1. The line l_1 has equation $2x + 4y - 3 = 0$

The line l_2 has equation $y = mx + 7$, where m is a constant.

Given that l_1 and l_2 are perpendicular,

(a) find the value of m.

(2)

The lines l_1 and l_2 meet at the point P.

(b) Find the x coordinate of P.

(2)

a) $2x + 4y - 3 = 0 \Rightarrow 4y = 3 - 2x$

$\Rightarrow y = \frac{3}{4} - \frac{1}{2}x$.

$\Rightarrow m_{l_2} = 2$.

b) $y = 2x + 7$, $y = -\frac{1}{2}x + \frac{3}{4}$.

$2x_p + 7 = -\frac{1}{2}x_p + \frac{3}{4}$.

$\Rightarrow \frac{5}{2}x_p = -\frac{25}{4}$

$\Rightarrow x_p = -\frac{5}{2}$.

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2. Find, using algebra, all real solutions to the equation

(i) $16a^2 = 2\sqrt{a}$

(4)

(ii) $b^4 + 7b^2 - 18 = 0$

(4)

i) $16a^2 = 2\sqrt{a}$.

$\Rightarrow \quad a^{3/2} = 1/8, \; 0.$

$\Rightarrow \quad a = 1/4, \; 0$

ii) Let $c = b^2$.

$c^2 + 7c - 18 = 0$

$(c+9)(c-2) = 0$

$\Rightarrow c = -9, 2.$

$b^2 = -9, 2.$

$b^2 \neq -9$, so $b^2 = 2$

$\Rightarrow b = \pm\sqrt{2}$.

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3. (a) Given that k is a constant, find

$$\int \left(\frac{4}{x^3} + kx \right) \mathrm{d}x$$

simplifying your answer.

(3)

(b) Hence find the value of k such that

$$\int_{0.5}^{2} \left(\frac{4}{x^3} + kx \right) \mathrm{d}x = 8$$

(3)

a) $\int 4x^{-3} + kx \quad dx$

$= -2x^{-2} + \frac{k}{2}x^2 + C.$

$= \frac{-2}{x^2} + \frac{1}{2}kx^2 + C.$

b) $\int_{0.5}^{2} 4x^{-3} + kx \quad dx$

$= \left[\frac{-2}{x^2} + \frac{1}{2}kx^2 \right]_{0.5}^{2}$

$= \left(-\frac{1}{2} + 2k\right) - \left(-8 + \frac{1}{8}k\right)$

$= \frac{15}{2} + \frac{15}{8}k \quad = 8.$

$\Rightarrow \frac{15}{8}k = \frac{1}{2}.$

$\Rightarrow \quad k = \frac{4}{15}.$

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4. A tree was planted in the ground.
Its height, H metres, was measured t years after planting.

Exactly 3 years after planting, the height of the tree was 2.35 metres.
Exactly 6 years after planting, the height of the tree was 3.28 metres.

Using a linear model,

(a) find an equation linking H with t.

(3)

The height of the tree was approximately 140 cm when it was planted.

(b) Explain whether or not this fact supports the use of the linear model in part (a).

(2)

a) $H = mt + c$

$2.35 = 3m + c$
$3.28 = 6m + c$

$\Rightarrow 3m = 0.93$
$\Rightarrow m = 0.31$
$\Rightarrow c = 1.42.$

$\Rightarrow H = 0.31t + 1.42$

b) $140\text{cm} = 1.4\text{m}.$

$1.4 \approx 1.42$, so this supports the model.

5. A curve has equation

$$y = 3x^2 + \frac{24}{x} + 2 \qquad x > 0$$

(a) Find, in simplest form, $\frac{dy}{dx}$

(3)

(b) Hence find the exact range of values of x for which the curve is increasing.

(2)

a) $y = 3x^2 + \frac{24}{x} + 2$

$\frac{dy}{dx} = 6x - \frac{24}{x^2}$

b) $\frac{dy}{dx} > 0 \Rightarrow 6x - \frac{24}{x^2} > 0$

$\Rightarrow 6x^3 - 24 > 0$

$\Rightarrow x^3 > 4.$

$\Rightarrow x > \sqrt[3]{4}.$

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6.

