Please check the examination detalls below before entering your candidate information


## Tuesday 21 May 2019

\section*{| Morning (Time: 1 hour 30 minutes) | Paper Reference 1MA1/1H |
| :--- | :--- |}

## Mathematics

## Paper 1 (Non-Calculator) Higher Tier

You must have: Ruler graduated in centimetres and millimetres, protractor, pair of compasses, pen, HB pencil, eraser.
Tracing paper may be used.

## Instructions

- Use black ink or ball-point pen.
- Fill in the boxes at the top of this page with your name, centre number and candidate number.
- Answer all questions.
- Answer the questions in the spaces provided - there may be more space than you need.
- You must show all your working.

- Diagrams are NOT accurately drawn, unless otherwise indicated.
- Calculators may not be used.


## Information

- The total mark for this paper is 80
- The marks for each question are shown in brackets - use this as a guide as to how much time to spend on each question.


## Advice

- Read each question carefully before you start to answer it.
- Keep an eye on the time.
- Try to answer every question.
- Check your answers if you have time at the end.


## Answer ALL questions. <br> Write your answers in the spaces provided. <br> You must write down all the stages in your working.

1 There are only blue cubes, red cubes and yellow cubes in a box.
The table shows the probability of taking at random a blue cube from the box.

| Colour | blue | red | yellow |
| :--- | :---: | :---: | :---: |
| Probability | 0.2 | 0.4 | 0.4 |

The number of red cubes in the box is the same as the number of yellow cubes in the box.
(a) Complete the table.

$$
\frac{1-0.2}{2}=0.4 \text { each }
$$

There are 12 blue cubes in the box.
(b) Work out the total number of cubes in the box.

$$
\frac{12}{0.2}=60
$$

2 Deon needs 50 g of sugar to make 15 biscuits.
She also needs
three times as much flour as sugar
two times as much butter as sugar
Deon is going to make 60 biscuits.
(a) Work out the amount of flour she needs.

$$
\begin{aligned}
& \frac{60}{15}=4 \text {. We need } 4 \text { times as much of each } \\
& \text { ingredient. } \\
& \text { Flour: } 50 \mathrm{~g} \times 3 \times 4=600 \mathrm{~g}
\end{aligned}
$$

Deon has to buy all the butter she needs to make 60 biscuits.
She buys the butter in 250 g packs.
(b) How many packs of butter does Deon need to buy?

$$
\begin{aligned}
50 g \times 2 \times 4 & =400 \mathrm{~g} \\
\frac{400}{250} & =1.6 \\
& \Rightarrow 2 \text { packs needed. }
\end{aligned}
$$

3 Find the highest common factor (HCF) of 72 and 90


4 'The diagram shows the plan, front elevation and side elevation of a solid shape, drawn on a centimetre grid.



Front elevation


Side elevation

In the space below, draw a sketch of the solid shape.
Give the dimensions of the solid on your sketch.

(Total for Question 4 is $\mathbf{2}$ marks)

5


Shape A can be transformed to shape B by a reflection in the $x$－axis followed by a translation $\binom{c}{d}$

Find the value of $c$ and the value of $d$ ．

$$
\begin{aligned}
& c=-6 \\
& d=\quad-1
\end{aligned}
$$

6 A shop sells packs of black pens, packs of red pens and packs of green pens.
There are
2 pens in each pack of black pens
5 pens in each pack of red pens
6 pens in each pack of green pens
On Monday,

$$
\begin{aligned}
& \text { number of packs } \\
& \text { of black pens sold }
\end{aligned}: \begin{aligned}
& \text { number of packs } \\
& \text { of red pens sold }
\end{aligned}: \begin{aligned}
& \text { number of packs } \\
& \text { of green pens sold }
\end{aligned}=7: 3: 4
$$

A total of 212 pens were sold.
Work out the number of green pens sold.

