MODEL SOLUTIONS

AQA

Please write clearly in block capitals.

Centre number	
Candidate number	
Surname	
Forename(s)	
Candidate signature	

AS MATHEMATICS

Paper 1

Wednesday 15 May 2019 Morning Time allowed: 1 hour 30 minutes

Materials
- You must have the AQA Formulae for A-level Mathematics booklet.
- You should have a graphical or scientific calculator that meets the requirements of the specification.

Instructions
- Use black ink or black ball-point pen. Pencil should only be used for drawing.
- Fill in the boxes at the top of this page.
- Answer **all** questions.
- You must answer each question in the space provided for that question. If you require extra space, use an AQA supplementary answer book; do **not** use the space provided for a different question.
- Show all necessary working; otherwise marks for method may be lost.
- Do all rough work in this book. Cross through any work that you do not want to be marked.

Information
- The marks for questions are shown in brackets.
- The maximum mark for this paper is 80.

Advice
- Unless stated otherwise, you may quote formulae, without proof, from the booklet.
- You do not necessarily need to use all the space provided.

For Examiner's Use	
Question	Mark
1	
2	
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TOTAL	

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Section A

Answer **all** questions in the spaces provided.

1 State the number of solutions to the equation $\tan 4\theta = 1$ for $0° < \theta < 180°$

Circle your answer.

[1 mark]

1 2 4 8

2 Dan believes that

for every positive integer n, at least one of $2^n - 1$ and $2^n + 1$ is prime.

Which value of n shown below is a counter example to Dan's belief?

Circle your answer.

[1 mark]

$n = 3$ $n = 4$ $n = 5$ $n = 6$

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3 It is given that $(x+1)$ and $(x-3)$ are two factors of $f(x)$, where

$$f(x) = px^3 - 3x^2 - 8x + q$$

3 (a) Find the values of p and q.

[3 marks]

$x = -1$ is a solution of $f(x) = 0$

$\hookrightarrow p(-1)^3 - 3(-1)^2 - 8(-1) + q = 0$

$\Rightarrow -p - 3 + 8 + q = 0$

$\Rightarrow p = q + 5$ ①

$x = 3$ is also a solution of $f(x) = 0$

$\hookrightarrow p(3)^3 - 3(3)^2 - 8(3) + q = 0$

$\Rightarrow 27p - 27 - 24 + q = 0$

$\Rightarrow 27p = 51 - q$ ②

Sub ① into ②

$27(q+5) = 51 - q$

$27q + 135 = 51 - q$

$28q = -84$

$\boxed{q = -3}$

Sub. back into ①

$\boxed{p = -3 + 5 = 2}$

3 (b) Fully factorise $f(x)$.

[2 marks]

$f(x) = 2x^3 - 3x^2 - 8x - 3$

$(x+1)(x-3) = x^2 - 2x - 3$

$(2x+1)(x^2 - 2x - 3) = 2x^3 - 4x^2 + x^2 - 2x - 6x - 3$

$= 2x^3 - 3x^2 - 8x - 3$

$(x+1)(x-3)(2x+1)$

Turn over ▶

Do not write outside the box

4 Show that $\frac{\sqrt{6}}{\sqrt{3}-\sqrt{2}}$ can be expressed in the form $m\sqrt{n}+n\sqrt{m}$, where m and n are integers.

Fully justify your answer.

[4 marks]