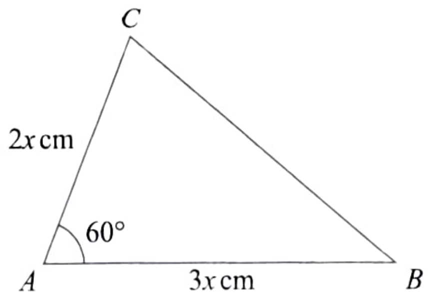


Figure 1

Figure 1 shows a sketch of a triangle ABC with $AB = 3x$ cm, $AC = 2x$ cm and angle $CAB = 60°$

Given that the area of triangle ABC is $18\sqrt{3}$ cm^2

(a) show that $x = 2\sqrt{3}$

(3)

(b) Hence find the exact length of BC, giving your answer as a simplified surd.

(3)

a) $\text{Area} = \frac{1}{2}\, 2x\, 3x \sin 60° = 18\sqrt{3}$

$\Rightarrow \quad \frac{3\sqrt{3}\,x^2}{2} = 18\sqrt{3}$

$\Rightarrow \quad x^2 = 12$

$\Rightarrow \quad x = +\sqrt{12} = 2\sqrt{3}$

b) $BC^2 = (6\sqrt{3})^2 + (4\sqrt{3})^2 - (2 \times 4\sqrt{3} \times 6\sqrt{3} \cos 60°)$

$= 108 + 48 - (144 \times \frac{1}{2})$

$= 84.$

$\Rightarrow \quad BC = \sqrt{84} = 2\sqrt{21}$ cm.

7. The curve C has equation

$$y = \frac{k^2}{x} + 1 \qquad x \in \mathbb{R},\ x \neq 0$$

where k is a constant.

(a) Sketch C stating the equation of the horizontal asymptote. **(3)**

The line l has equation $y = -2x + 5$

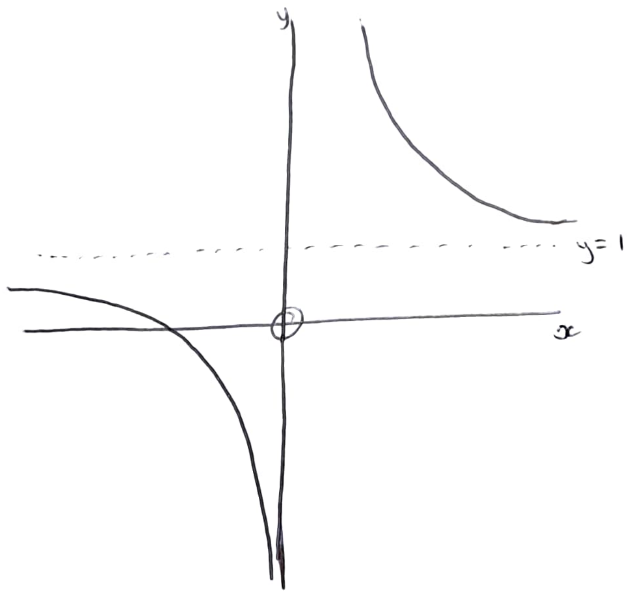
(b) Show that the x coordinate of any point of intersection of l with C is given by a solution of the equation

$$2x^2 - 4x + k^2 = 0$$

(2)

(c) Hence find the exact values of k for which l is a tangent to C. **(3)**

a)



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Question 7 continued

b) $y = -2x+5 = k^2 \frac{1}{x} + 1$

$\Rightarrow -2x^2 + 5x = k^2 + x$

$\Rightarrow 2x^2 - 4x + k^2 = 0$

c) $b^2 - 4ac = 0$

$\Rightarrow 16 - 8k^2 = 0$

$\Rightarrow k^2 = 2$

$\Rightarrow k = \pm\sqrt{2}$.

