A-LEVEL
MATHEMATICS
7357/1
Paper 1

Mark scheme
June 2019

Version: 1.0 Final

196A73571/MS

Mark schemes are prepared by the Lead Assessment Writer and considered, together with the relevant questions, by a panel of subject teachers. This mark scheme includes any amendments made at the standardisation events which all associates participate in and is the scheme which was used by them in this examination. The standardisation process ensures that the mark scheme covers the students' responses to questions and that every associate understands and applies it in the same correct way. As preparation for standardisation each associate analyses a number of students' scripts. Alternative answers not already covered by the mark scheme are discussed and legislated for. If, after the standardisation process, associates encounter unusual answers which have not been raised they are required to refer these to the Lead Examiner.

It must be stressed that a mark scheme is a working document, in many cases further developed and expanded on the basis of students' reactions to a particular paper. Assumptions about future mark schemes on the basis of one year's document should be avoided; whilst the guiding principles of assessment remain constant, details will change, depending on the content of a particular examination paper.

Further copies of this mark scheme are available from aqa.org.uk

Mark scheme instructions to examiners

General

The mark scheme for each question shows:
- the marks available for each part of the question
- the total marks available for the question
- marking instructions that indicate when marks should be awarded or withheld including the principle on which each mark is awarded. Information is included to help the examiner make his or her judgement and to delineate what is creditworthy from that not worthy of credit
- a typical solution. This response is one we expect to see frequently. However credit must be given on the basis of the marking instructions.

If a student uses a method which is not explicitly covered by the marking instructions the same principles of marking should be applied. Credit should be given to any valid methods. Examiners should seek advice from their senior examiner if in any doubt.

Key to mark types

M	mark is for method
R	mark is for reasoning
A	mark is dependent on M or m marks and is for accuracy
B	mark is independent of M or m marks and is for method and accuracy
E	mark is for explanation
F	follow through from previous incorrect result

Key to mark scheme abbreviations

CAO	correct answer only
CSO	correct solution only
ft	follow through from previous incorrect result
‘their’	Indicates that credit can be given from previous incorrect result
AWFW	anything which falls within
AWRT	anything which rounds to
ACF	any correct form
AG	answer given
SC	special case
OE	or equivalent
NMS	no method shown
PI	possibly implied
SCA	substantially correct approach
sf	significant figure(s)
dp	decimal place(s)

AS/A-level Maths/Further Maths assessment objectives

AO		Description
AO1	AO1.1a	Select routine procedures
	AO1.1b	Correctly carry out routine procedures
	AO1.2	Accurately recall facts, terminology and definitions
AO2	AO2.1	Construct rigorous mathematical arguments (including proofs)
	AO2.2a	Make deductions
	AO2.2b	Make inferences
	AO2.3	Assess the validity of mathematical arguments
	AO2.4	Explain their reasoning
	AO2.5	Use mathematical language and notation correctly
AO3	AO3.1a	Translate problems in mathematical contexts into mathematical processes
	AO3.1b	Translate problems in non-mathematical contexts into mathematical processes
	AO3.2a	Interpret solutions to problems in their original context
	AO3.2b	Where appropriate, evaluate the accuracy and limitations of solutions to problems
	AO3.3	Translate situations in context into mathematical models
	AO3.4	Use mathematical models
	AO3.5a	Evaluate the outcomes of modelling in context
	AO3.5b	Recognise the limitations of models
	AO3.5c	Where appropriate, explain how to refine models

Examiners should consistently apply the following general marking principles

No Method Shown

Where the question specifically requires a particular method to be used, we must usually see evidence of use of this method for any marks to be awarded.

Where the answer can be reasonably obtained without showing working and it is very unlikely that the correct answer can be obtained by using an incorrect method, we must award **full marks**. However, the obvious penalty to students showing no working is that incorrect answers, however close, earn **no marks**.

Where a question asks the student to state or write down a result, no method need be shown for full marks.

