MODEL SOLUTIONS

AQA

AS
MATHEMATICS
Paper 2

Wednesday 22 May 2019   Morning   Time allowed: 1 hour 30 minutes

Materials
- You must have the AQA Formulae for A-level Mathematics booklet.
- You should have a graphical or scientific calculator that meets the requirements of the specification.

Instructions
- Use black ink or black ball-point pen. Pencil should only be used for drawing.
- Fill in the boxes at the top of this page.
- Answer **all** questions.
- You must answer each question in the space provided for that question. If you require extra space, use an AQA supplementary answer book; do **not** use the space provided for a different question.
- Show all necessary working; otherwise marks for method may be lost.
- Do all rough work in this book. Cross through any work that you do not want to be marked.

Information
- The marks for questions are shown in brackets.
- The maximum mark for this paper is 80.

Advice
- Unless stated otherwise, you may quote formulae, without proof, from the booklet.
- You do not necessarily need to use all the space provided.

For Examiner's Use	
Question	Mark
1	
2	
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9	
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11	
12	
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14	
15	
16	
TOTAL	

Section A

Answer **all** questions in the spaces provided.

1   Find the gradient of the curve $y = e^{-3x}$ at the point where it crosses the y-axis.

Circle your answer.

[1 mark]

       $\boxed{-3}$ -1 1 3

2   Find the centre of the circle $x^2 + y^2 + 4x - 6y = 12$

Tick (✓) **one** box.

[1 mark]

$(-2, -3)$ ☐

$(-2, 3)$ ☑

$(2, -3)$ ☐

$(2, 3)$ ☐

3 It is given that $\sin \theta = -0.1$ and $180° < \theta < 270°$

Find the exact value of $\cos \theta$

[2 marks]

$$\sin^2 \theta + \cos^2 \theta = 1$$

$$(-0.1)^2 + \cos^2 \theta = 1$$

$$\cos^2 \theta = 0.99$$

$$\cos \theta = -\frac{3\sqrt{11}}{10}$$

4 Show that, for $x > 0$

$$\log_{10} \frac{x^4}{100} + \log_{10} 9x - \log_{10} x^3 \equiv 2(-1 + \log_{10} 3x)$$

[4 marks]

LHS: $\log_{10} \frac{x^4}{100} + \log_{10} 9x - \log_{10} x^3$

$$= 4\log_{10} x - \log_{10} 100 + \log_{10} 9 + \log_{10} x - 3\log_{10} x$$

$$= 2\log_{10} x - 2\log_{10} 10 + 2\log_{10} 3$$

$$= 2(\log_{10} x - \log_{10} 10 + \log_{10} 3)$$
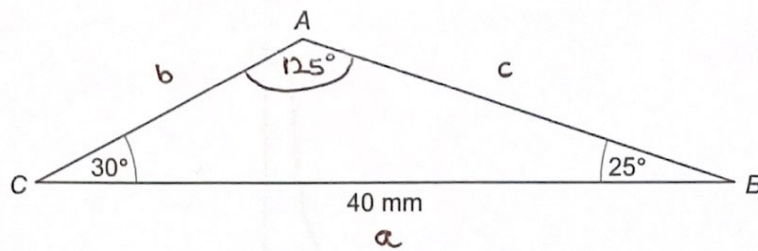
$$= 2(\log_{10} x - 1 + \log_{10} 3)$$

$$= 2(-1 + \log_{10} 3x)$$

Turn over for the next question

Turn over ▶

5   A triangular prism has a cross section *ABC* as shown in the diagram below.

Angle *ABC* = 25°

Angle *ACB* = 30°

BC = 40 millimetres.

The length of the prism is 300 millimetres.

Calculate the volume of the prism, giving your answer to three significant figures.
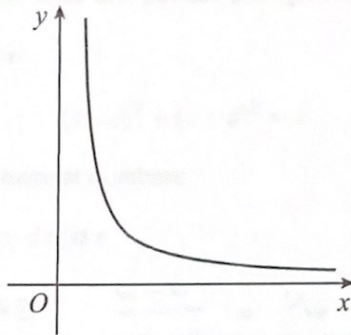
[4 marks]

$A = 180 - 30 - 25 = 125°$

$\dfrac{c}{\sin 30} = \dfrac{40}{\sin 125} \Rightarrow c = \dfrac{40 \sin 30}{\sin 125} = 24.415...$

$\text{Area} = \frac{1}{2} \times 40 \times (24.415...) \times \sin 25$

$= 206.4 \, mm^2$

$\text{Volume} = 206.4 \times 300 = 61900 \, mm^3$

6 A curve has equation $y = \dfrac{2}{x\sqrt{x}}$

The region enclosed between the curve, the x-axis and the lines $x = 1$ and $x = a$ has area 3 units.

Given that $a > 1$, find the value of a.

Fully justify your answer.

