Write your name here

Surname	Other names

Pearson Edexcel Level 3 GCE

Centre Number | Candidate Number

Mathematics

Advanced

Paper 3: Statistics and Mechanics

Sample Assessment Material for first teaching September 2017

Time: 2 hours

Paper Reference **9MA0/03**

You must have:
Mathematical Formulae and Statistical Tables, calculator

Total Marks

Candidates may use any calculator permitted by Pearson regulations. Calculators must not have the facility for algebraic manipulation, differentiation and integration, or have retrievable mathematical formulae stored in them.

Instructions

- Use **black** ink or ball-point pen.
- If pencil is used for diagrams/sketches/graphs it must be dark (HB or B).
- **Fill in the boxes** at the top of this page with your name, centre number and candidate number.
- There are **two** sections in this question paper. Answer **all** the questions in Section A and **all** the questions in Section B.
- Answer the questions in the spaces provided – *there may be more space than you need.*
- You should show sufficient working to make your methods clear. Answers without working may not gain full credit.
- Answers should be given to three significant figures unless otherwise stated.

Information

- A booklet 'Mathematical Formulae and Statistical Tables' is provided.
- There are 10 questions in this question paper. The total mark for this paper is 100.
- The marks for **each** question are shown in brackets – *use this as a guide as to how much time to spend on each question.*

Advice

- Read each question carefully before you start to answer it.
- Try to answer every question.
- Check your answers if you have time at the end.
- If you change your mind about an answer cross it out and put your new answer and any working out underneath.

Turn over

S54261A

1/1/1/1/1/1/1/

S 5 4 2 6 1 A 0 1 2 5

Pearson

DO NOT WRITE IN THIS AREA

SECTION A: STATISTICS

Answer ALL questions. Write your answers in the spaces provided.

1. The number of hours of sunshine each day, y, for the month of July at Heathrow are summarised in the table below.

Hours	$0 \leqslant y < 5$	$5 \leqslant y < 8$	$8 \leqslant y < 11$	$11 \leqslant y < 12$	$12 \leqslant y < 14$
Frequency	12	6	8	3	2

A histogram was drawn to represent these data. The $8 \leqslant y < 11$ group was represented by a bar of width 1.5 cm and height 8 cm.

(a) Find the width and the height of the $0 \leqslant y < 5$ group.

(3)

(b) Use your calculator to estimate the mean and the standard deviation of the number of hours of sunshine each day, for the month of July at Heathrow.
Give your answers to 3 significant figures.

(3)

The mean and standard deviation for the number of hours of daily sunshine for the same month in Hurn are 5.98 hours and 4.12 hours respectably.
Thomas believes that the further south you are the more consistent should be the number of hours of daily sunshine.

(c) State, giving a reason, whether or not the calculations in part (b) support Thomas' belief.

(2)

(d) Estimate the number of days in July at Heathrow where the number of hours of sunshine is more than 1 standard deviation above the mean.

(2)

Helen models the number of hours of sunshine each day, for the month of July at Heathrow by $N(6.6, 3.7^2)$.

(e) Use Helen's model to predict the number of days in July at Heathrow when the number of hours of sunshine is more than 1 standard deviation above the mean.

(2)

(f) Use your answers to part (d) and part (e) to comment on the suitability of Helen's model.

(1)

(a) Area for $8 \leqslant y < 11$: $8\text{cm} \times 1.5\text{cm} = 12\text{cm}^2$
Frequency " : 8 hours

So area for $0 \leq y < 5$: 18cm^2.
width = 2.5cm so height = 7.2cm.

DO NOT WRITE IN THIS AREA

DO NOT WRITE IN THIS AREA

Question 1 continued

(b) $\bar{y} = \dfrac{205.5}{31} \approx 6.63$

$\sigma_y = \sqrt{\dfrac{1785.25}{31} - \bar{y}^2} = \sqrt{13.6446} \approx 3.69$

(c) Standard deviation decreases for Heathrow, so its data is more reliable.

Hurn is south of Heathrow so it does not support his belief.

(d) $\bar{x} + \sigma \approx 10.3$

$\left(\dfrac{11 - 10.3}{3} \times 8\right) + 5 \approx 6.8\dot{6}$.

(e) $P(H > 10.3) \approx 0.15865$.

$31 \times 0.15865 \approx 4.9$ days.

(f) 4.9 days $< 6.8\dot{6}$ days so the model may not be suitable.

(Total for Question 1 is 13 marks)

2. A meteorologist believes that there is a relationship between the daily mean windspeed, w kn, and the daily mean temperature, t °C. A random sample of 9 consecutive days is taken from past records from a town in the UK in July and the relevant data is given in the table below.

t	13.3	16.2	15.7	16.6	16.3	16.4	19.3	17.1	13.2
w	7	11	8	11	13	8	15	10	11

The meteorologist calculated the product moment correlation coefficient for the 9 days and obtained $r = 0.609$

(a) Explain why a linear regression model based on these data is unreliable on a day when the mean temperature is 24 °C

(1)

(b) State what is measured by the product moment correlation coefficient.

(1)

(c) Stating your hypotheses clearly test, at the 5% significance level, whether or not the product moment correlation coefficient for the population is greater than zero.

(3)

Using the same 9 days a location from the large data set gave $\bar{t} = 27.2$ and $\bar{w} = 3.5$

(d) Using your knowledge of the large data set, suggest, giving your reason, the location that gave rise to these statistics.

