Do not write outside the box

Section A

Answer **all** questions in the spaces provided.

1 $f(x) = \arcsin x$

State the maximum possible domain of f

Tick (✓) **one** box.

[1 mark]

- [x] $\{x \in \mathbb{R} : -1 \leq x \leq 1\}$
- [ ] $\left\{x \in \mathbb{R} : -\frac{\pi}{2} \leq x \leq \frac{\pi}{2}\right\}$
- [ ] $\{x \in \mathbb{R} : -\pi \leq x \leq \pi\}$
- [ ] $\{x \in \mathbb{R} : -90 \leq x \leq 90\}$

2 Find the value of $\dfrac{100!}{98! \times 3!}$

Circle your answer.

[1 mark]

$\dfrac{50}{147}$ 1650 3300 161 700

3 Given $u_1 = 1$, determine which one of the formulae below defines an increasing sequence for $n \geq 1$

Circle your answer.

[1 mark]

$u_{n+1} = 1 + \dfrac{1}{u_n}$ $u_n = 2 - 0.9^{n-1}$ $u_{n+1} = -1 + 0.5u_n$ $u_n = 0.9^{n-1}$

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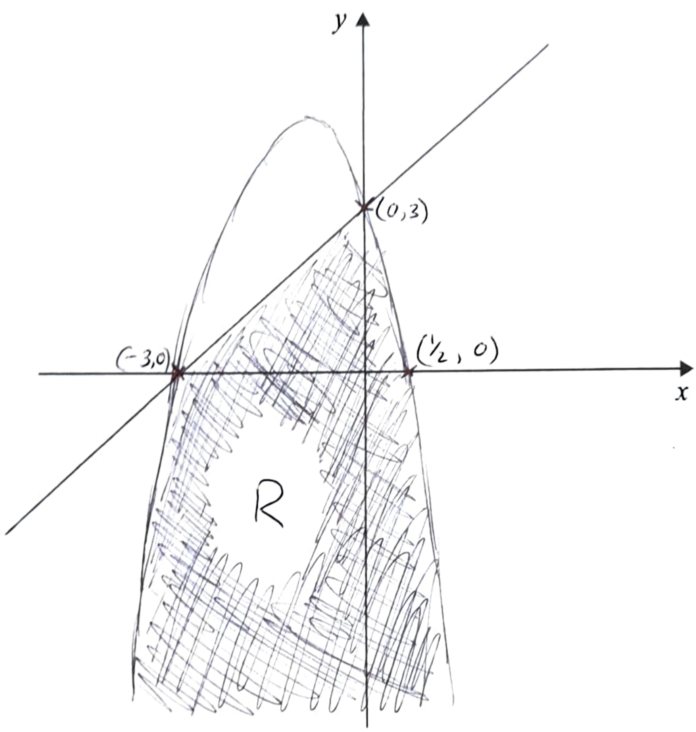
4 Sketch the region defined by the inequalities

$$y \leq (1 - 2x)(x + 3) \text{ and } y - x \leq 3$$

Clearly indicate your region by shading it in and labelling it *R*.

[3 marks]



Turn over for the next question

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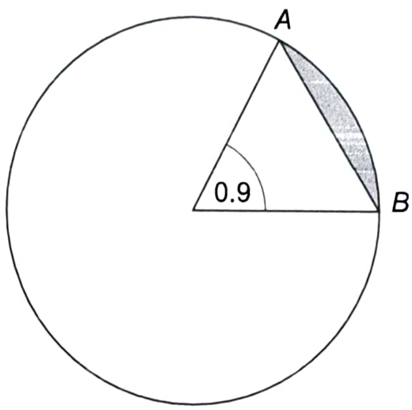
5 A circle has equation $x^2 + y^2 - 6x - 8y = 264$

AB is a chord of the circle.

The angle at the centre of the circle, subtended by AB, is 0.9 radians, as shown in the diagram below.



Find the area of the minor segment shaded on the diagram.