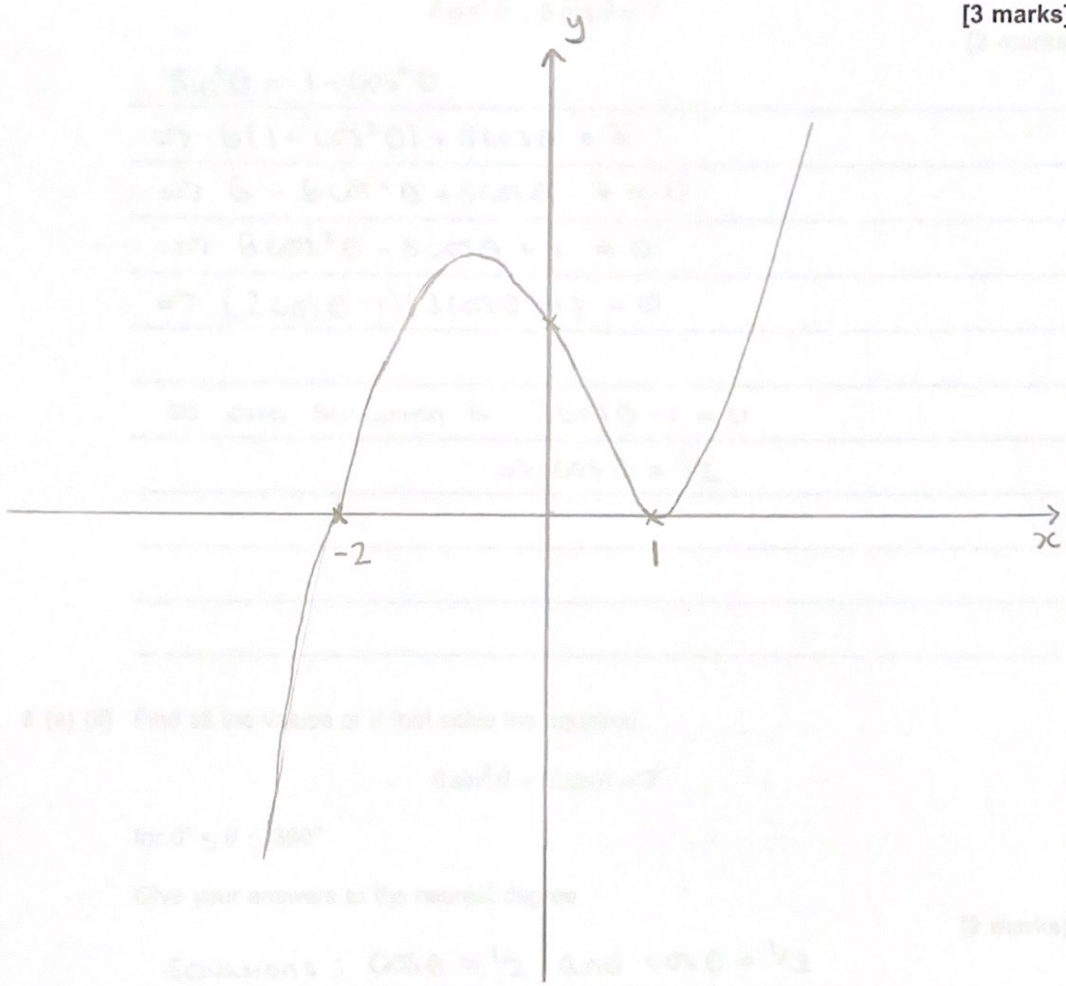
$$\frac{\sqrt{6}}{\sqrt{3}-\sqrt{2}} \times \frac{\sqrt{3}+\sqrt{2}}{\sqrt{3}+\sqrt{2}} = \frac{\sqrt{18}+\sqrt{12}}{3-2} = 3\sqrt{2}+2\sqrt{3}$$

Do not write outside the box

5 (a) Sketch the curve $y = g(x)$ where

$$g(x) = (x+2)(x-1)^2$$

[3 marks]



5 (b) Hence, solve $g(x) \leq 0$

[2 marks]

$x \leqslant -2$

$x = 1$

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6 (a) (i) Show that $\cos\theta = \dfrac{1}{2}$ is one solution of the equation

$$6\sin^2\theta + 5\cos\theta = 7$$

[2 marks]

$\sin^2\theta = 1 - \cos^2\theta$

$\Rightarrow 6(1 - \cos^2\theta) + 5\cos\theta = 7$

$\Rightarrow 6 - 6\cos^2\theta + 5\cos\theta - 7 = 0$

$\Rightarrow 6\cos^2\theta - 5\cos\theta + 1 = 0$

$\Rightarrow (2\cos\theta - 1)(3\cos\theta - 1) = 0$

so one solution is $2\cos\theta - 1 = 0$

$\Rightarrow \cos\theta = 1/2$

6 (a) (ii) Find all the values of θ that solve the equation

$$6\sin^2\theta + 5\cos\theta = 7$$

for $0° \leq \theta \leq 360°$

Give your answers to the nearest degree.