Where the permitted calculator has functions which reasonably allow the solution of the question directly, the correct answer without working earns **full marks**, unless it is given to less than the degree of accuracy accepted in the mark scheme, when it gains **no marks**.

Otherwise we require evidence of a correct method for any marks to be awarded.

Diagrams

Diagrams that have working on them should be treated like normal responses. If a diagram has been written on but the correct response is within the answer space, the work within the answer space should be marked. Working on diagrams that contradicts work within the answer space is not to be considered as choice but as working, and is not, therefore, penalised.

Work erased or crossed out

Erased or crossed out work that is still legible and has not been replaced should be marked. Erased or crossed out work that has been replaced can be ignored.

Choice

When a choice of answers and/or methods is given and the student has not clearly indicated which answer they want to be marked, mark positively, awarding marks for all of the student's best attempts. Withhold marks for final accuracy and conclusions if there are conflicting complete answers or when an incorrect solution (or part thereof) is referred to in the final answer.

Q	Marking instructions	AO	Mark	Typical solution
1	Circles the correct response	1.1b	B1	$-4\log_{10}\left(\sqrt{a}\right)$
	Total		**1**	

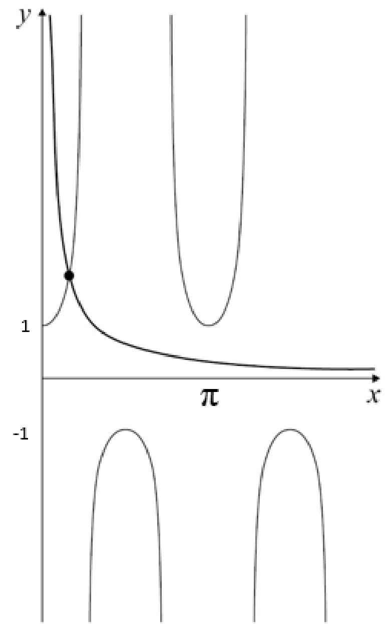
Q	Marking instructions	AO	Mark	Typical solution
2	Circles the correct response	1.1b	B1	$\frac{\mathrm{d}y}{\mathrm{d}x} = ke^{kx}$
	Total		**1**	

Q	Marking instructions	AO	Mark	Typical solution
3	Circles the correct response	1.1b	B1	6.4 cm^2
	Total		**1**	