Give your answer to three significant figures.

[5 marks]

$(x-3)^2 - 9 + (y-4)^2 - 16 = 264$

$\Rightarrow (x-3)^2 + (y-4)^2 = 289$

$r = 17$

Area of sector of circle : $\frac{1}{2} \times 17^2 \times 0.9$

$= 130.05$

Area of triangle : $\frac{1}{2} \times 17^2 \times \sin 0.9$

$= 113.19$

$130.05 - 113.19 = 16.86 = 16.9.$

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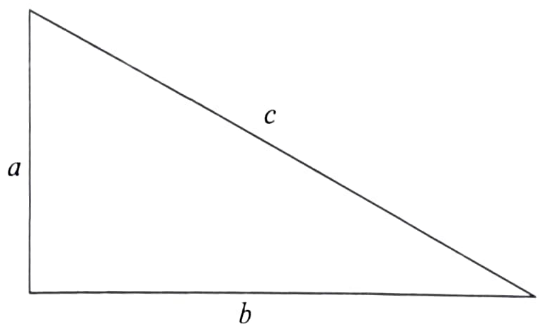
6 The three sides of a right-angled triangle have lengths a, b and c, where $a, b, c \in \mathbb{Z}$



6 (a) State an example where a, b and c are all even.

[1 mark]

$a=6$, $b=8$, $c=10$.

6 (b) Prove that it is **not** possible for all of a, b and c to be odd.

[3 marks]

Assume a and b odd.

$(2m+1)^2 + (2n+1)^2 = c^2$

$= 4m^2+4m+1 + \cancel{4}n^2 + 4n+1 = 2(2m^2+2n^2+2m+2n+1)$

so c^2 is even, so c is even.

If c is odd, then c^2 is odd. Then either a^2 or b^2 must be even, with the other being odd. So either a or b must be odd. Then not all three of a, b and c can be odd.

Turn over for the next question

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7 (a) Express $\frac{4x+3}{(x-1)^2}$ in the form $\frac{A}{x-1} + \frac{B}{(x-1)^2}$

[3 marks]

$\frac{4x+3}{(x-1)^2} = \frac{A}{x-1} + \frac{B}{(x-1)^2}$

$A(x-1) + B = 4x+3$

$\Rightarrow A = 4, B = 7$

$\frac{4x+3}{(x-1)^2} = \frac{4}{x-1} + \frac{7}{(x-1)^2}$,

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7 (b) Show that

$$\int_3^4 \frac{4x+3}{(x-1)^2}\,dx = p + \ln q$$

where p and q are rational numbers.

[5 marks]

$$\int_3^4 \frac{4}{x-1}\,dx + \int_3^4 \frac{7}{(x-1)^2}\,dx$$

$$= \Big[4\ln(x-1)\Big]_3^4 + \left[\frac{-7}{x-1}\right]_3^4$$

$$= \left(4\ln 3 - \frac{7}{3}\right) - \left(4\ln 2 - \frac{7}{2}\right)$$

$$= 4\ln\frac{3}{2} + \frac{7}{6}$$

$$= \frac{7}{6} + \ln\frac{81}{16}$$

Turn over for the next question

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8 A student is conducting an experiment in a laboratory to investigate how quickly liquids cool to room temperature.

A beaker containing a hot liquid at an initial temperature of 75 °C cools so that the temperature, θ °C, of the liquid at time t minutes can be modelled by the equation

$$\theta = 5(4 + \lambda e^{-kt})$$

where λ and k are constants.

After 2 minutes the temperature falls to 68 °C.

8 (a) Find the temperature of the liquid after 15 minutes.

Give your answer to three significant figures.