(Total for Question 7 is 8 marks)

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8. (a) Find the first 3 terms, in ascending powers of x, of the binomial expansion of

$$\left(2+\frac{3x}{4}\right)^6$$

giving each term in its simplest form.

(4)

(b) Explain how you could use your expansion to estimate the value of 1.925^6
You do not need to perform the calculation.

(1)

a) $\left(2+\frac{3}{4}x\right)^6 = 2^6 + \left({}^6C_1 \times 2^5 \times \frac{3}{4}x\right) + \left({}^6C_2 \times 2^4 \times \left[\frac{3}{4}x\right]^2\right)$

$= 64 + 144x + 135x^2$.

b) Set $2+\frac{3}{4}x = 1.925$.

$\frac{3}{4}x = -0.075$

$\Rightarrow x = -0.1$.

Use $x = -0.1$ in the expansion above.

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9. A company started mining tin in Riverdale on 1st January 2019.

A model to find the total mass of tin that will be mined by the company in Riverdale is given by the equation

$$T = 1200 - 3(n - 20)^2$$

where T tonnes is the total mass of tin mined in the n years after the start of mining.

Using this model,

(a) calculate the mass of tin that will be mined up to 1st January 2020, **(1)**

(b) deduce the maximum total mass of tin that could be mined, **(1)**

(c) calculate the mass of tin that will be mined in 2023. **(2)**

(d) State, giving reasons, the limitation on the values of n. **(2)**

a) $1200 - 3(1-20)^2 = 117$ tonnes

b) 1200 tonnes.

c) $(1200 - 3(5-20)^2) - (1200 - 3(4-20)^2)$

$= 525 - 432$

$= 93$ tonnes.

d) The model is only valid for $n \leq 20$, the total amount mined cannot decrease.

10. A circle C has equation

$$x^2 + y^2 - 4x + 8y - 8 = 0$$

(a) Find

(i) the coordinates of the centre of C,

(ii) the exact radius of C.

(3)

The straight line with equation $x = k$, where k is a constant, is a tangent to C.

(b) Find the possible values for k.

(2)

a) i) $(x-2)^2 - 4 + (y+4)^2 - 16 - 8 = 0$

$\Rightarrow (x-2)^2 + (y+4)^2 = 28.$

Centre at $(2, -4)$.

ii) Radius $= \sqrt{28}$.

b) $k = 2 \pm \sqrt{28}$.

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11.

$$f(x) = 2x^3 - 13x^2 + 8x + 48$$

(a) Prove that $(x - 4)$ is a factor of $f(x)$.

(2)

(b) Hence, using algebra, show that the equation $f(x) = 0$ has only two distinct roots.

(4)

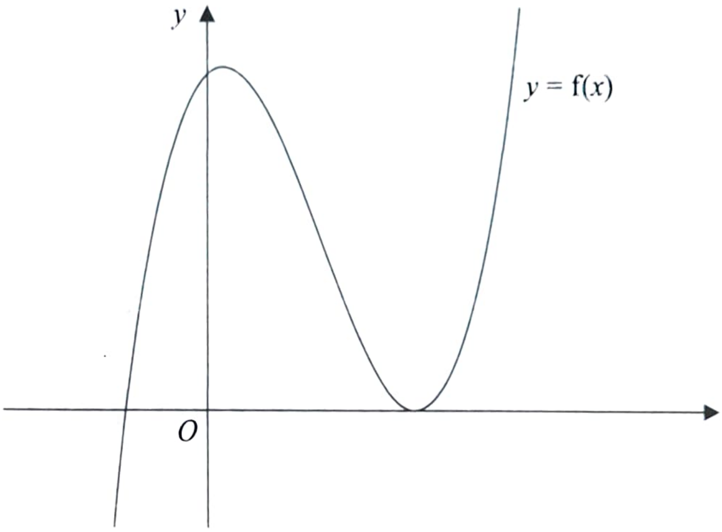

Figure 2

Figure 2 shows a sketch of part of the curve with equation $y = f(x)$.

