks for questions are shown in brackets.
- The maximum mark for this paper is 100.

Advice

- Unless stated otherwise, you may quote formulae, without proof, from the booklet.
- You do not necessarily need to use all the space provided.

For Examiner's Use		
Question	Mark	
1		
2		
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5		
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8		
9		
10		
11		
12		
13		
14		
TOTAL		



Answer all questions in the spaces provided.

1 Which of the integrals below is **not** an improper integral?

Circle your answer.

[1 mark]

$$\int_{0}^{\infty} e^{-x} dx$$

$$\int_0^\infty e^{-x} dx \qquad \int_0^2 \frac{1}{1-x^2} dx \qquad \int_0^1 \sqrt{x} dx \qquad \int_0^1 \frac{1}{\sqrt{x}} dx$$

$$\int_0^1 \sqrt{x} \, dx$$

$$\int_0^1 \frac{1}{\sqrt{x}} \, \mathrm{d}x$$

2 Which one of the matrices below represents a rotation of 90° about the x-axis? Circle your answer.

[1 mark]

$$\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & -1 \end{bmatrix}$$

$$\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & -1 \end{bmatrix} \qquad \begin{bmatrix} -1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \qquad \begin{bmatrix} 1 & 0 & 0 \\ 0 & 0 & 1 \\ 0 & 1 & 0 \end{bmatrix} \qquad \begin{bmatrix} 1 & 0 & 0 \\ 0 & 0 & -1 \\ 0 & 1 & 0 \end{bmatrix}$$

$$\begin{bmatrix} 1 & 0 & 0 \\ 0 & 0 & 1 \\ 0 & 1 & 0 \end{bmatrix}$$

$$\begin{bmatrix} 1 & 0 & 0 \\ 0 & 0 & -1 \\ 0 & 1 & 0 \end{bmatrix}$$

3 The quadratic equation $ax^2 + bx + c = 0$ ($a, b, c \in \mathbb{R}$) has real roots α and β .

One of the four statements below is incorrect.

Which statement is incorrect?

Tick (✓) one box.

[1 mark]

$$c = 0 \Rightarrow \alpha = 0 \text{ or } \beta = 0$$

$$c=a\Rightarrow \alpha$$
 is the reciprocal of β

$$b < 0$$
 and $c < 0 \Rightarrow \alpha > 0$ and $\beta > 0$

$$b = 0 \Rightarrow \alpha = -\beta$$

Turn over for the next question

4	It is given that 1 – 31 is one root of the quartic equation	
	$z^4 - 2z^3 + pz^2 + rz + 80 = 0$	
	where p and r are real numbers.	
4 (a)	Express $z^4 - 2z^3 + pz^2 + rz + 80$ as the product of two quadratic factors with real coefficients.	[4 marks]
	-	



4 (b)	Find the value of p and the value of r .	[2 marks]
	Turn over for the next question	



5	H_1 is the locus of points such that the distance from the point (5, 0) is distance from the line $x = 2$	twice the
5 (a)	Show that the equation of H_{1} can be written in the form	
	$(x-1)^2 - \frac{y^2}{q} = r$	
	where q and r are integers.	[5 marks]



	$x^2 - y^2 = 4$		
Describe fully a sequen	Describe fully a sequence of two transformations which maps the graph of ${\cal H}$		
the graph of H_1			[4
			•



- **6** Let w be the root of the equation $z^7=1$ that has the smallest argument α in the interval $0<\alpha<\pi$
- **6 (a)** Prove that w^n is also a root of the equation $z^7 = 1$ for any integer n.

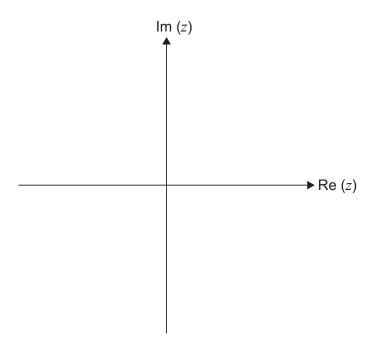
[1 mark]

6 (b) Prove that $1 + w + w^2 + w^3 + w^4 + w^5 + w^6 = 0$

[2 marks]

6 (c) Show the positions of w, w^2 , w^3 , w^4 , w^5 , and w^6 on the Argand diagram below.