$$
\begin{gathered}
(2 \times 7):(3 \times 5):(4 \times 6)=14: 15: 24 \\
14 \times 15+24=53
\end{gathered}
$$

$$
\text { Green: } \quad 4 \times 6 \times \frac{212}{53}=96
$$

7 Here are two rectangles.


$$
\begin{aligned}
& Q R=10 \mathrm{~cm} \\
& B C=P Q
\end{aligned}
$$

The perimeter of $A B C D$ is 26 cm
The area of $P Q R S$ is $45 \mathrm{~cm}^{2}$
Find the length of $A B$.
Area $P Q R S=45=10 x$

$$
\Rightarrow x=\frac{45}{10}=4.5 \mathrm{~cm}
$$

Perimeter $A B C D: 26=4.5+4.5+2 y$

$$
y=\frac{26-9}{2}=8.5 \mathrm{~cm}
$$

8 (a) Work out an estimate for the value of $\sqrt{63.5 \times 101.7}$

$$
\approx \sqrt{64 \times 100}=\sqrt{64} \sqrt{100}=8 \times 10=80
$$

$(2.3)^{6}=148$ correct to 3 significant figures.
(b) Find the value of $(0.23)^{6}$ correct to 3 significant figures.

(c) Find the value of $5^{-2}$

9 Work out $3 \frac{1}{2} \times 1 \frac{3}{5}$
Give your answer as a mixed number in its simplest form.

$$
\frac{7}{2} \times \frac{8}{5}=\frac{56}{10}=5 \frac{6}{10}=5 \frac{3}{5}
$$

10 The graphs with equations $3 y+2 x=\frac{1}{2}$ and $2 y-3 x=-\frac{113}{12}$ have been drawn on the grid below.


Using the graphs, find estimates of the solutions of the simultaneous equations

$$
\begin{aligned}
& 3 y+2 x=\frac{1}{2} \\
& 2 y-3 x=-\frac{113}{12} \\
& x=\quad 2.25 \\
& y=\ldots \quad-1.30
\end{aligned}
$$

11 A bus company recorded the ages, in years, of the people on coach A and the people on coach B.

Here are the ages of the 23 people on coach A.

| 41 | 42 | 44 | 48 | 52 | 53 | 53 | 53 | 56 | 57 | 57 | 59 |
| :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- |
| 60 | 61 | 63 | 64 | 64 | 66 | 67 | 69 | 74 | 77 | 79 |  |

(a) Complete the table below to show information about the ages of the people on coach A .

| Median | 59 |
| :--- | :---: |
| Lower quartile | 53 |
| Upper quartile | 66 |
| Least age | 41 |
| Greatest age | 79 |

Here is some information about the ages of the people on coach B.

| Median | 70 |
| :--- | :---: |
| Lower quartile | 54 |
| Upper quartile | 73 |
| Least age | 42 |
| Greatest age | 85 |

Richard says that the people on coach A are younger than the people on coach B.
(b) Is Richard correct?

You must give a reason for your answer.
Yes, all statistics are laver for coach A.

Richard says that the people on coach A vary more in age than the people on coach B.
(c) Is Richard correct?

You must give a reason for your answer.
No, the $I Q R$ for coach $A$ is 13 , but is 19 for coach B. $\qquad$ A smaller $\qquad$ ar $\qquad$ indic odes that the variance $\qquad$ in age is $\qquad$ smaller on coach A. (1)
(Total for Question 11 is $\mathbf{4}$ marks)
12 Here are three spheres.


P


Q

R

The volume of sphere $\mathbf{Q}$ is $50 \%$ more than the volume of sphere $\mathbf{P}$.
The volume of sphere $\mathbf{R}$ is $50 \%$ more than the volume of sphere $\mathbf{Q}$.
Find the volume of sphere $\mathbf{P}$ as a fraction of the volume of sphere $\mathbf{R}$.

$$
\begin{aligned}
& \frac{3}{2} P=Q \\
& \frac{3}{2} Q=R . \\
& \Rightarrow \frac{9}{4} P=R . \\
& \Rightarrow \frac{P}{R}=\frac{4}{9}
\end{aligned}
$$

13 Given that $n$ can be any integer such that $n>1$, prove that $n^{2}-n$ is never an odd number.

$$
n^{2}-n=n(n-1)
$$

$n$ and $n-1$ are consecutive integers, so, ane must be add, and the other, even.