[7 marks]

$75 = 5(4 + \lambda) \Rightarrow \lambda = 11.$

$68 = 5(4 + 11e^{-2k})$

$e^{-2k} = \dfrac{\frac{68}{5} - 4}{11}$

$\Rightarrow k = -\frac{1}{2} \ln \dfrac{\frac{68}{5} - 4}{11} = 0.0681$

$\theta = 5(4 + 11e^{-15 \times 0.0681})$

$= 39.813 = 39.8\,°C.$

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8 (b) (i) Find the room temperature of the laboratory, giving a reason for your answer.
[2 marks]

$5(4+11\times 0) = 5\times 4 = 20°C$

As t becomes ~~~~ infinitely large, θ approaches room temperature.

8 (b) (ii) Find the time taken in minutes for the liquid to cool to 1 °C above the room temperature of the laboratory.
[2 marks]

$5(4+11e^{-0.0681t}) = 21.$

$20 + 55e^{-0.0681t} = 21$

$\Rightarrow e^{-0.0681t} = \frac{1}{55}$

$\Rightarrow t = \frac{1}{-0.0681} \ln \frac{1}{55}$

$= 58.84$

8 (c) Explain why the model might need to be changed if the experiment was conducted in a different place.
[1 mark]

The room temperature may not be the same as in the lab.

Turn over for the next question

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9 A curve has equation

$$x^2y^2 + xy^4 = 12$$

9 (a) Prove that the curve does not intersect the coordinate axes.

[2 marks]

$x=0: \quad 0y^2 + 0y^4 = 12 \Rightarrow 0 = 12.$

$y=0: \quad 0x^2 + 0x = 12 \Rightarrow 0 = 12$

Neither result is possible, so the curve does not intersect the coordinate axes.

9 (b) (i) Show that $\dfrac{dy}{dx} = -\dfrac{2xy + y^3}{2x^2 + 4xy^2}$

[5 marks]

$2xy^2 + 2x^2y\frac{dy}{dx} + y^4 + 4xy^3\frac{dy}{dx} = 0.$

$2xy^2 + y^4 = -(2x^2y + 4xy^3)\frac{dy}{dx}$

$\Rightarrow \dfrac{dy}{dx} = \dfrac{-(2xy^2 + y^4)}{2x^2y + 4xy^3}.$

$= \dfrac{-(2xy + y^3)}{2x^2 + 4xy^2}.$

Do not write outside the box

9 (b) (ii) Prove that the curve has no stationary points.

[4 marks]

$\frac{dy}{dx} = 0$. Assume there are/is stationary point(s).

$-\frac{2xy + y^3}{2x^2y + 4xy^2} = 0$.

$\Rightarrow 2xy + y^3 = 0 \Rightarrow -2x = y^2 \geq 0$.

$\Rightarrow x < 0$.

$-2x^3 + 4x^3 = 2x^3 = 12$.

$\Rightarrow x^3 = 6$.

But $x < 0$, so this is impossible, so there are no stationary points.

9 (b) (iii) In the case when $x > 0$, find the equation of the tangent to the curve when $y = 1$

[4 marks]

$y = 1 \Rightarrow x^2 + x - 12 = 0$.

$\Rightarrow (x - 3)(x + 4) = 0 \quad \Rightarrow x = 3, -4$.

$x > 0 \Rightarrow x = 3$

$x = 3, y = 1$ gives

$\frac{dy}{dx} = \frac{-(6+1)}{18 + 12} = \frac{-7}{30}$

$y - 1 = \frac{-7}{30}(x - 3)$

$\Rightarrow y = \frac{-7}{30}x + \frac{17}{10}$.