[2 marks]



6 (d)	Prove that		
		$\cos\frac{2\pi}{7} + \cos\frac{4\pi}{7} + \cos\frac{6\pi}{7} = -\frac{1}{2}$	[4 marks]



7	Three planes have equations
	(4k+1)x - 3y + (k-5)z = 3
	(k-1)x + (3-k)y + 2z = 1
	7x - 3y + 4z = 2
7 (a)	The planes do not meet at a unique point.
	Show that $k=4.5$ is one possible value of k , and find the other possible value of k . [3 marks]



7 (b)	For each value of k found in part (a), identify the configuration of the given planes.		
	In each case fully justify your answer, stating whether or not the equation planes form a consistent system.	ions of the	
		[4 marks	



8	The three roots of the equation	
	$4x^3 - 12x^2 - 13x + k = 0$	
	where k is a constant, form an arithmetic sequence.	
	Find the roots of the equation.	[6 marks]



Turn over for the next question

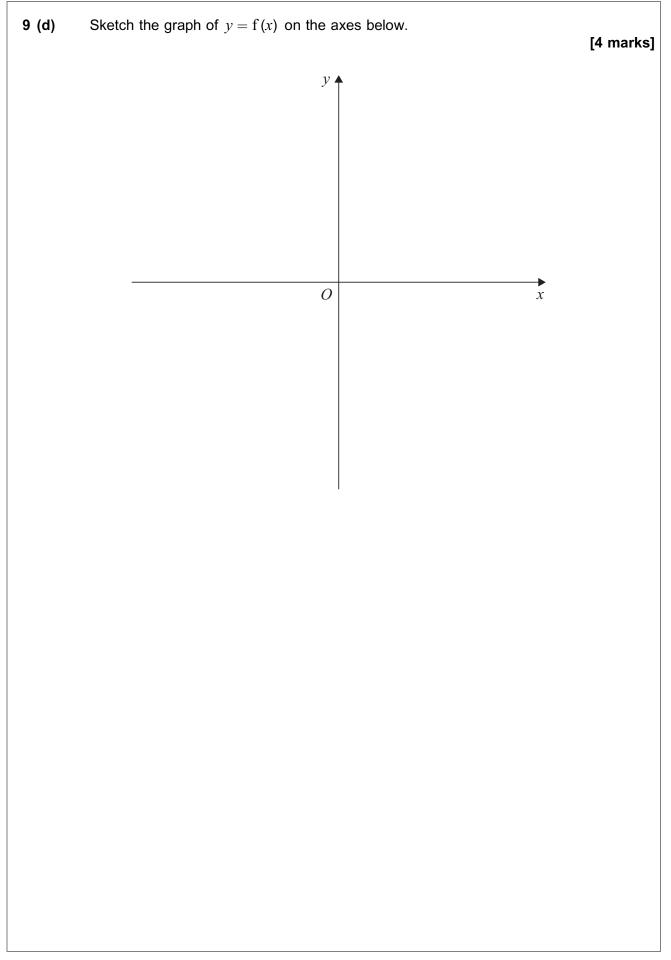


9	The function f is defined by	
	$f(x) = \frac{x(x+3)}{x+4} \qquad (x \in \mathbb{R}, x \neq -4)$	
9 (a)	Find the interval (a, b) in which $f(x)$ does not take any values.	
	Fully justify your answer.	
		[5 marks]
		 -



(b)	Find the coordinates of the two stationary points of the graph of $y = f(x)$	[2 mark
(c)	Show that the graph of $y = f(x)$ has an oblique asymptote and find its equ	ation. [2 mark
	Question 9 continues on the next page	







Turn over for the next question DO NOT WRITE ON THIS PAGE ANSWER IN THE SPACES PROVIDED

dy = 2y	$=\frac{x+3}{x(x-1)(x^2+3)}$	(x > 1)	
$dx^{\top} x$	$-\frac{1}{x(x-1)(x^2+3)}$	$(\lambda > 1)$	
			[8 marks



10 (b)	Find the particular solution for which $y = 0$ when $x = 3$	
	Give your answer in the form $y = f(x)$	
		[2 marks]



11 The lines l_1 , l_2 and l_3 are defined as follows.

$$l_1: \left(\mathbf{r} - \begin{bmatrix} 1 \\ 5 \\ -1 \end{bmatrix}\right) \times \begin{bmatrix} -2 \\ 1 \\ -3 \end{bmatrix} = \mathbf{0}$$

$$l_2: \left(\mathbf{r} - \begin{bmatrix} -3\\2\\7 \end{bmatrix}\right) \times \begin{bmatrix} 2\\-1\\3 \end{bmatrix} = \mathbf{0}$$

$$l_3: \left(\mathbf{r} - egin{bmatrix} -5 \\ 12 \\ -4 \end{bmatrix} \right) imes egin{bmatrix} 4 \\ 0 \\ 9 \end{bmatrix} = \mathbf{0}$$

11 (a) (i) Explain	n how you knov	v that two of the	lines are parallel.
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		[1 mark]



11 (a) (II)	Show that the perpendicular distance between these two parallel lines is 7.95 units, correct to three significant figures.				
		[5 marks]			



11 (b)	Show that the lines l_1 and l_3 meet, and find the coordinates of their point intersection.			
		[5 marks]		