[2 marks]

Solutions : $\cos\theta = 1/2$ and $\cos\theta = 1/3$

$\theta = 60°, 300°$

$\theta = 70.5°, 289.5°$

Do not write outside the box

6 (b) Hence, find all the solutions of the equation

$$6\sin^2 2\theta + 5\cos 2\theta = 7$$

for $0° \leq \theta \leq 360°$

Give your answers to the nearest degree.

[2 marks]

$\theta = 30°, 35.25°, 150°, 144.75°$

$210°, 215.25°, 330°, 324.75°$

Turn over for the next question

Do not write outside the box

7 Given that $y \in \mathbb{R}$, prove that

$$(2 + 3y)^4 + (2 - 3y)^4 \geq 32$$

Fully justify your answer.

[6 marks]

$(2+3y)^4 = 2^4 + 4 \times 2^3 \times (3y) + 6 \times 2^2 \times (3y)^2 + 4 \times 2 \times (3y)^3$
$+ (3y)^4$
$= 16 + 96y + 216y^2 + 216y^3 + 81y^4$

$(2-3y)^4 = 2^4 + 4 \times 2^3 \times (-3y) + 6 \times 2^2 \times (-3y)^2 + 4 \times 2 \times (-3y)^3$
$+ \cancel{4 \times 2 \times} (-3y)^4$
$= 16 - 96y + 216y^2 - 216y^3 + 81y^4$

so $(2+3y)^4 + (2-3y)^4$
$= 32 + 432y^2 + 162y^4$
$y^2 \geq 0$ and $y^4 \geq 0 \ \forall y$
$\therefore (2+3y)^4 + (2-3y)^4 \geq 32.$

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8 Prove that the curve with equation

$$y = 2x^5 + 5x^4 + 10x^3 - 8$$

has **only one** stationary point, stating its coordinates.

[6 marks]

$dy/dx = 10x^4 + 20x^3 + 30x^2$

Let $dy/dx = 0 \Rightarrow 10x^4 + 20x^3 + 30x^2 = 0$

$\Rightarrow 10x^2(x^2 + 2x + 3) = 0$

$\Rightarrow x = 0$ or $x^2 + 2x + 3 = 0$

$x^2 + 2x + 3 = 0$ has no real roots

as $b^2 - 4ac = 4 - 12 = -8$

Therefore the only stationary point is at $x = 0, y = -8$

$(0, -8)$

Turn over for the next question

Do not write outside the box

9 A curve cuts the x-axis at (2, 0) and has gradient function

$$\frac{dy}{dx} = \frac{24}{x^3}$$

9 (a) Find the equation of the curve.

[4 marks]

$\int \frac{24}{x^3}\,dx = \int 24x^{-3}\,dx = -12x^{-2} + c$

$-12(2)^{-2} + c = 0$

$-12 \times 1/4 + c =$

$c = 3$

$y = -\frac{12}{x^2} + 3$

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9 (b) Show that the perpendicular bisector of the line joining $A(-2, 8)$ to $B(-6, -4)$ is the normal to the curve at $(2, 0)$

[6 marks]

Midpoint of AB $= (-4, 2)$

Gradient of AB $= \dfrac{8-(-4)}{-2-(-6)} = \dfrac{12}{4} = 3$

Gradient of perpendicular bisector $= -1/3$

Equation: $y - \cancel{8}\,2 = -1/3(x+4)$

$\Rightarrow y = -1/3\,x + 2/3$

Sub in $x = 2$: $y = -1/3(2) + 2/3 = 0$

$\therefore$ passes through $(2, 0)$

Gradient of $y = -\dfrac{12}{x^2} + 3$ at $(2, 0)$:

is $\dfrac{24}{2^3} = 3$, so normal gradient $= -\dfrac{1}{3}$.

