Write your name here

Surname	Other names
MODEL SOLUTION	

Pearson Edexcel Level 3 GCE

Centre Number	Candidate Number

Mathematics

Advanced Subsidiary
Paper 1: Pure Mathematics

Sample Assessment Material for first teaching September 2017 **Time: 2 hours**	Paper Reference **8MA0/01**

You must have: Mathematical Formulae and Statistical Tables, calculator	Total Marks

Candidates may use any calculator permitted by Pearson regulations. Calculators must not have the facility for algebraic manipulation, differentiation and integration, or have retrievable mathematical formulae stored in them.

Instructions

- Use **black** ink or ball-point pen.
- If pencil is used for diagrams/sketches/graphs it must be dark (HB or B).
- **Fill in the boxes** at the top of this page with your name, centre number and candidate number.
- Answer **all** the questions and ensure that your answers to parts of questions are clearly labelled.
- Answer the questions in the spaces provided – *there may be more space than you need.*
- You should show sufficient working to make your methods clear. Answers without working may not gain full credit.
- Answers should be given to three significant figures unless otherwise stated.

Information

- A booklet 'Mathematical Formulae and Statistical Tables' is provided.
- There are 17 questions in this question paper. The total mark for this paper is 100.
- The marks for each question are shown in brackets – *use this as a guide as to how much time to spend on each question.*

Advice

- Read each question carefully before you start to answer it.
- Try to answer every question.
- Check your answers if you have time at the end.
- If you change your mind about an answer, cross it out and put your new answer and any working underneath.

Turn over

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Pearson

Answer ALL questions. Write your answers in the spaces provided.

1. The line l passes through the points A (3, 1) and B (4, −2).

   Find an equation for l.

   (3)

$y_2 - y_1 = m(x_2 - x_1)$

$-2 - 1 = m(4 - 3)$

$m = -3$.

$y = -3x + c$.

$y_1 = -3x_1 + c$.

$1 = 3 \times 3 + c$

$1 = -9 + c$

$c = 10$.

$y = -3x + 10$.

(Total for Question 1 is 3 marks)

2. The curve C has equation

$$y = 2x^2 - 12x + 16$$

Find the gradient of the curve at the point P (5, 6).

(Solutions based entirely on graphical or numerical methods are not acceptable.)

(4)

$y = 2x^2 - 12x + 16$.

$\frac{dy}{dx} = 4x - 12$.

$\frac{dy}{dx}$ at $x = 5$: $4 \times 5 - 12 = 20 - 12 = 8$.

$\Rightarrow$ Gradient at (5, 6) is 8.

(Total for Question 2 is 4 marks)

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3. Given that the point A has position vector $3\mathbf{i} - 7\mathbf{j}$ and the point B has position vector $8\mathbf{i} + 3\mathbf{j}$,

(a) find the vector $\overrightarrow{AB}$

(2)

(b) Find $|\overrightarrow{AB}|$. Give your answer as a simplified surd.

(2)

a) $\overrightarrow{AB} = 8i + 3j - (3i - 7j)$
$= 8i - 3i + 3j + 7j$
$= 5i + 10j$.

b) $|\overrightarrow{AB}| = \sqrt{5^2 + 10^2}$
$= \sqrt{25 + 100}$
$= \sqrt{125}$
$= \sqrt{5 \times 25}$
$= \sqrt{25}\sqrt{5}$
$= 5\sqrt{5}$.

(Total for Question 3 is 4 marks)

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4.

$$f(x) = 4x^3 - 12x^2 + 2x - 6$$

(a) Use the factor theorem to show that $(x - 3)$ is a factor of $f(x)$.

(2)

(b) Hence show that 3 is the only real root of the equation $f(x) = 0$

(4)

a) Factor theorem: $(x-k)$ is a factor of $f(x)$ if and only if $f(k) = 0$.
So $(x-3)$ is a factor of $f(x)$ if and only if $f(3) = 0$.
$f(3) = 4\times3^3 - 12\times3^2 + 2\times3 - 6$
$f(3) = 4\times27 - 12\times9 + 6 - 6$
$f(3) = 108 - 108 + 6 - 6$
$f(3) = 0$.
Hence, $(x-3)$ is a factor of $f(x)$.

b)
$$\begin{array}{r|l} & 4x^2 \quad\quad + 2 \\ x-3 & 4x^3 - 12x^2 + 2x - 6 \\ & \underline{4x^3 - 12x^2} \\ & \quad\quad 0 + 2x - 6 \\ & \quad\quad\quad \underline{2x - 6} \\ & \quad\quad\quad\quad 0 \end{array}$$

Hence $f(x) = (x-3)(4x^2+2)$.
Consider $4x^2 + 2$
$b^2 - 4ac =$
$0 - 4\times4\times2 =$
$-32 < 0$
No real roots.
3 is the only real root.