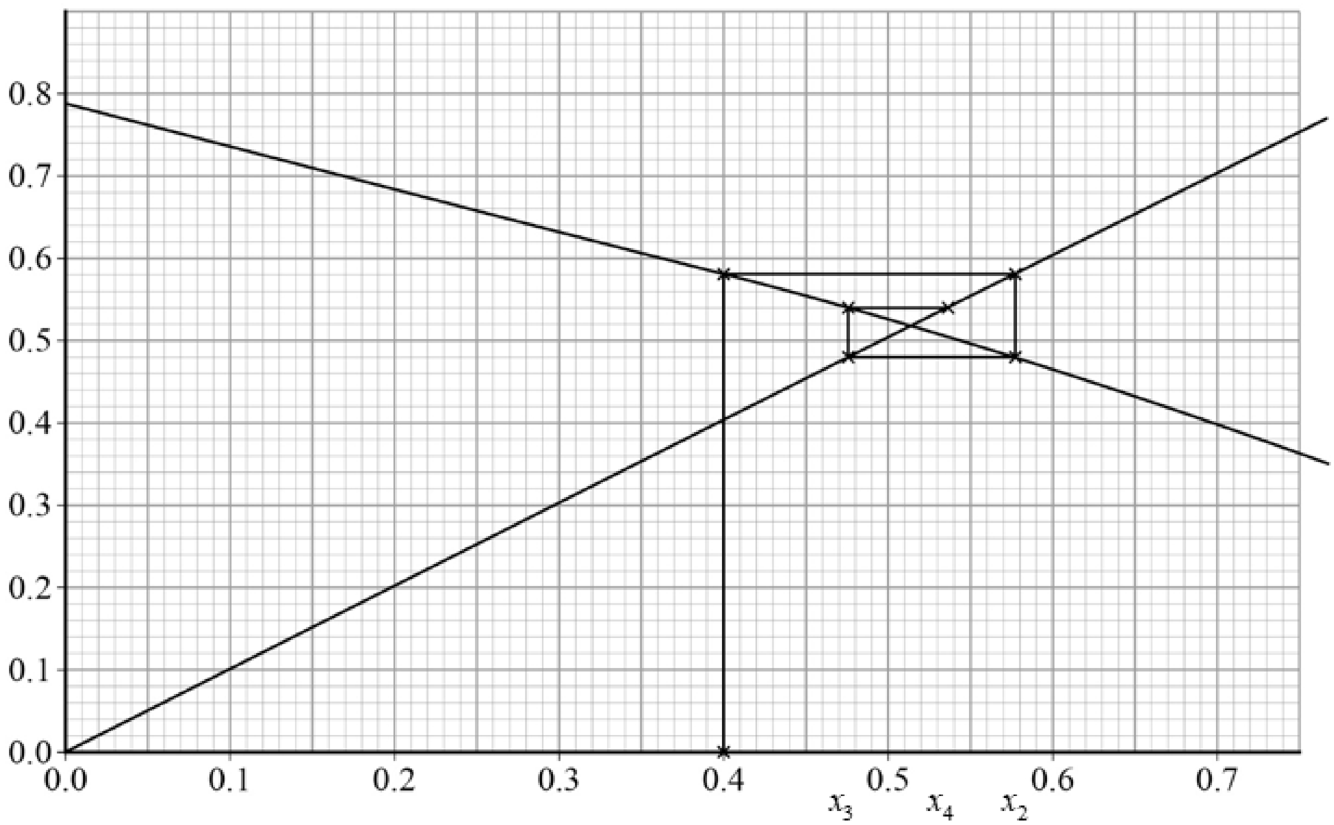
Q	Marking instructions	AO	Mark	Typical solution
4	Uses negative reciprocal to obtain equation with correct gradient	3.1a	M1	$-4x+5y=k$ $x=1$ $\Rightarrow 5+4y=17$ $\Rightarrow y=3$ $k=-4\times1+5\times3=11$ $5y-4x=11$ $y=\frac{4}{5}x+\frac{11}{5}$
	Obtains correct x coordinate of midpoint Or obtains correct equations of lines through A and B perpendicular to AB $5y-4x=31.5 \qquad 5y-4x=-9.5$ OE	1.1b	B1	
	Substitutes their mid-point value of x to obtain value of y coordinate of midpoint (not in terms of *a* or *b*) Or Finds a value for their $\frac{a+b}{2}$ Or Finds k by adding correct equations of lines through A and B perpendicular to AB Or equating intercepts.	1.1a	M1	
	Obtains correct equation **ACF** Eg $y=\frac{4}{5}x+c, \quad c=2.2$ **ISW** once correct answer seen.	1.1b	A1	
	Total		**4**	

Q	Marking instructions	AO	Mark	Typical solution
5(a)	Uses S_n =260 for arithmetic sequence with *n*=16 to form a correct equation PI by $8(2a+15d)=260$	1.1a	M1	$\frac{16}{2}(2a+(16-1)d)=260$ $8(2a+15d)=260$ $2(2a+15d)=65$ $4a+30d=65$
	Completes rigorous argument with correct algebraic manipulation to show required result **Must** see at least one line of simplification after $8(2a+15d)=260$ before given answer.	2.1	R1	
5(b)	Forms a second equation in *a* and *d* using $S_{60}=315$ and solves simultaneously to find *a* or *d*	3.1a	M1	$30(2a+59d)=315$ $20a+590d=105$ $a=20$ $d=-0.5$ $S_{41}=\frac{41}{2}(2\times 20-40\times 0.5)=410$
	Obtains correct *a* and *d*	1.1b	A1	
	Uses their *a* and *d* to obtain their value of $S_{41}=41a+820d$ Follow through provided one of their a or d is correct.	1.1b	A1F	
5(c)	Explains that values of U_n are positive n < 41 Or Explains that values of U_n are negative for $n>41$ Or Uses quadratic manipulation or differentiation of formula for S_n to obtain n = 40.5 CSO	2.4	M1	The terms before the 41st term are all positive. The terms after the 41st term are all negative so the sum of the first 41 terms must be a maximum value.
	Completes a valid argument explaining all terms positive before 41 and negative after 41 Or Completes argument linking 40.5 with the sum to 40 terms and the sum to 41 terms. CSO	2.1	R1	
	Total		**7**	