Their product must be even.
(Total for Question 13 is 2 marks)

14 Find the exact value of $\tan 30^{\circ} \times \sin 60^{\circ}$
Give your answer in its simplest form.

$$
\begin{aligned}
\tan 30^{\circ} \times \sin 60^{\circ} & =\tan 30^{\circ} \times \cos 30^{\circ} \\
& =\sin 30^{\circ} \\
& =\frac{1}{2}
\end{aligned}
$$

Alternatively,

$$
\begin{aligned}
& \text { Alternative } \\
& \qquad \tan 30^{\circ}=\frac{1}{\sqrt{3}}, \sin 60^{\circ}=\frac{\sqrt{3}}{2} \\
& \tan 30^{\circ} \times \sin 60^{\circ}= \\
& \frac{1}{\sqrt{3}} \times \frac{\sqrt{3}}{2}=\frac{1}{2}
\end{aligned}
$$

(Total for Question 14 is $\mathbf{2}$ marks)

15 The diagram shows a solid shape.
The shape is a cone on top of a hemisphere.


The height of the cone is 10 cm .
The base of the cone has a diameter of 6 cm .
The hemisphere has a diameter of 6 cm .
The total volume of the shape is $k \pi \mathrm{~cm}^{3}$, where $k$ is an integer.
Work out the value of $k$.
Hemisphere: $\frac{2}{3} \times \pi \times 3^{3}=18 \pi$.

$$
\text { Cone: } \frac{1}{3} \times \pi \times 3^{2} \times 10=30 \pi
$$

$$
18 \pi+30 \pi=48 \pi
$$

$$
k=48
$$

16 There are three dials on a combination lock.
Each dial can be set to one of the numbers $1,2,3,4,5$
The three digit number 553 is one way the dials can be set, as shown in the diagram.

(a) Work out the number of different three digit numbers that can be set for the combination lock.

$$
5^{3}=125
$$

(b) How many of the possible three digit numbers have three different digits?

$$
5 \times 4 \times 3=60
$$

17 Given that

$$
x^{2}:(3 x+5)=1: 2
$$

find the possible values of $x$.

$$
\begin{aligned}
& 2 x^{2}=3 x+5 . \\
\Rightarrow & 2 x^{2}-3 x-5=0 \\
& (2 x-5)(x+1)=0 \\
\Rightarrow & x=-1,5 / 2
\end{aligned}
$$

$$
x=-1,5 / 2
$$

(Total for Question 17 is $\mathbf{4}$ marks)

18 (a) Express $\sqrt{3}+\sqrt{12}$ in the form $a \sqrt{3}$ where $a$ is an integer.

$$
\begin{aligned}
\sqrt{3}+\sqrt{12}=\sqrt{3}+\sqrt{4 \times 3} & =\sqrt{3}+2 \sqrt{3} \\
& =3 \sqrt{3}
\end{aligned}
$$

(b) Express $\left(\frac{1}{\sqrt{3}}\right)^{7}$ in the form $\frac{\sqrt{b}}{c}$ where $b$ and $c$ are integers.

$$
\sqrt{3}^{7}=27 \sqrt{3}
$$

$$
\left(\frac{1}{\sqrt{3}}\right)^{7}=\frac{1}{27 \sqrt{3}} x=\frac{1}{27 \sqrt{3}} \times \frac{\sqrt{3}}{\sqrt{3}}=\frac{\sqrt{3}}{81}
$$

19 Given that $\quad x^{2}-6 x+1=(x-a)^{2}-b \quad$ for all values of $x$,
(i) find the value of $a$ and the value of $b$.