(Total for Question 4 is 6 marks)

5. Given that

$$f(x) = 2x + 3 + \frac{12}{x^2}, \quad x > 0$$

show that $\int_1^{2\sqrt{2}} f(x)dx = 16 + 3\sqrt{2}$

(5)

$$f(x) = 2x + 3 + \frac{12}{x^2}.$$

$$\int_1^{2\sqrt{2}} 2x + 3 + \frac{12}{x^2}\, dx =$$

$$\int_1^{2\sqrt{2}} 2x + 3 + 12x^{-2}\, dx =$$

$$\left[x^2 + 3x - 12x^{-1}\right]_1^{2\sqrt{2}} =$$

$$(2\sqrt{2})^2 + 3 \times 2\sqrt{2} - \frac{12}{2\sqrt{2}} - 1^2 - 3 \times 1 + 12 \times 1^{-1}$$

$$= 4 \times 2 + 6\sqrt{2} - \frac{12\sqrt{2}}{4} - 1 - 3 + 12$$

$$= 8 - 1 - 3 + 12 + 6\sqrt{2} - 3\sqrt{2}$$

$$= 16 + 3\sqrt{2}.$$

(Total for Question 5 is 5 marks)

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6. Prove, from first principles, that the derivative of $3x^2$ is $6x$.

(4)

$$\frac{d}{dx}(3x^2) = \lim_{h\to 0} \frac{3(x+h)^2 - 3x^2}{h}$$
$$= \lim_{h\to 0} \frac{3(x^2+2hx+h^2) - 3x^2}{h}$$
$$= \lim_{h\to 0} \frac{3x^2+6hx+h^2-3x^2}{h}$$
$$= \lim_{h\to 0} \frac{6hx+h^2}{h}$$
$$= \lim_{h\to 0} 6x+h$$
$$= 6x.$$

(Total for Question 6 is 4 marks)

7. (a) Find the first 3 terms, in ascending powers of x, of the binomial expansion of $\left(2-\frac{x}{2}\right)^7$, giving each term in its simplest form.

(4)

(b) Explain how you would use your expansion to give an estimate for the value of 1.995^7

(1)

a) $\left(2-\frac{x}{2}\right)^7 =$

$\binom{7}{0}2^7\left(\frac{x}{2}\right)^0 - \binom{7}{1}2^6\left(\frac{x}{2}\right)^1 + \binom{7}{2}2^5\left(\frac{x}{2}\right)^2 =$

$\frac{7!}{7!0!}\times 128 - \frac{7!}{6!1!}\times 64 \times \frac{x}{2} + \frac{7!}{5!2!}\times 32 \times \frac{x^2}{4} =$

$128 - 7\times 32x + 21\times 8x^2 =$

$128 - 224x + 168x^2$

b) Estimate comes from $2-\frac{x}{2} = 1.995$.

$\frac{x}{2} = 0.005$

$x = 0.01$.

So estimate is:

$128 - 224\times 0.01 + 168\times 0.01^2 =$

$128 - 2.24 + 0.0168 =$

125.7768.

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8.

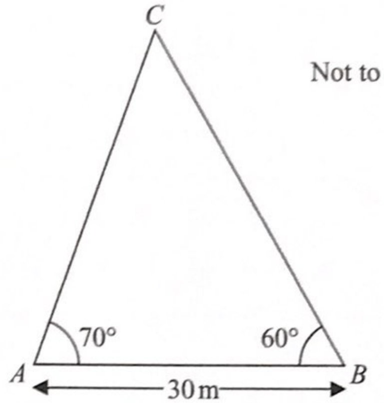

Figure 1

A triangular lawn is modelled by the triangle ABC, shown in Figure 1. The length AB is to be 30 m long.