Q	Marking Instructions	AO	Marks	Typical Solution
6(a)	Deduces the range of f Accept $f(x) \geq \frac{1}{2}$, $y \geq \frac{1}{2}$ *or* $[0.5, \infty)$ OE	AO2.2a	**B1**	$\{y : y \geq \frac{1}{2}\}$
6(b)(i)	Rearranges formula, isolating squared term with at least one correct step seen.	1.1a	**M1**	$y = \frac{1}{2}(x^2 + 1)$ $2y = x^2 + 1$ $2y - 1 = x^2$ $x = \sqrt{2y-1}$ $f^{-1}(x) = \sqrt{2x-1}$ $x \geq \frac{1}{2}$
	Obtains inverse function in any correct form.	1.1b	**A1**	
	Obtains correct inverse function using $f^{-1}(x) = ...$ and states correct domain	2.5	**A1**	
6 (b)(ii)	States correct range Accept $f^{-1}(x) \geq 0$ OE	1.1b	**B1**	$\{y : y \geq 0\}$
6(c)	Recalls correct transformation	1.2	**B1**	Reflection in $y = x$
6(d)	Forms equation using two of the three expressions $x = \frac{x^2+1}{2} = \sqrt{2x-1}$ allow their $\sqrt{2x-1}$ PI by correct answer	3.1a	**M1**	$x = \frac{x^2+1}{2}$ (1, 1)
	Obtains x=1 and y=1 CSO	1.1b	**A1**	
	Total		**8**	

Q	Marking instructions	AO	Mark	Typical solution
7(a)	Sketches graph of $y=\frac{1}{x}$ Must not cross axes Correct asymptotes	1.2	B1	
	Sketches graph of $y=\sec kx$ up to first asymptote	1.1a	M1	
	Draws fully correct graphs in **first quadrant,** intersecting at one point with sec 2x up to asymptote at x=$\frac{5\pi}{4}$. Ignore fourth quadrant/negative y. Condone missing labels on y-axis.	1.1b	A1	
7(b)	Rearranges to the form $\mathrm{f}(x)=0$ and evaluates $\mathrm{f}(x)$ at 0.4 and 0.6 Can evaluate at two values either side of the root 0.515 in the interval [0.4,0.6]	1.1a	M1	$\frac{1}{x}=\sec 2x \Rightarrow \frac{1}{x}-\sec 2x=0$ $\mathrm{f}\left(x\right)=\frac{1}{x}-\sec 2x$ $\mathrm{f}\left(0.4\right)=1.06..>0$ $\mathrm{f}\left(0.6\right)=-1.09..<0$ Hence the solution lies between 0.4 and 0.6
	Completes rigorous argument with any reference to change of sign. Must see evidence of correct evaluation accepting values correct to 1 sf. If function notation used it must be defined.	2.1	R1	
7(c)	Uses $\sec 2x=\frac{1}{\cos 2x}$ to obtain a correct equation in $\cos 2x$ eg, $\frac{1}{x}=\frac{1}{\cos 2x}$ or $1=\frac{x}{\cos 2x}$	1.1a	M1	$\frac{1}{x}=\sec 2x$ $\frac{1}{x}=\frac{1}{\cos 2x}$ $x=\cos 2x$ $2x=\cos^{-1} x$ $x=\frac{1}{2}\cos^{-1} x$
	Completes rearrangement Must see $\cos^{-1} x=2x$ before given answer	2.1	R1	