Turn over ▶

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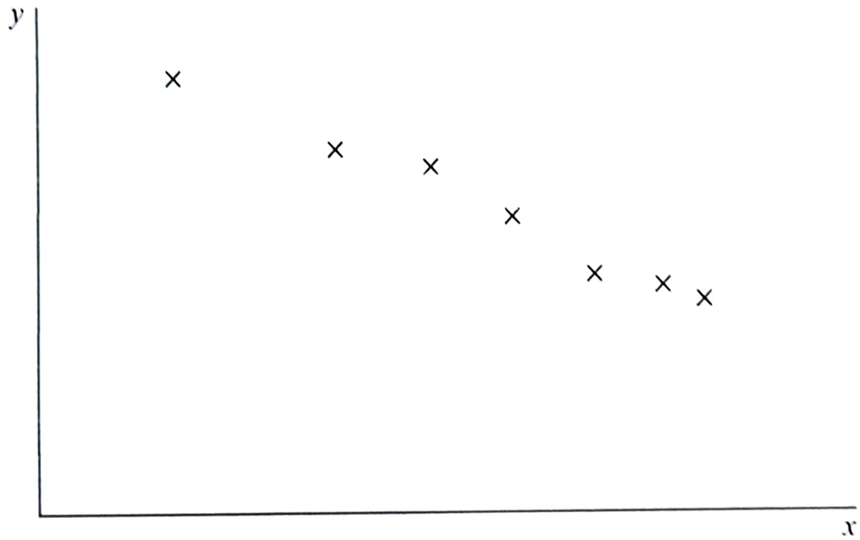
Section B

Answer **all** questions in the spaces provided.

10 Which of the options below best describes the correlation shown in the diagram below?



Tick (✓) **one** box.

[1 mark]

moderate positive	☐
strong positive	☐
moderate negative	☐
strong negative	✓

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11 Lenny is one of a team of people interviewing shoppers in a town centre.

He is asked to survey 50 women between the ages of 18 and 29

Identify the name of this type of sampling.

Circle your answer.

[1 mark]

simple random   stratified   quota   systematic

Turn over for the next question

Do not write outside the box

12 Amelia decides to analyse the heights of members of her school rowing club.

The heights of a random sample of 10 rowers are shown in the table below.

Rower	Jess	Nell	Liv	Neve	Ann	Tori	Maya	Kath	Darcy	Jen
Height (cm)	162	169	172	156	146	161	159	164	157	160

12 (a) Any value more than 2 standard deviations from the mean may be regarded as an outlier.

Verify that Ann's height is an outlier.

Fully justify your answer.

[4 marks]

$\bar{x} = 160.6$, $\sigma_x = 6.81.$

$160.6 - 2 \times 6.81 = 146.98$

$146 < 146.98$

Ann is an outlier.

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12 (b) Amelia thinks she may have written down Ann's height incorrectly.

If Ann's height were discarded, state with a reason what, if any, difference this would make to the mean and standard deviation.

[2 marks]

The mean height would increase (as Ann's height is below the mean).

The standard deviation would decrease (as Ann's height is an outlier).

Turn over for the next question

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13 Patrick is practising his skateboarding skills. On each day, he has 30 attempts at performing a difficult trick.

Every time he attempts the trick, there is a probability of 0.2 that he will fall off his skateboard.

Assume that the number of times he falls off on any given day may be modelled by a binomial distribution.

13 (a) (i) Find the mean number of times he falls off in a day.

[1 mark]

$30 \times 0.2 = 6.$

13 (a) (ii) Find the variance of the number of times he falls off in a day.

[1 mark]

$30 \times 0.2 \times (1-0.2) = 4.8.$

13 (b) (i) Find the probability that, on a particular day, he falls off exactly 10 times.

[2 marks]

$P(X=10) = \binom{30}{10} 0.2^{10}\ 0.8^{20} = 0.0355.$

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13 (b) (ii) Find the probability that, on a particular day, he falls off 5 or more times.

[3 marks]

$\mathbb{P}(X \geq 5) = 1 - \mathbb{P}(X \leq 4)$

$= 1 - 0.2552332547$

$\approx 1 - 0.255$

$= 0.745.$

13 (c) Patrick has 30 attempts to perform the trick on each of 5 consecutive days.

13 (c) (i) Calculate the probability that he will fall off his skateboard at least 5 times on each of the 5 days.