[5 marks]

$$y = \frac{2}{x\sqrt{x}} = 2x^{-\frac{3}{2}}$$

$$\rightarrow \quad 3 = \int_1^a 2x^{-3/2}\, dx = \left[-4x^{-1/2} \right]_1^a$$

$$\Rightarrow \quad \left(-4(a)^{-1/2} \right) - \left(-4(1)^{-1/2} \right) = 3$$

$$\Rightarrow \quad -4a^{-1/2} + 4 = 3$$

$$\Rightarrow \quad a^{-1/2} = 1/4$$

$$\Rightarrow \quad a = 16$$

Turn over ▶

7   The points A $(a, 3)$ and B $(10, 6)$ lie on a circle.

AB is a diameter of the circle and passes through the point $(2, 4)$

The circle has equation

$$(x - c)^2 + (y - d)^2 = e$$

where c, d and e are rational numbers.

Find the values of a, c, d and e.

[6 marks]

Gradient AB : $\dfrac{4-6}{2-10} = \frac{1}{4}$

So $\dfrac{3-6}{a-10} = \frac{1}{4}$ $\Rightarrow -3 = \frac{a}{4} - \frac{10}{4}$

$\Rightarrow a = -2$

$|AB| = 12^2 +$

Midpoint of $AB = \left(\dfrac{-2+10}{2}, \dfrac{3+6}{2}\right) = (4, 4.5)$

So $c = 4, d = 4.5$

Radius$^2 = 6^2 + 1.5^2 = 38.25$

So $e = 38.25$

$a = -2$

$c = 4$

$d = 4.5$

$e = 38.25$

8   A curve has equation

$$y = x^3 + px^2 + qx - 45$$

The curve passes through point R (2, 3)

The gradient of the curve at R is 8

8 (a)   Find the value of p and the value of q.

[5 marks]

Sub in R : $3 = 2^3 + p2^2 + 2q - 45$

$\Rightarrow 3 = 8 + 4p + 2q - 45$

$\Rightarrow 40 = 4p + 2q$ ①

$dy/dx = 3x^2 + 2px + q$

$dy/dx = 8$ when $x = 2$ also

So $3(2)^2 + 2p(2) + q = 8$

$\Rightarrow 12 + 4p + q = 8$

$\Rightarrow 4p = -4 - q$ ②

Sub ② into ① : $40 = (-4 - q) + 2q$

$\Rightarrow 40 = -4 + q$

$\Rightarrow q = 44$

sub $q = 44$ into ② : $4p = -4 - 44$

$\Rightarrow p = \dfrac{-48}{4} = -12$

$\boxed{\begin{array}{l} p = -12 \\ q = 44 \end{array}}$

8 (b) Calculate the area enclosed between the normal to the curve at R and the coordinate axes.

[5 marks]

Normal gradient $= -\frac{1}{8}$

Normal equation : $y - 3 = -\frac{1}{8}(x - 2)$

$$\Rightarrow y = -\frac{1}{8}x + \frac{13}{4}$$

when $x = 0$, $y = \frac{13}{4}$

when $y = 0$, $x = 26$

Area $= \frac{1}{2} \times 26 \times \frac{13}{4} = 42.25$

9 A curve C has equation $y = f(x)$ where

$$f(x) = (x-2)(x-3)^2$$

9 (a) Find the exact coordinates of the turning points of C.

Determine the nature of each turning point.

Fully justify your answer.

[8 marks]

$$f(x) = (x-2)(x-3)^2 = (x-2)(x^2-6x+9)$$

$$= x^3 - 6x^2 + 9x - 2x^2 + 12x - 18$$

$$= x^3 - 8x^2 \overset{+21x}{\cancel{18x}} - 18$$

$$f'(x) = 3x^2 - 16x + 21$$

let $f'(x) = 0 \rightarrow 3x^2 - 16x + 21 = 0$

$x = 3$ and $7/3$

$$f''(x) = 6x - 16$$

$f''(3) = 2 > 0 \rightarrow$ min point

$f''(7/3) = -2 < 0 \rightarrow$ max point

Turning points : $(3, 0)$ min

$(7/3, \frac{4}{27})$ max

9 (b) State the coordinates of the turning points of the curve

$$y = f(x + 1) - 4$$

[2 marks]

$(2, -4)$ and $\left(\frac{4}{3}, -\frac{104}{27}\right)$

Turn over for the next question

Turn over ▶

10 As part of an experiment, Zena puts a bucket of hot water outside on a day when the outside temperature is 0°C.

She measures the temperature of the water after 10 minutes and after 20 minutes. Her results are shown below.

Time (minutes)	10	20
Temperature (degrees Celsius)	30	12

Zena models the relationship between θ, the temperature of the water in °C, and t, the time in minutes, by

$$\theta = A \times 10^{-kt}$$

where A and k are constants.

10 (a) Using $t = 0$, explain how the value of A relates to the experiment.

[1 mark]

A is the starting temperature.

10 (b) Show that

$$\log_{10} \theta = \log_{10} A - kt$$

[1 mark]

$\log_{10} \theta = \log_{10} \left(A \times 10^{-kt} \right)$

$= \log_{10} A + \log_{10} 10^{-kt}$

$= \log_{10} A - kt \log_{10} 10$

$= \log_{10} A - kt$

10 (c) Using Zena's results, calculate the values of A and k.