7 (d)(i)	Obtains any one correct value to at least 3 decimal places, ignoring labels.	1.1a	M1	$x_2 = 0.5796$ $x_3 = 0.4763$ $x_4 = 0.5372$
	Obtains x_2, x_3 and x_4 correct to 4 decimal places If no labels only accept the three correct answers in the correct order with no extras seen beyond x_4 **CAO**	1.1b	A1	
7 (d)(ii)	Draws correct cobweb diagram Condone missing vertical line x = 0.4	1.1a	M1	See diagram below
	Shows positions of x_2, x_3 and x_4 with clear indication of positioning on x-axis not on y=x	1.1b	A1	
	Total		**11**	



Q	Marking instructions	AO	Mark	Typical solution
8(a)	Obtains one correct value	1.1b	B1	$P(3)=27$ $P(10)=1000$
	Obtains both correct values	1.1b	B1	
8(b)	Forms cubic equation replacing $P(n)=n^3$ (condone k^3) PI correct answer	2.5	M1	$n^3=1.25\times10^8$ $n=500$
	Obtains 500 CSO	1.1b	A1	
	Total		**4**	

Q	Marking instructions	AO	Mark	Typical solution
9	Begins proof by contradiction. This may be evidenced by: stating assumption at the start “the sum is rational” Or Sight of “contradiction” later as part of argument.	3.1a	M1	Assume m is rational and n is irrational and their sum is rational. Then $\frac{a}{b}+n=\frac{c}{d}$ Where a, b, c and d are all integers. $n=\frac{c}{d}-\frac{a}{b}$ $=\frac{bc-ad}{bd}$ $\therefore$ *n* is rational, which is a contradiction. So the original statement is false and the sum of a rational and irrational must be irrational.
	Forms an equation of the form rational + irrational = rational with the rationals written algebraically $\frac{a}{b}+n=\frac{c}{d}$ n must clearly be irrational and not written as an algebraic fraction and not a specific value.	2.5	M1	
	Manipulates their equation to show that n is rational	1.1b	A1	
	Explains or demonstrates why there is a contradiction	2.4	E1	
	Completes rigorous argument to prove the required result including correct initial assumptions Where a, b, c and d are all integers.	2.1	R1	
	Total		**5**	

Q	Marking instructions	AO	Mark	Typical solution
10	Models the rate of change of volume with a differential equation of the correct form. With respect to time, not contradicted.	3.3	B1	$\frac{dv}{dt} = k$
	Obtains $4\pi r^2$ by differentiation.	1.1b	B1	$\frac{dv}{dr} = 4\pi r^2$
	Uses the chain rule to connect rates of change substituting their expressions for dv/dt and dv/dr. Or Integrates to obtain expression for v=kt+c Then differentiates wrt r dv/dr=kdt/dr Substitutes their expression for dv/dr	3.1b	M1	$\frac{dv}{dt} = \frac{dv}{dr} \times \frac{dr}{dt}$
	Completes argument, obtaining a correct expression for $\frac{dr}{dt}$ and concluding that $\frac{dr}{dt} \propto \frac{1}{r^2}$ OE statement	2.1	R1	$k = 4\pi r^2 \times \frac{dr}{dt}$ $\frac{dr}{dt} = \frac{k}{4\pi r^2}$ $\therefore \frac{dr}{dt} \propto \frac{1}{r^2}$
	Total		**4**	