Given that angle $BAC = 70°$ and angle $ABC = 60°$,

(a) calculate the area of the lawn to 3 significant figures.

(4)

(b) Why is your answer unlikely to be accurate to the nearest square metre?

(1)

a) $ACB = 180 - 70 - 60 = 50$.

$\frac{AC}{\sin 60} = \frac{30}{\sin 50}$.

$AC = \frac{30 \sin 60}{\sin 50}$

Area $= \frac{1}{2} ab \sin C$

Area $= \frac{1}{2} \times 30 \times AC \times \sin(70)$.

Area $= \frac{1}{2} \times 30 \times \frac{30 \sin 60 \sin 70}{\sin 50}$

Area $= 478 m^2$.

b) The angles and side length are not given to enough significant figures for an area that accurate.

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9. Solve, for $360° \leqslant x < 540°$,

$$12\sin^2 x + 7\cos x - 13 = 0$$

Give your answers to one decimal place.

(Solutions based entirely on graphical or numerical methods are not acceptable.)

(5)

$12\sin^2 x + 7\cos x - 13 = 0$

$12(1-\cos^2 x) + 7\cos x - 13 = 0$

$12 - 12\cos^2 x + 7\cos x - 13 = 0$

$-12\cos^2 x + 7\cos x - 1 = 0$

$12\cos^2 x - 7\cos x + 1 = 0$

$(4\cos x - 1)(3\cos x - 1) = 0$

$4\cos x = 1$ $\qquad$ $3\cos x = 1$

$\cos x = \frac{1}{4}$ $\qquad$ $\cos x = \frac{1}{3}$

$x = 75.5°$ $\qquad$ $x = 70.5°$

Add 360 to be in range:

$x = 435.5°$ $\qquad$ $x = 430.5°$

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10. The equation $kx^2 + 4kx + 3 = 0$, where k is a constant, has no real roots.

Prove that

$$0 \leqslant k < \frac{3}{4}$$

(4)

No real roots : consider discriminant :

$b^2 - 4ac < 0$

$(4k)^2 - 4 \times k \times 3 < 0$.

$16k^2 - 12k < 0$

$4k^2 - 3k < 0$

$k(4k-3) < 0$.

$k = 0 \qquad k = \frac{3}{4}$.

Hence, $0 < k < \frac{3}{4}$.

$k = \frac{3}{4}$ gives real roots .

$k = 0$ gives $3 = 0$ which is not true .

Hence, $0 \leq k < \frac{3}{4}$.

(Total for Question 10 is 4 marks)

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11. (a) Prove that for all positive values of x and y

$$\sqrt{xy} \leqslant \frac{x+y}{2}$$

(2)

(b) Prove by counter example that this is not true when x and y are both negative.

(1)

a) If x and y both positive, then

$(\sqrt{x} - \sqrt{y})^2 \geq 0$.

$\sqrt{x}^2 - 2\sqrt{x}\sqrt{y} + \sqrt{y}^2 \geq 0$.

$x - 2\sqrt{xy} + y \geq 0$.

$x + y \geq 2\sqrt{xy}$

$\sqrt{xy} \leq \frac{x+y}{2}$.

b) $x = -4$, $y = -1$.

$\sqrt{(-4)\times(-1)} = \sqrt{4} = 2$.

$\frac{x+y}{2} = \frac{-4-1}{2} = -\frac{5}{2} = -2.5 < 2$.

So the result is not true.

(Total for Question 11 is 3 marks)

12. A student was asked to give the exact solution to the equation

$$2^{2x+4} - 9(2^x) = 0$$

The student's attempt is shown below:

$$2^{2x+4} - 9(2^x) = 0$$

$$2^{2x} + 2^4 - 9(2^x) = 0$$

$$\text{Let } \quad 2^x = y$$

$$y^2 - 9y + 8 = 0$$

$$(y - 8)(y - 1) = 0$$

$$y = 8 \text{ or } y = 1$$

$$\text{So } x = 3 \text{ or } x = 0$$

(a) Identify the two errors made by the student.

(2)

(b) Find the exact solution to the equation.

(2)

a) The student identifies $2^{2x+4} = 2^{2x} + 2^4$, which is not correct. In fact, $2^{2x+4} = 2^{2x} \times 2^4$.