[2 marks]

$0.745^5 = 0.229$

13 (c) (ii) Explain why it may be unrealistic to use the same value of 0.2 for the probability of falling off for all 5 days.

[1 mark]

Probability is likely to decrease as Patrick improves.

Turn over for the next question

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14 A survey was conducted into the health of 120 teachers.

The survey recorded whether or not they had suffered from a range of four health issues in the past year.

In addition, their physical exercise level was categorised as low, medium or high.

50 teachers had a low exercise level, 40 teachers had a medium exercise level and 30 teachers had a high exercise level.

The results of the survey are shown in the table below.

	Low exercise	Medium exercise	High exercise
Back trouble	14	7	10
Stress	38	14	5
Depression	9	2	1
Headache/Migraine	4	5	5

14 (a) Find the probability that a randomly selected teacher:

14 (a) (i) suffers from back trouble and has a high exercise level;

[1 mark]

$\frac{10}{120} = \frac{1}{12}$.

14 (a) (ii) suffers from depression.

[2 marks]

$\frac{9+2+1}{120} = \frac{12}{120} = \frac{1}{10}$.

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14 (a) (iii) suffers from stress, given that they have a low exercise level.

[2 marks]

$\frac{38}{50} = \frac{76}{100} = \frac{19}{25}$.

14 (b) For teachers in the survey with a low exercise level, explain why the events 'suffers from back trouble' and 'suffers from stress' are **not** mutually exclusive.

[2 marks]

$14 + 38 = 52$.

$52 > 50$.

If they were mutually exclusive, there would be more LE teachers with back trouble or stress than LE teachers. This is impossible.

Turn over for the next question

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15 Jamal, a farmer, claims that the larger the rainfall, the greater the yield of wheat from his farm.

He decides to investigate his claim, at the 5% level of significance.

He measures the rainfall in centimetres and the yield in kilograms for a random sample of ten years.

He correctly calculates the product moment correlation coefficient between rainfall and yield for his sample to be 0.567

The table below shows the critical values for correlation coefficients for a sample size of 10 for different significance levels, for both 1- and 2-tailed tests.

1-tailed test significance level	5%	2.5%	1%	0.5%
2-tailed test significance level	10%	5%	2%	1%
Critical value	0.549	0.632	0.716	0.765

Determine what Jamal's conclusion to his investigation should be, justifying your answer.

[3 marks]

$0.567 > 0.549.$

There is sufficient evidence to suggest that a larger amount of rainfall causes a greater wheat yield.

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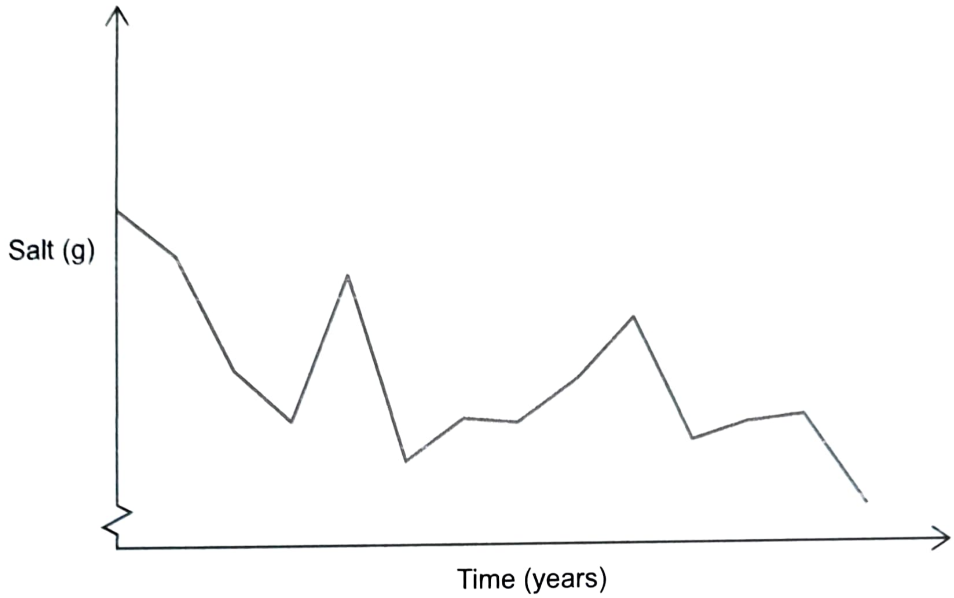
16 (a) The graph below shows the amount of salt, in grams, purchased per person per week in England between 2001–02 and 2014, based upon the Large Data Set.