Q	Marking instructions	AO	Mark	Typical solution
11	Replaces $h=0$ with $h \to 0$ or better seen anywhere	2.3	M1	For gradient of curve at *A*, let $h \to 0$ then
	Uses limit notation fully correctly Accept $\frac{\sin(\mathrm{h})}{h} \to 0$ here Accept full limit notation here	2.5	A1	$\frac{\cos(h)-1}{h} \to 0$ and $\frac{\sin(h)}{h} \to 1$
	$\frac{\sin(\mathrm{h})}{h} = 1$ seen OE eg $\sin(h) = h$	2.3	B1	
	Writes last line explicitly as $\sin\left(\frac{\pi}{2}\right)\times 0 + \cos\left(\frac{\pi}{2}\right)\times 1 = 0$ Accept $1\times 0 + 0\times 1 = 0$	2.2a	B1	Hence the gradient of the curve at A is given by $\sin\left(\frac{\pi}{2}\right)\times 0 + \cos\left(\frac{\pi}{2}\right)\times 1 = 0$
	Total		**4**	

Q	Marking instructions	AO	Mark	Typical solution
12(a)	Uses appropriate trig identity to form quadratic equation **in single trigonometrical** term. Condone $2(\pm 1 \pm \operatorname{cosec}^2 x) + 2\operatorname{cosec}^2 x = 1 + 4\operatorname{cosec} x$	1.1a	M1	$2\cot^2 x + 2\operatorname{cosec}^2 x = 1 + 4\operatorname{cosec} x$ $2(\operatorname{cosec}^2 x - 1) + 2\operatorname{cosec}^2 x = 1 + 4\operatorname{cosec} x$ $4\operatorname{cosec}^2 x - 4\operatorname{cosec} x - 3 = 0$
	Completes rigorous argument to show the required result	2.1	R1	
12(b)	Solves quadratic and Obtains one of $\operatorname{cosec} x = \frac{3}{2}$ or $\operatorname{cosec} x = -\frac{1}{2}$ OE	1.1b	B1	$\operatorname{cosec} x = \frac{3}{2}$ or $\operatorname{cosec} x = -\frac{1}{2}$ reject since $\lvert\operatorname{cosec} x\rvert \geq 1$ $\cot^2 x = \left(\frac{3}{2}\right)^2 - 1 = \frac{5}{4}$ $\tan x = -\frac{2\sqrt{5}}{5}$ Since *x* is obtuse
	Explains why their spurious solution(s) is rejected referring to the range of cosec or sine with explicit comparison to ± 1 May accept later rejection for valid reason ie sq root of negative OE	2.4	E1F	
	Uses trig identity or right-angled triangle/Pythagoras or given equation with their exact value of cosec *x or sin x to obtain an exact value of tan x* value used must satisfy $\lvert\operatorname{cosec} x\rvert \geq 1$ OE	1.1a	M1	
	Completes rigorous argument to find correct exact magnitude of tan *x* *ACF*	2.1	R1	
	Deduces tan x is negative. May be seen anywhere without contradiction by a positive final answer.	2.2a	B1	
	Total		**7**	

Q	Marking instructions	AO	Mark	Typical solution
13	Chooses an appropriate technique to differentiate accept any evidence of product rule or quotient rule	3.1a	M1	$x \neq 0$ as y is undefined $\frac{\mathrm{d}y}{\mathrm{d}x} = \frac{3e^{3x-5}x^2 - 2xe^{3x-5}}{x^4}$ At a turning point $\frac{\mathrm{d}y}{\mathrm{d}x} = 0$ $\Rightarrow \frac{3e^{3x-5}x^2 - 2xe^{3x-5}}{x^4} = 0$ $\Rightarrow 3e^{3x-5}x^2 - 2xe^{3x-5} = 0$ $\Rightarrow (3x-2)xe^{3x-5} = 0$ $\Rightarrow x = \frac{2}{3}, \quad \mathrm{e}^{3x-5} = 0$ $\mathrm{e}^{3x-5} \neq 0$ $\therefore$ there is only one stationary point.
	Differentiates e^{3x-5} correctly	1.1b	B1	
	Obtains correct $\frac{\mathrm{d}y}{\mathrm{d}x}$ ACF	1.1b	A1	
	Explains that stationary points occur when $\frac{\mathrm{d}y}{\mathrm{d}x} = 0$	2.4	E1	
	Equates their $\frac{\mathrm{d}y}{\mathrm{d}x}$ to zero and solves their equation with at least one correct line of correct rearrangement. Resulting in a value for x.	1.1a	M1	
	Deduces their factor $\mathrm{e}^{3x-5} \neq 0$	2.2a	B1F	
	Completes argument to show exactly one stationary point at $x = \frac{2}{3}$. Must include consideration of $x \neq 0$ somewhere.	2.1	R1	
	Total		**7**	