The student claims $2^4 = 8$, but $2^4 = 16$.

b) $2^{2x+4} - 9 \times 2^x = 0$

$2^{2x} 2^4 - 9 \times 2^x = 0$

$16 \times 2^{2x} - 9 \times 2^x = 0$

$y = 2^x$

$y^2 = 2^{2x}$

$16y^2 - 9y = 0$

$y(16y - 9) = 0$

$y = 0$

$2^x = 0$

No solutions

$y = \frac{9}{16}$

$2^x = \frac{9}{16}$

$x \log(2) = \log\left(\frac{9}{16}\right)$

$x = (\log(9) - \log(16)) / \log(2)$

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13. (a) Factorise completely $x^3 + 10x^2 + 25x$

(2)

(b) Sketch the curve with equation

$$y = x^3 + 10x^2 + 25x$$

showing the coordinates of the points at which the curve cuts or touches the x-axis.

(2)

The point with coordinates $(-3, 0)$ lies on the curve with equation

$$y = (x + a)^3 + 10(x + a)^2 + 25(x + a)$$

where a is a constant.

(c) Find the two possible values of a.

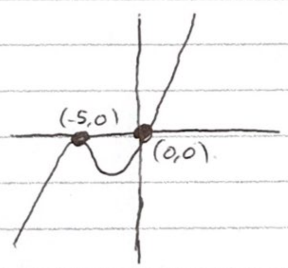
(3)

a) $x^3 + 10x^2 + 25x =$

$x(x^2 + 10x + 25) =$

$x(x+5)^2$

b)



c) This represents a translation of the curve by a to the left.

$a = -2$ $a = 3$

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14.

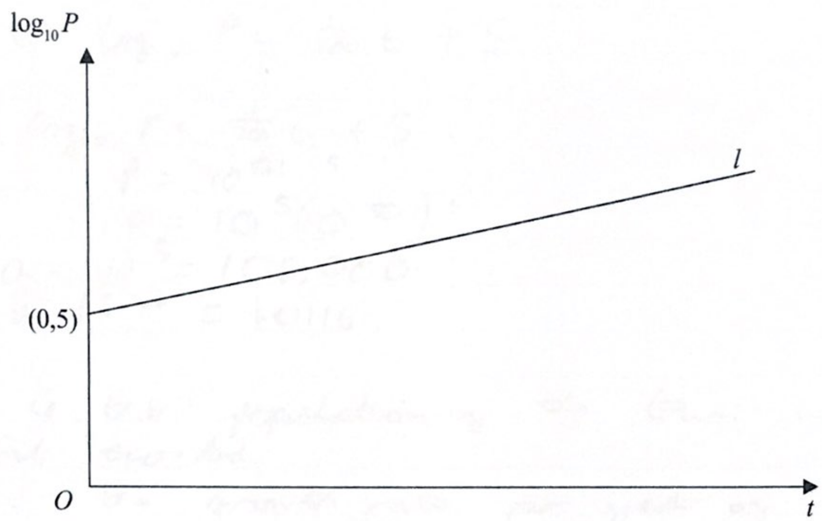

Figure 2

A town's population, P, is modelled by the equation $P = ab^t$, where a and b are constants and t is the number of years since the population was first recorded.

The line l shown in Figure 2 illustrates the linear relationship between t and $\log_{10}P$ for the population over a period of 100 years.

The line l meets the vertical axis at (0, 5) as shown. The gradient of l is $\frac{1}{200}$.

(a) Write down an equation for l.

(2)

(b) Find the value of a and the value of b.

(4)

(c) With reference to the model interpret

(i) the value of the constant a,

(ii) the value of the constant b.

(2)

(d) Find

(i) the population predicted by the model when $t = 100$, giving your answer to the nearest hundred thousand,

(ii) the number of years it takes the population to reach 200 000, according to the model.

(3)

(e) State two reasons why this may not be a realistic population model.

(2)

Question 14 continued

a) $l: \log_{10} P = \frac{1}{200} t + 5$

b) $\log_{10} P = \frac{1}{200} t + 5$.

$P = 10^{\frac{1}{200}t + 5}$

$P = 10^5 (10^{\frac{1}{200}})^t$.