$\therefore$ The perpendicular bisector is the normal.

Turn over ▶

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10 On 18 March 2019 there were 12 hours of daylight in Inverness.

On 16 June 2019, 90 days later, there will be 18 hours of daylight in Inverness.

Jude decides to model the number of hours of daylight in Inverness, N, by the formula

$$N = A + B\sin t°$$

where t is the number of days after 18 March 2019.

10 (a) (i) State the value that Jude should use for A.

[1 mark]

$A = 12$

10 (a) (ii) State the value that Jude should use for B.

[1 mark]

$B = 6$

10 (a) (iii) Using Jude's model, calculate the number of hours of daylight in Inverness on 15 May 2019, 58 days after 18 March 2019.

[1 mark]

$N = 12 + 6 \sin 58 = 17.1$ hours

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10 (a) (iv) Using Jude's model, find how many days during 2019 will have at least 17.4 hours of daylight in Inverness.

[4 marks]

$17.4 = 12 + 6\sin t$

$\Rightarrow \sin t = 0.9$

$t = 64$

$t = 116$

$\boxed{53 \text{ days}}$

10 (a) (v) Explain why Jude's model will become inaccurate for 2020 and future years.

[1 mark]

Jude's model will repeat after 360 days, but a year has 365 day in it.

10 (b) Anisa decides to model the number of hours of daylight in Inverness with the formula

$$N = A + B\sin\left(\frac{360}{365}t\right)^{\circ}$$

Explain why Anisa's model is better than Jude's model.

[1 mark]

Anisa's model will repeat after 365 days due to $\frac{360}{365}$.

Turn over ▶

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Section B

Answer **all** questions in the spaces provided.

11 A ball moves in a straight line and passes through two fixed points, A and B, which are 0.5 m apart.

The ball is moving with a constant acceleration of $0.39\,\text{m s}^{-2}$ in the direction AB.

The speed of the ball at A is $1.9\,\text{m s}^{-1}$

Find the speed of the ball at B.

Circle your answer.

[1 mark]

$2\,\text{m s}^{-1}$ $\quad$ $3.2\,\text{m s}^{-1}$ $\quad$ $3.8\,\text{m s}^{-1}$ $\quad$ $4\,\text{m s}^{-1}$

12 A particle P, of mass m kilograms, is attached to one end of a light inextensible string.

The other end of this string is held at a fixed position, O.

P hangs freely, in equilibrium, vertically below O.

Identify the statement below that correctly describes the tension, T newtons, in the string as m varies.

Tick (✓) **one** box.

[1 mark]

T varies along the string, with its greatest value at O	☐
T varies along the string, with its greatest value at P	☐
$T = 0$ because the system is in equilibrium	☐
T is directly proportional to m	✓

Do not write outside the box

13 A car, starting from rest, is driven along a horizontal track.

The velocity of the car, $v\,\mathrm{m\,s^{-1}}$, at time t seconds, is modelled by the equation

$$v = 0.48t^2 - 0.024t^3 \text{ for } 0 \le t \le 15$$

13 (a) Find the distance the car travels during the first 10 seconds of its journey.

[3 marks]

$S = \int_0^{10} 0.48t^2 - 0.024t^3 \, dt$

$= \left[0.16t^3 - 0.006t^4\right]_0^{10}$

$= 0.16(10)^3 - 0.006(10)^4$

$= 100 \text{ m}$

13 (b) Find the maximum speed of the car.

Give your answer to three significant figures.

[4 marks]

$\frac{dv}{dt} = 0.96t - 0.072t^2$

Let $0.96t - 0.072t^2 = 0$

$\Rightarrow t = 0$ or $(0.96 - 0.072t) = 0$

$\Rightarrow t = \frac{0.96}{0.072} = \frac{40}{3}$

When $t = \frac{40}{3}$, $v = 28.4\,\mathrm{ms^{-1}}$

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13 (c) Deduce the range of values of t for which the car is modelled as decelerating.