Q	Marking instructions	AO	Mark	Typical solution
14 (a)(i)	States correct number of ordinates	1.2	B1	5
14 (a)(ii)	Obtains at least 4 correct y values (condone 7.5… for y_4) and correct h	1.1b	B1	x: 0, 1, 2, 3, 4; y: 0, 1, 3.2, 5.4, 7.52941
	Substitutes their y values into trapezium rule with correct number of strips. Condone missing 0 May see working on graph 0.5+2.1+4.3+6.4647. (Exact value 1136/85)	1.1a	M1	$Area = \frac{1}{2}\times1\times\left(0+7.529+2\left(1+3.2+5.4\right)\right)$ $=13.36$
	Obtains correct area NMS correct answer award full marks **CAO**	1.1b	A1	
14(b)	Selects substitution $u = x^2+1$ or $u = x^2$ and obtains $\frac{du}{dx} = 2x$ or writes the integrand in the form $Ax + \frac{Bx}{x^2+1}$	3.1a	M1	$u = x^2+1 \Rightarrow \frac{du}{dx} = 2x$ $\int_1^{17} \frac{2x^3}{u} \times \frac{1}{2x} du$ $= \int_1^{17} \frac{x^2}{u} du$ $= \int_1^{17} \frac{u-1}{u} du$ $= \int_1^{17} 1 - \frac{1}{u} du$ $= \left[u - \ln u\right]_1^{17}$ $= \left(17 - \ln 17\right) - \left(1 - \ln 1\right)$ $= 16 - \ln 17$
	Obtains $\int_1^{17} 1 - \frac{1}{u} du$ OE Or $\int_0^4 2x - \frac{2x}{x^2+1} dx$ Ignore limits	1.1b	A1	
	Integrates their expression obtaining an ln term correctly.	1.1a	M1	
	Obtains fully correct integral $u - \ln u$ or $x^2 - \ln\left(x^2+1\right)$ Condone missing limits.	1.1b	A1	
	Completes fully correct argument Substituting correct limits for their method to show the correct required result with correct notation with **AG**	2.1	R1	
14(c)	Explains that as n increases the approximation found will tend to the value of $\int_0^4 \frac{2x^3}{x^2+1} dx$ OE	2.4	E1	Area $\to 16 - \ln 17$
	Total		**10**	

Q	Marking Instructions	AO	Mark	Typical Solution
15 (a)(i)	Uses model with t=0 to find correct value of h *AWRT 5.9*	AO3.4	B1	5.88metres
15 (a)(ii)	Uses v=0 to set up quadratic equation for t	AO3.4	M1	$0 = 4 - (\frac{2t}{3} - 2)^2$
	Obtains t = 6 PI correct answer	AO1.1b	A1	t = 6
	Interprets their lowest positive solution correctly NMS can score 3	AO3.2a	A1F	8 am
15 (a)(iii)	Obtains correct h for their positive t provided h < 5.88 Accept 0.12 If given to more decimal places AWFW 0.115 to 0.116 FT their t allow negative h	AO3.4	B1F	0.12m
15(b)	Identifies t=3 (for maximum velocity)	AO3.1b	B1	t = 3
	Substitutes their t into model for h PI their answer	3.4	M1	$h = 3 - 2\sqrt[3]{3-3}$ h = 3 metres
	Finds correct height **with units**	3.2a	A1	
15(c)	Explains that the validity of the model is limited by time.	3.5b	B1	The model breaks down after one cycle of the tide.
	Explains that the height continues to decrease after 6 hours (or after the first cycle or first low tide). Or explains there are no other times when v=0	3.5a	B1	After 6 hours the model shows the height continues to decrease.
	Total		**10**	