$a = 10^5 = 100,000$

$b = 10^{\frac{1}{200}} = 1.0116$

c) i) a is the population of the town when first recorded.

ii) b is the growth rate per year of the population of the town.

d) i) $P = 100,000 \times 1.0116^{100}$

$P = 300,000$ to nearest 100,000.

ii) $200,000 = 100,000 \times (10^{\frac{1}{200}})^t$

$2 = (10^{\frac{1}{200}})^t$.

$\log 2 = t \log (10^{\frac{1}{200}})$

$\log 2 = \frac{1}{200} t \log (10)$

$t = \frac{200 \log 2}{\log 10}$

$t = 60.2$ (to 3sf).

e) Population might not grow at the same rate each year.

The model predicts unlimited growth.

(Total for Question 14 is 13 marks)

15.

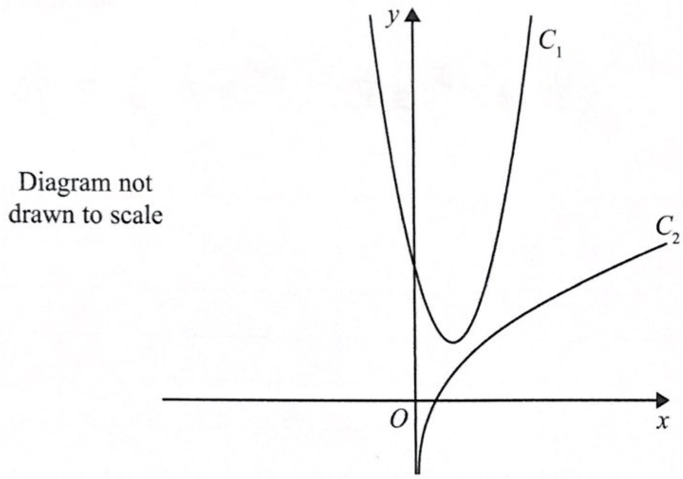

Figure 3

The curve C_1, shown in Figure 3, has equation $y = 4x^2 - 6x + 4$.

The point $P\left(\frac{1}{2}, 2\right)$ lies on C_1

The curve C_2, also shown in Figure 3, has equation $y = \frac{1}{2}x + \ln(2x)$.

The normal to C_1 at the point P meets C_2 at the point Q.

Find the exact coordinates of Q.

(*Solutions based entirely on graphical or numerical methods are not acceptable.*)

(8)

$y = 4x^2 - 6x + 4$.

$\frac{dy}{dx} = 8x - 6$.

$8(\frac{1}{2}) - 6 = 4 - 6 = -2$

Normal gradient at P is $\frac{1}{2}$.

$y - y_1 = m(x - x_1)$

$y - 2 = \frac{1}{2}(x - \frac{1}{2})$.

$y - 2 = \frac{1}{2}x - \frac{1}{4}$.

$y = \frac{1}{2}x + \frac{7}{4}$.

$y = \frac{1}{2}x + \ln(2x)$.

$\frac{1}{2}x + \frac{7}{4} = \frac{1}{2}x + \ln(2x)$.

$\frac{7}{4} = \ln(2x)$.

$2x = e^{\frac{7}{4}}$

$x = \frac{1}{2}e^{\frac{7}{4}}$,

$y = \frac{1}{2}(\frac{1}{2}e^{\frac{7}{4}}) + \frac{7}{4}$.

$y = \frac{1}{4}e^{\frac{7}{4}} + \frac{7}{4}$.

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Question 15 continued

$$Q = \left(\tfrac{1}{2}e^{3/4},\ \tfrac{1}{4}e^{7/4} + \tfrac{7}{4} \right).$$

(Total for Question 15 is 8 marks)

16.

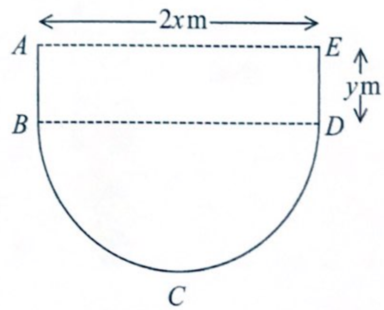

Figure 4

Figure 4 shows the plan view of the design for a swimming pool.

The shape of this pool $ABCDEA$ consists of a rectangular section $ABDE$ joined to a semicircular section BCD as shown in Figure 4.