(c) Deduce, giving reasons for your answer, the number of real roots of the equation

$$2x^3 - 13x^2 + 8x + 46 = 0$$

(2)

Given that k is a constant and the curve with equation $y = f(x + k)$ passes through the origin,

(d) find the two possible values of k.

(2)

a) $f(4) = 2(4)^3 - 13(4)^2 + 8(4) + 48$

$= 128 - 208 + 32 + 48$

$= 0.$

$f(4) = 0 \Rightarrow x - 4$ is a factor.

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Question 11 continued

b) $2x^3 - 13x^2 + 8x + 48$

$= (x-4)(2x^2 - 5x - 12)$

$= (x-4)(2x+3)(x-4)$

$= (x-4)^2(2x+3).$

$(x-4)^2(2x+3) = 0$ has two roots,

$x = 4, -\frac{3}{2}.$

c) 3. This equation is $f(x)-2$, so the point $x=4$ is no longer a stationary point of inflection. Two new solutions are created in place of this one.

d) $k = 4, -\frac{3}{2}.$

12. (a) Show that

$$\frac{10\sin^2\theta - 7\cos\theta + 2}{3 + 2\cos\theta} \equiv 4 - 5\cos\theta$$

(4)

(b) Hence, or otherwise, solve, for $0 \leqslant x < 360°$, the equation

$$\frac{10\sin^2 x - 7\cos x + 2}{3 + 2\cos x} = 4 + 3\sin x$$

(3)

a) $10\sin^2\theta = 10 - 10\cos^2\theta.$

$$\frac{10 - 10\cos^2\theta - 7\cos\theta + 2}{3 + 2\cos\theta}$$

$$= \frac{-10\cos^2\theta - 7\cos\theta + 12}{3 + 2\cos\theta}$$

$$= \frac{(-5\cos\theta + 4)\cancel{(2\cos\theta + 3)}}{\cancel{3 + 2\cos\theta}}$$

$$= 4 - 5\cos\theta.$$

b) $4 - 5\cos x = 4 + 3\sin x.$

$\Rightarrow -5\cos x = 3\sin x$

$\Rightarrow \tan x = -\frac{5}{3}.$

$\Rightarrow x = 121°, 301°$

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13.

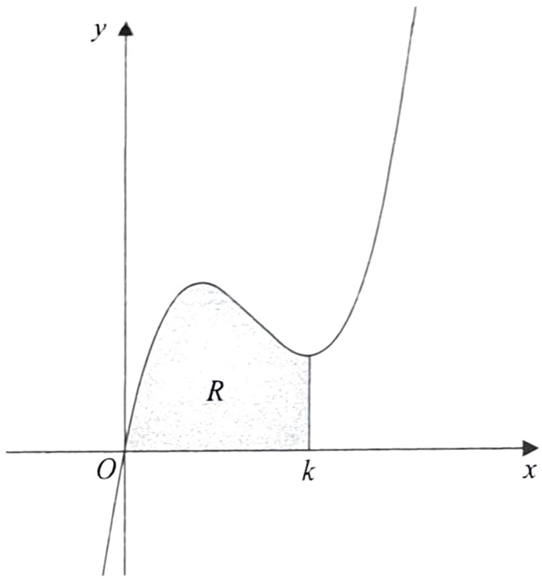


Figure 3

Figure 3 shows a sketch of part of the curve with equation

$$y = 2x^3 - 17x^2 + 40x$$

The curve has a minimum turning point at $x = k$.

The region R, shown shaded in Figure 3, is bounded by the curve, the x-axis and the line with equation $x = k$.

Show that the area of R is $\dfrac{256}{3}$

(*Solutions based entirely on graphical or numerical methods are not acceptable.*)

(7)

$y = 2x^3 - 17x^2 + 40x$

$\Rightarrow \dfrac{dy}{dx} = 6x^2 - 34x + 40 = 0$

$\Rightarrow 3x^2 - 17x + 20 = 0$

$\Rightarrow (3x - 5)(x - 4) = 0$

$\Rightarrow x = \frac{5}{3}, 4.$

$k = x = 4$, as the other turning point has been considered / passed already.