[4 marks]

$30 = A \times 10^{-10k} \Rightarrow \log_{10} 30 = \log_{10} A - 10k$

$12 = A \times 10^{-20k} \Rightarrow \log_{10} 12 = \log_{10} A - 20k$

$$\Rightarrow \log_{10} \frac{30}{12} = 10k$$

$$\Rightarrow k = \frac{1}{10} \log_{10}(2.5)$$

$$= 0.0398$$

$$A = 75$$

10 (d) Zena states that the temperature of the water will be less than 1°C after 45 minutes.

Determine whether the model supports this statement.

[3 marks]

$$\theta = 75 \times 10^{-0.0398 \times 45}$$

$$= 1.213\ldots$$

So model does not support Zena's Statement.

10 (e) Explain why Zena's model is unlikely to accurately give the value of θ after 45 minutes.

[1 mark]

Outside temperature may have changed after 45 minutes.

Turn over ▶

Section B

Answer **all** questions in the spaces provided.

11 A survey is undertaken to find out the most popular political party in London.

The first 1100 available people from London are surveyed.

Identify the name of this type of sampling.

Circle your answer.

[1 mark]

simple random   (opportunity)   stratified   quota

12 Manny is studying the price and number of pages of a random sample of books.

He calculates the value of the product moment correlation coefficient between the price and number of pages in each book as 1.05

Which of the following best describes the value 1.05?

Tick (✓) **one** box.

[1 mark]

definitely correct   ☐

probably correct   ☐

probably incorrect   ☐

definitely incorrect   ☑

13 Denzel wants to buy a car with a propulsion type **other than** petrol or diesel.

He takes a sample, from the Large Data Set, of the CO_2 emissions, in g/km, of cars with one particular propulsion type.

The sample is as follows

| 82 | 13 | 96 | 49 | 96 | 92 | 70 | 81 |

13 (a) Using your knowledge of the Large Data Set, state which propulsion type this sample is for, giving a reason for your answer.

[2 marks]

Electric / Petrol

⌐→ only category with this many values.

13 (b) Calculate the mean of the sample.

[1 mark]

72.375

13 (c) Calculate the standard deviation of the sample.

[1 mark]

28.7

13 (d)   Denzel claims that the value 13 is an outlier.

13 (d) (i)   Any value more than 2 standard deviations from the mean can be regarded as an outlier.

Verify that Denzel's claim is correct.

[1 mark]

$$72.4 - 2 \times 28.7 = 15 > 13$$

13 (d) (ii)   State what effect, if any, removing the value 13 from the sample would have on the standard deviation.

[1 mark]

Standard deviation would decrease.

Turn over for the next question

Turn over ▶

14 A probability distribution is given by

$$P(X = x) = c(4 - x), \text{ for } x = 0, 1, 2, 3$$

where c is a constant.

14 (a) Show that $c = \dfrac{1}{10}$

[2 marks]

$$4c + 3c + 2c + c = 1$$

$$10c = 1$$

$$c = \frac{1}{10}$$

14 (b) Calculate $P(X \geq 1)$

[2 marks]

$$3c + 2c + c = 6c$$

$$P(X \geq 1) = 0.6$$

15 Two independent events, A and B, are such that

$$P(A) = 0.2$$

$$P(A \cup B) = 0.8$$

15 (a) (i) Find $P(B)$

[4 marks]

$P(A) \times P(B) = P(A \cap B)$ ~~not~~

~~$P(B) = 1 - 0.8$ etc.~~

$P(A \cup B) = P(A) + P(B) - P(A \cap B)$

$0.8 = 0.2 + P(B) - 0.2 \, P(B)$

$0.6 = 0.8 \, P(B)$

$P(B) = 0.75$

15 (a) (ii) Find $P(A \cap B)$

[1 mark]

$P(A) \times P(B) = P(A \cap B) = 0.2 \times 0.75 = 0.15$

15 (b) State, with a reason, whether or not the events A and B are mutually exclusive.

[1 mark]

$P(A \cap B) \neq 0$ so A and B are not mutually exclusive.

16 Andrea is the manager of a company which makes mobile phone chargers.

In the past, she had found that 12% of all chargers are faulty.

16 (a) Andrea decides to move the manufacture of chargers to a different factory.

Andrea tests 60 of the new chargers and finds that 4 chargers are faulty.

Investigate, at the 10% level of significance, whether the proportion of faulty chargers has reduced.

[7 marks]

$H_0: p = 0.12$ where p = proportion of faulty chargers.

$H_1: p < 0.12$

Under H_0: $X \sim B(60, 0.12)$

$P(X \leq 4) = 0.139$

$0.139 > 0.1$

Therefore we accept H_0, as there is insufficient evidence to suggest that the proportion of faulty chargers has reduced.

16 (b) State, **in context**, two assumptions that are necessary for the distribution that you have used in part **(a)** to be valid.

[2 marks]

1) The probability of a faulty charger is fixed.

2) A charger being faulty is independent of any other charge being faulty.

END OF QUESTIONS