Q	Marking instructions	AO	Mark	Typical solution
16(a)	Chooses an appropriate technique to differentiate accept any evidence of product rule or quotient rule	3.1a	M1	$\frac{dy}{dx} = -e^{-x}(\sin x + \cos x) + e^{-x}(\cos x - \sin x)$ $= -2e^{-x} \sin x$
	Differentiates fully correctly	1.1b	A1	
	Obtains fully correct simplified answer.	1.1b	A1	
16(b)	Uses their result from (a) in the form of $Be^{-x} \sin x$ showing an understanding of the fundamental theorem of calculus Condone missing constant.	3.1a	M1	$\int(-2e^{-x} \sin x)dx = e^{-x}(\sin x + \cos x) + k$ $\therefore \int(e^{-x} \sin x)dx = -\frac{1}{2}e^{-x}(\sin x + \cos x) + c$
	Obtains $\frac{1}{B}e^{-x}(\sin x + \cos x)$	2.1	R1F	

16 (c)(i)	Writes the area as $\frac{1}{B}\left[e^{-x}\left(\sin x+\cos x\right)\right]_0^{\pi}$ Condone missing limits	3.1a	M1	$\int_0^{\pi}\left(e^{-x}\sin x\right)dx=-\frac{1}{2}\left[e^{-x}\left(\sin x+\cos x\right)\right]_0^{\pi}$ $=-\frac{1}{2}[e^{-\pi}\left(\sin\pi+\cos\pi\right)$ $-e^{0}\left(\sin 0+\cos 0\right)]$ $=\frac{e^{-\pi}+1}{2}$
	Deduces correct limits and substitutes correctly	2.2a	A1	
	Obtains correct exact value from correct answer in part(b) CSO	1.1b	A1	
16 (c)(ii)	Substitutes correct limits for A_2 Into their $\frac{1}{B}\left[e^{-x}\left(\sin x+\cos x\right)\right]_0^{\pi}$ Or Writes $A_2=\pm\int_{\pi}^{2\pi}\left(e^{-x}\sin x\right)dx$ and uses the substitution $u=x-\pi$	1.1a	M1	$\int_{\pi}^{2\pi}\left(e^{-x}\sin x\right)dx=-\frac{1}{2}\left[e^{-x}\left(\sin x+\cos x\right)\right]_{\pi}^{2\pi}$ $=-\frac{e^{-\pi}+1}{2}e^{-\pi}$ Area $=\frac{e^{-\pi}+1}{2}e^{-\pi}$ $\frac{A_2}{A_1}=\frac{\frac{e^{-\pi}+1}{2}e^{-\pi}}{\frac{e^{-\pi}+1}{2}}=e^{-\pi}$
	Obtains correct exact area for $\pm A_2$ CSO Or Makes complete substitution $A_2=\pm\int_0^{\pi}\left(e^{-(u+\pi)}\sin\left(u+\pi\right)\right)du$	1.1b	A1	
	Forms required ratio using their exact A1 and A2, may be unsimplified Or Extracts factor of $e^{-\pi}$ and uses $\sin\left(u+\pi\right)=-\sin u$ To obtain $A_2=-e^{-\pi}\int_0^{\pi}e^{-u}\sin u\,du$	1.1a	M1	
	Completes rigorous argument, with correct limits and negatives handled correctly CSO	2.1	R1	
16 (c)(iii)	Deduces that the areas form a geometric series Accept any indication of this being geometric series	2.2a	B1	$\frac{a}{1-r}=\frac{e^{-\pi}+1}{2}\times\frac{1}{1-e^{-\pi}}$ $=\frac{1+e^{\pi}}{2\left(e^{\pi}-1\right)}$
	Uses $\frac{A_1}{1-e^{-\pi}}$	3.1a	M1	
	Obtains a value for the geometric series first term using their (c)(i)	1.1a	B1F	
	Completes rigorous argument to achieve required result in correct form. CSO AG	2.1	R1	
	Total		**16**	