Question 13 continued

$$\int_0^4 2x^3 - 17x^2 + 40x \, dx$$

$$= \left[\frac{1}{2}x^4 - \frac{17}{3}x^3 + 20x^2\right]_0^4$$

$$= 128 - \frac{1088}{3} + 320$$

$$= \frac{256}{3}.$$

14. The value of a car, £V, can be modelled by the equation

$$V = 15\,700e^{-0.25t} + 2300 \qquad t \in \mathbb{R},\ t \geqslant 0$$

where the age of the car is t years.

Using the model,

(a) find the initial value of the car.
(1)

Given the model predicts that the value of the car is decreasing at a rate of £500 per year at the instant when $t = T$,

(b) (i) show that

$$3925e^{-0.25T} = 500$$

(ii) Hence find the age of the car at this instant, giving your answer in years and months to the nearest month.

(*Solutions based entirely on graphical or numerical methods are not acceptable.*)
(6)

The model predicts that the value of the car approaches, but does not fall below, £A.

(c) State the value of A.
(1)

(d) State a limitation of this model.
(1)

a) £18000

b) i) $\frac{dV}{dt} = -3925e^{-0.25t} = -500.$

$\Rightarrow 3925e^{-0.25t} = 500$

ii) $e^{-0.25t} = \frac{500}{3925}$

$-0.25t = \ln \frac{500}{3925}$

$\Rightarrow t = -4\ln \frac{500}{3925} =$ 8years, 3 months.

Question 14 continued

c) £2300

d) Other factors affect the value of the car, i.e. condition, mileage, etc.

15. Given $n \in \mathbb{N}$, prove that $n^3 + 2$ is not divisible by 8

(4)

n even :

$n = 2k$:

$$(2k)^3 + 2$$

$$= \underbrace{8k^3}_{\text{div. by } 8} + \underbrace{2}_{\text{not div. by } 8}.$$

n odd :

$n = 2k+1$:

$$(2k+1)^3 + 2$$

$$= \underbrace{8k^3 + 12k + 6k}_{\text{even}} \underbrace{+3}_{\text{odd}}$$

$n^3 + 2$ is odd for n odd,
so it is not divisible by 8.

$\Rightarrow$ $n^3 + 2$ not divisible by 8.

16. (i) Two non-zero vectors, **a** and **b**, are such that

$$|\mathbf{a} + \mathbf{b}| = |\mathbf{a}| + |\mathbf{b}|$$

Explain, geometrically, the significance of this statement.

(1)

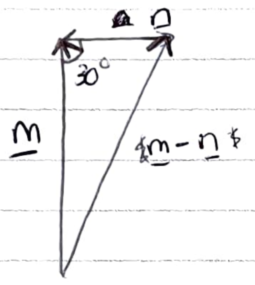
(ii) Two different vectors, **m** and **n**, are such that $|\mathbf{m}| = 3$ and $|\mathbf{m} - \mathbf{n}| = 6$
The angle between vector **m** and vector **n** is 30°

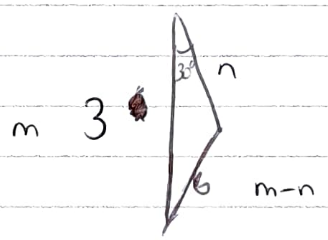
Find the angle between vector **m** and vector **m** – **n**, giving your answer, in degrees, to one decimal place.

(4)

i) $\underline{a}$ and $\underline{b}$ must lie in the same direction.

ii)





$$\frac{6}{\sin 30^\circ} = \frac{3}{\sin\theta}$$

$\Rightarrow \quad \sin\theta = \frac{1}{2}\sin 30^\circ$

$\Rightarrow \quad \theta = 14.48°$

$180° - (30° + 14.48°) = 135.52°$

$\Rightarrow 135.5°$