[2 marks]

$\frac{40}{3} < t \leq 15$, as max speed is at $t = \frac{40}{3}$

Turn over for the next question

Do not write outside the box

14 Two particles, A and B, lie at rest on a smooth horizontal plane.

A has position vector $\mathbf{r}_A = (13\mathbf{i} - 22\mathbf{j})$ metres
B has position vector $\mathbf{r}_B = (3\mathbf{i} + 2\mathbf{j})$ metres

14 (a) Calculate the distance between A and B.

[2 marks]

Distance $= \sqrt{(13-3)^2 + (-22-2)^2}$

$= 26$ metres

14 (b) Three forces, $\mathbf{F}_1$, $\mathbf{F}_2$ and $\mathbf{F}_3$ are applied to particle A, where

$$\mathbf{F}_1 = (-2\mathbf{i} + 4\mathbf{j}) \text{ newtons}$$

$$\mathbf{F}_2 = (6\mathbf{i} - 10\mathbf{j}) \text{ newtons}$$

Given that A remains at rest, explain why $\mathbf{F}_3 = (-4\mathbf{i} + 6\mathbf{j})$ newtons

[1 mark]

$(-2\underline{i} + 4\underline{j}) + (6\underline{i} - 10\underline{j}) = 4\underline{i} - 6\underline{j}$

so $F_3 = -4\underline{i} + 6\underline{j}$ to ensure the ~~total~~ resultant force applied to A is 0 newtons.

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14 (c) A force of $(5\mathbf{i} - 12\mathbf{j})$ newtons, is applied to B, so that B moves, from rest, in a straight line towards A.

B has a mass of 0.8 kg

14 (c) (i) Show that the acceleration of B towards A is $16.25\,\text{m}\,\text{s}^{-2}$

[2 marks]

$F = ma$

$|F| = \sqrt{5^2 + (-12)^2} = 13\text{ N}$

$13 = 0.8a$

$a = 16.25\text{ ms}^{-2}$

14 (c) (ii) Hence, find the time taken for B to reach A.

Give your answer to two significant figures.

[2 marks]

$s = ut + \frac{1}{2}at^2$

$\Rightarrow 26 = 1/2 \times 16.25 \times t^2$

$\Rightarrow 26 = 8.125t^2$

$\Rightarrow t^2 = 3.2$

$\Rightarrow t = 1.8$ seconds

Turn over ▶

Do not write outside the box

15 A tractor and its driver have a combined mass of m kilograms.

The tractor is towing a trailer of mass $4m$ kilograms in a straight line along a horizontal road.

The tractor and trailer are connected by a horizontal tow bar, modelled as a light rigid rod.

A driving force of 11 080 N and a total resistance force of 160 N act on the tractor.

A total resistance force of 600 N acts on the trailer.

The tractor and the trailer have an acceleration of $0.8\,\text{m}\,\text{s}^{-2}$

15 (a) Find m.

[3 marks]

$F = ma$

$11080 - 160 - 600 = 0.8(4m + m)$

$10320 = 4m$

$m = 2580$ Kg

15 (b) Find the tension in the tow bar.

[2 marks]

$T - 600 = 0.8 \times 4 \times 2580$

$T = 8856$

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15 (c) At the instant the speed of the tractor reaches $18\,\text{km}\,\text{h}^{-1}$ the tow bar breaks.

The total resistance force acting on the trailer remains constant.

Starting from the instant the tow bar breaks, calculate the time taken until the speed of the trailer reduces to $9\,\text{km}\,\text{h}^{-1}$

[4 marks]

$-600 = 10320a$ $\qquad 18\,\text{Kmh}^{-1} = 18000\,\text{mh}^{-1}$

$\Rightarrow a = \frac{-5}{86}\,\text{ms}^{-2}$ $\qquad = 5\,\text{ms}^{-1}$

so $9\,\text{Kmh}^{-1} = 2.5\,\text{ms}^{-1}$

$V = u + at$

$2.5 = 5 + \left(\frac{-5}{86}\right)t$

$\Rightarrow t = 43$ seconds.

END OF QUESTIONS