Given that $AE = 2x$ metres, $ED = y$ metres and the area of the pool is $250\,\text{m}^2$,

(a) show that the perimeter, P metres, of the pool is given by

$$P = 2x + \frac{250}{x} + \frac{\pi x}{2}$$

(4)

(b) Explain why $0 < x < \sqrt{\frac{500}{\pi}}$

(2)

(c) Find the minimum perimeter of the pool, giving your answer to 3 significant figures.

(4)

a) BCD is semicircle.
Perimeter of circle is πd.
So perimeter of semicircle is ~~..~~ $\frac{1}{2}\pi d$.
$d = 2x$.
Perimeter of semicircle $= \pi x$.
Perimeter of rectangle is $2x + 2y$.
$P = 2x + 2y + \pi x$.

Area $= 2xy + \frac{1}{2}\pi \cancel{(..)}\, x^2 = 250$

$\cancel{2xy} \quad y + \frac{1}{4}\pi x = \frac{125}{x}$.

$y = \frac{125}{x} - \frac{1}{4}\pi x$.

$P = 2x + 2\left(\frac{125}{x} - \frac{1}{4}\pi x\right) + \pi x$.

$P = 2x + \frac{250}{x} - \frac{1}{2}\pi x + \pi x$.

$P = 2x + \frac{250}{x} + \frac{1}{2}\pi x$.

b) $x > 0$ otherwise Area $= 0$.

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Question 16 continued

Area minimised at $y=0$.

$\frac{1}{2}\pi x^2 = 250$ at max x.

$\pi x^2 = 500$

$x^2 = \frac{500}{\pi}$

$x = \sqrt{\frac{500}{\pi}}$.

Hence, $0 < x < \sqrt{\frac{500}{\pi}}$.

c) $P = 2x + \frac{250}{x} + \frac{\pi x}{2}$.

$\frac{dP}{dx} = 0$ at minimum.

$2 - \frac{250}{x^2} + \frac{\pi}{2} = 0$.

$2 + \frac{\pi}{2} = \frac{250}{x^2}$.

$x^2 = \frac{250}{2+\frac{\pi}{2}}$.

$x^2 = \frac{500}{4+\pi}$.

$x = \sqrt{\frac{500}{4+\pi}}$.

$P = 2\sqrt{\frac{500}{4+\pi}} + \frac{250}{\sqrt{\frac{500}{4+\pi}}} + \frac{\pi}{2}\sqrt{\frac{500}{4+\pi}}$.

$P = 59.8m$.

(Total for Question 16 is 10 marks)

17. A circle C with centre at $(-2, 6)$ passes through the point $(10, 11)$.

(a) Show that the circle C also passes through the point $(10, 1)$.

(3)

The tangent to the circle C at the point $(10, 11)$ meets the y axis at the point P and the tangent to the circle C at the point $(10, 1)$ meets the y axis at the point Q.

(b) Show that the distance PQ is 58 explaining your method clearly.

(7)

a) radius $= \sqrt{(10-(-2))^2 + (11-6)^2} = \sqrt{12^2+5^2}$

$= \sqrt{144+25}$

$= \sqrt{169}$

$= 13$

Distance $(-2,6), (10,1) =$

$\sqrt{(10-(-2))^2 + (1-6)^2} =$

$\sqrt{12^2 + 5^2} =$

$\sqrt{144+25} =$

$\sqrt{169} =$

$13 =$ radius

Hence, circle passes through $(0,1)$.

b) Gradient of radius $= \frac{11-6}{10-(-2)} = \frac{5}{12}$

Gradient of tangent $= \frac{-12}{5}$

$y - y_1 = m(x - x_1)$

$y - 11 = -\frac{12}{5}(x-10)$

$y - 11 = -\frac{12}{5}x + 24$

$y = -\frac{12}{5}x + 35$

$x = 0$ at y-axis

$P = (0, 35)$

Gradient of radius $= \frac{1-6}{10-(-2)} = \frac{-5}{12}$

Gradient of tangent $= \frac{12}{5}$

$y - y_1 = m(x - x_1)$

$y - 1 = \frac{12}{5}(x - 10)$

$y - 1 = \frac{12}{5}x - 24$

$y = \frac{12}{5}x - 23$

$x = 0$ at y axis.

$Q = (0, -23)$.

$35 - (-23) = 58$ so P and Q are 58 units apart.

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