Meera and Gemma are arguing about what this graph shows.

Meera believes that the amount of salt consumed by people decreased greatly during this period.

Gemma says that this is not the case.

Using your knowledge of the Large Data Set, give **two** reasons why Gemma may be correct.

[2 marks]

y-axis does not start at zero.

The graph is for salt purchased as an individual product, it does not necessarily apply to amount of salt consumed.

16 (b) It is known that the mean amount of sugar purchased per person in England in 2014 was 78.9 grams, with a standard deviation of 25.0 grams.

In 2018, a sample of 918 people had a mean of 80.4 grams of sugar purchased per person.

Investigate, at the 5% level of significance, whether the mean amount of sugar purchased per person in England has changed between 2014 and 2018.

Assume that the survey data is a random sample taken from a normal distribution and that the standard deviation has remained the same.

[6 marks]

$H_0: \mu = 78.9$, $H_1: \mu \neq 78.9$

$$\frac{80.4 - 78.9}{25/\sqrt{918}} = \frac{1.5}{25/\sqrt{918}} = 1.818.$$

Critical value is at $z = 1.96$

$1.818 < 1.96$

No reason to reject H_0.

Question 16 continues on the next page

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16 (c) Another test is performed to determine whether the mean amount of fat purchased per person has changed between 2014 and 2018.

At the 10% significance level, the null hypothesis is rejected.

With reference to the 10% significance level, explain why it is not necessarily true that there has been a change.

[2 marks]

There is a 10% that the null hypothesis was rejected incorrectly.

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17 Elizabeth's Bakery makes brownies.

It is known that the mass, X grams, of a brownie may be modelled by a normal distribution.

10% of the brownies have a mass less than 30 grams.

80% of the brownies have a mass greater than 32.5 grams.

17 (a) Find the mean and standard deviation of X.

[7 marks]

$\mathbb{P}\left(Z < \frac{30-\mu}{\sigma}\right) = 0.1$

$\mathbb{P}\left(Z > \frac{32.5-\mu}{\sigma}\right) = 0.8 \Rightarrow P\left(Z < \frac{32.5-\mu}{\sigma}\right) = 0.2$

$\Rightarrow \frac{30-\mu}{\sigma} = -1.2816, \quad \frac{32.5-\mu}{\sigma} = -0.8416$

$\Rightarrow 30-\mu = -1.2816\sigma, \quad 32.5-\mu = -0.8416\sigma.$

$\Rightarrow 2.5 = 0.44\sigma$

$\Rightarrow \sigma = \frac{2.5}{0.44} = 5.68$

$\Rightarrow \mu = 37.3.$

17 (b) (i) Find $P(X \neq 35)$

[1 mark]

1.

17 (b) (ii) Find $P(X < 35)$

[2 marks]

$\mathbb{P}(X < 35) = \mathbb{P}\left(Z < \frac{35 - 37.3}{5.68}\right)$

$= \mathbb{P}(Z < -0.4016)$

$= 0.3\cancel{4}44.$

17 (c) Brownies are baked in batches of 13.

Calculate the probability that, in a batch of brownies, no more than 3 brownies are less than 35 grams.

You may assume that the masses of brownies are independent of each other.

[2 marks]

$Y \sim B(13, 0.344)$

$\mathbb{P}(Y \leq 3) = 0.294.$

END OF QUESTIONS