Turn over for the next question DO NOT WRITE ON THIS PAGE ANSWER IN THE SPACES PROVIDED

2 3

	cosh ⁻¹	$\left(\frac{x}{a}\right) = \ln\left(\frac{x}{a}\right)$	$x + \sqrt{x^2}$	$\frac{a^2}{a^2}$	1	for $a > 0$	
		(4)	и)			[6 marks
							



12 (b)	The formulae health divide the integral of	
12 (b)	The formulae booklet gives the integral of $\frac{1}{\sqrt{x^2 - a^2}}$ as	
	$ \cosh^{-1}\left(\frac{x}{a}\right) \text{or} \ln\left(x + \sqrt{x^2 - a^2}\right) + c $	
	Ronald says that this contradicts the result given in part (a).	
	Explain why Ronald is wrong.	
	[2 n	narks]



13	Two light elastic strings each have one end attached to a particle ${\it B}$ of mass $3c{\rm kg}$, which rests on a smooth horizontal table.
	The other ends of the strings are attached to the fixed points A and C , which are 8 metres apart.
	ABC is a horizontal line.
	$A \downarrow \qquad \qquad B \qquad \qquad \downarrow C$
	String AB has a natural length of 4 metres and a stiffness of $5c$ newtons per metre.
	String BC has a natural length of 1 metre and a stiffness of c newtons per metre.
	The particle is pulled a distance of $\frac{1}{3}$ metre from its equilibrium position towards A , and released from rest.
13 (a)	Show that the particle moves with simple harmonic motion. [8 marks]



Find the sp equilibrium	peed of the particle position. Give yo	e when it is at a poir our answer to two sig	nt P , a distance $\frac{1}{4}$ metrognificant figures.	
				[4 ma

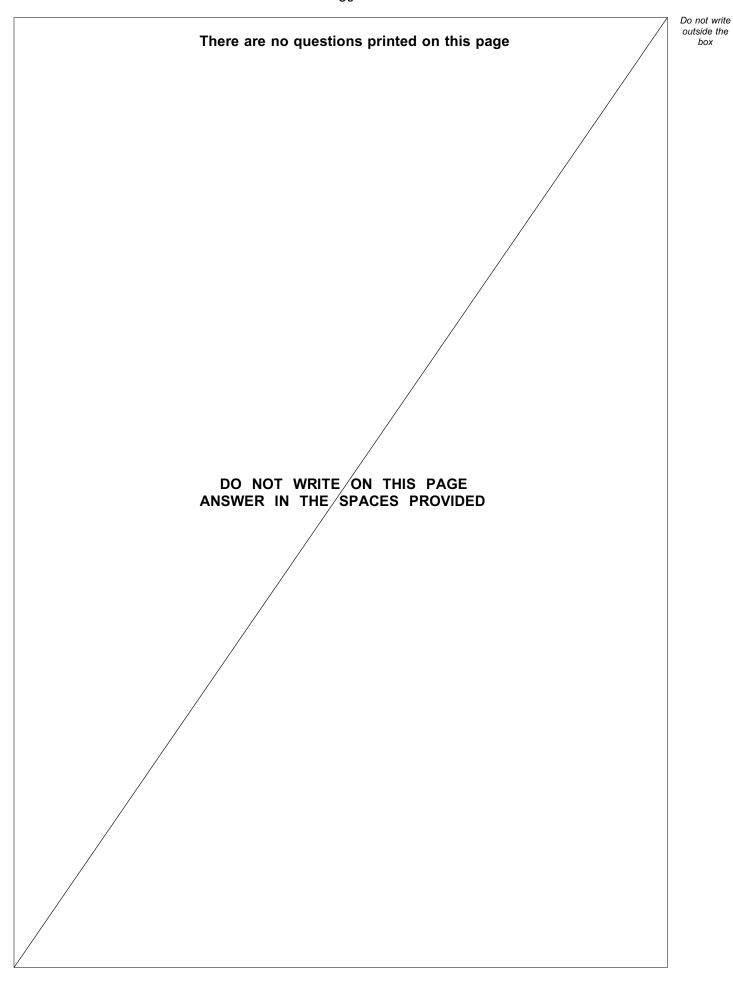


Giv	ven that
	$\sinh(A+B)=\sinh A\cosh B+\cosh A\sinh B$
	spress $\sinh (m+1)x$ and $\sinh (m-1)x$ in terms of $\sinh mx$, $\cosh mx$, $\sinh x$ and
CO	sh <i>x</i> [1 mark
На	ence find the sum of the series
110	$C_n = \cosh x + \cosh 2x + \dots + \cosh nx$
in ·	terms of $\sinh x$, $\sinh nx$ and $\sinh (n+1)x$
	[5 marks]



END OF QUESTIONS







Question number	Additional page, if required. Write the question numbers in the left-hand margin.



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