$$
\begin{aligned}
x^{2}-6 x+1 & =(x-3)^{2}-9+1 \\
& =(x-3)^{2}-8
\end{aligned}
$$

$$
\begin{aligned}
& a=\quad 3 \\
& b=\quad 8
\end{aligned}
$$

(ii) Hence write down the coordinates of the turning point on the graph of $y=x^{2}-6 x+1$
$20 h$ is inversely proportional to $p$ $p$ is directly proportional to $\sqrt{1}$
Given that $h=10$ and $t=144$ when $p=6$
find a formula for $h$ in terms of $t$

$$
\begin{aligned}
& h \alpha \frac{1}{P}, P \alpha \sqrt{t} \\
& \Rightarrow h=\frac{k}{p}, \quad P=K \sqrt{t} \\
& \Rightarrow 10=\frac{k}{6}, \quad K=\frac{1}{2} \\
& \Rightarrow 12=60, \quad \frac{120}{\sqrt{t}} \\
& h=\frac{60}{\frac{1}{2} \sqrt{t}}=\frac{12}{}
\end{aligned}
$$

$$
h=\frac{120}{\sqrt{t}}
$$

(Total for Question 20 is $\mathbf{4}$ marks)

21 The functions $f$ and $g$ are such that

$$
f(x)=3 x-1 \quad \text { and } \quad g(x)=x^{2}+4
$$

(a) Find $\mathrm{f}^{-1}(x)$

Let $\quad f(x)=y$.

$$
\begin{align*}
& y=3 x-1 \\
& y+1=3 x \\
& x=\frac{y+1}{3} \\
& \Rightarrow f^{-1}(x)=\frac{x+1}{3} \tag{2}
\end{align*}
$$

$$
f^{-1}(x)=\frac{x+1}{3}
$$

Given that $\mathrm{fg}(x)=2 \mathrm{gf}(x)$,
(b) show that $15 x^{2}-12 x-1=0$

$$
\begin{aligned}
& \text { show that } 15 x^{2}-12 x-1=0 \\
& f g(x)=2 g f(x) \\
& 3\left(x^{2}+4\right)-1=2\left((3 x-1)^{2}+4\right) \\
& \Rightarrow 3 x^{2}+11=2\left(9 x^{2}-6 x+5\right) \\
& 3 x^{2}+11=18 x^{2}-12 x+10 \\
& \Rightarrow 18 x^{2}-3 x^{2}-12 x+10-11=0 \\
& \Rightarrow 15 x^{2}-12 x-1=0
\end{aligned}
$$

22 There are only $r$ red counters and $g$ green counters in a bag.
A counter is taken at random from the bag.
The probability that the counter is green is $\frac{3}{7}$
The counter is put back in the bag.
2 more red counters and 3 more green counters are put in the bag.
A counter is taken at random from the bag.
The probability that the counter is green is $\frac{6}{13}$
Find the number of red counters and the number of green counters that were in the bag originally.

$$
\begin{aligned}
& \frac{9}{r+9}=\frac{3}{7} \\
& \text {, } \frac{9+3}{r+2+g+3}=\frac{6}{13} \\
& \Rightarrow 7 g=3(r+g) \\
& \Rightarrow 3 r=4 \mathrm{~g} . \\
& \Rightarrow 6 r=8 \mathrm{~g} . \\
& 8 g=7 g+9 \\
& \Rightarrow g=9 \text {. } \\
& \Rightarrow r=12 \\
& \Rightarrow \quad 6(r+g+5)=13(g+3) \\
& \begin{array}{c}
\Rightarrow 6 r+6 g+30=13 g+39 \\
\Rightarrow 6 r=7 g+9
\end{array} \\
& \begin{array}{c}
\Rightarrow 6 r+6 g+30=13 g+39 \\
\Rightarrow 6 r=7 g+9
\end{array} \\
& \text { ( })
\end{aligned}
$$

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