(1)

(a) Extrapolation is required, the model may not hold.

(b) The linear relationship between w and t.

(c) $H_0: \rho = 0$, $H_1: \rho > 0$

Critical value: 0.5822

$r = 0.609 > 0.5822$

So reject H_0, there is evidence that $\rho > 0$.

(d) Higher $\bar{t}$ suggests this is overseas, not Perth. Lower wind speed suggests this is inland, so Beijing.

DO NOT WRITE IN THIS AREA

DO NOT WRITE IN THIS AREA

DO NOT WRITE IN THIS AREA

3. A machine cuts strips of metal to length L cm, where L is normally distributed with standard deviation 0.5 cm.

Strips with length either less than 49 cm or greater than 50.75 cm **cannot** be used.

Given that 2.5% of the cut lengths exceed 50.98 cm,

(a) find the probability that a randomly chosen strip of metal **can** be used.

(5)

Ten strips of metal are selected at random.

(b) Find the probability fewer than 4 of these strips **cannot** be used.

(2)

A second machine cuts strips of metal of length X cm, where X is normally distributed with standard deviation 0.6 cm

A random sample of 15 strips cut by this second machine was found to have a mean length of 50.4 cm

(c) Stating your hypotheses clearly and using a 1% level of significance, test whether or not the mean length of all the strips, cut by the second machine, is greater than 50.1 cm

(5)

(a) $P(L > 50.98) = 0.025$

$$\frac{50.98 - \mu}{0.5} = 1.96 \Rightarrow \mu = 50.$$

$$P(49 < L < 50.75) = P(L < 50.75) - P(L < 49)$$
$$= 0.9104\ldots$$
$$= \underline{0.910}$$

(b) ~~P(S ≤ 3)~~ $S \sim B(10, 0.090)$

$$P(S \leq 3) = 0.991166\ldots = \underline{0.991}$$

(c) $H_0: \mu = 50.1$, $H_1: \mu > 50.1$

$$\bar{X} \sim N\left(50.1, \frac{0.6^2}{15}\right), \quad \bar{x} > 50.4$$

$$P(\bar{X} > 50.4) = 0.0264. > 0.01$$

So we cannot reject H_0, and that the mean length of strips is greater than 50.1.

4. Given that

$$P(A) = 0.35 \qquad P(B) = 0.45 \qquad \text{and} \qquad P(A \cap B) = 0.13$$

find

(a) $P(A' \mid B')$

(2)

(b) Explain why the events A and B are not independent.

(1)

The event C has $P(C) = 0.20$

The events A and C are mutually exclusive and the events B and C are statistically independent.

(c) Draw a Venn diagram to illustrate the events A, B and C, giving the probabilities for each region.

(5)

(d) Find $P([B \cup C]')$

(2)

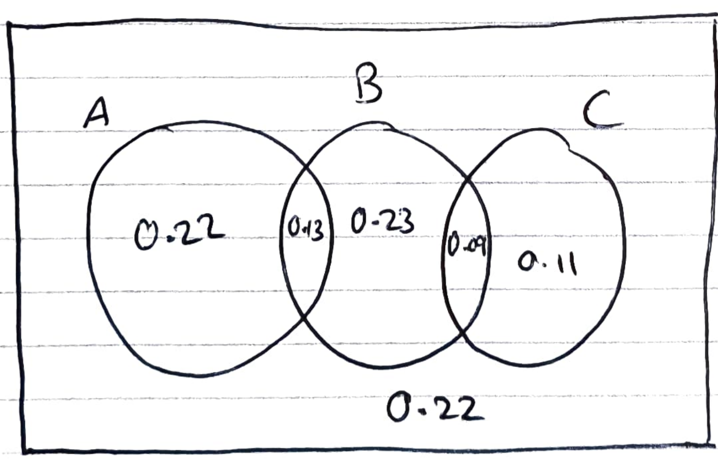
$1 - P(A) - P(B) + P(A \cap B)$

(a) $P(A' | B') = \dfrac{P(A' \cap B')}{P(B')} = \dfrac{0.33}{0.55}$

$= 0.6$

(b) $P(A)\,P(B) = \frac{63}{400} \neq P(A \cap B) = \frac{52}{400}$

(c)



5. A company sells seeds and claims that 55% of its pea seeds germinate.

(a) Write down a reason why the company should not justify their claim by testing all the pea seeds they produce.
(1)

A random selection of the pea seeds is planted in 10 trays with 24 seeds in each tray.

(b) Assuming that the company's claim is correct, calculate the probability that in at least half of the trays 15 or more of the seeds germinate.
(3)

(c) Write down two conditions under which the normal distribution may be used as an approximation to the binomial distribution.
(1)

A random sample of 240 pea seeds was planted and 150 of these seeds germinated.

(d) Assuming that the company's claim is correct, use a normal approximation to find the probability that at least 150 pea seeds germinate.
(3)

(e) Using your answer to part (d), comment on whether or not the proportion of the company's pea seeds that germinate is different from the company's claim of 55%
(1)

(a) They would be unable to sell any produce.

(b) $S \sim B(24, 0.55)$
$T \sim B(10, p)$

$p = P(S \geqslant 15) = 0.299126...$
So $P(T \geqslant 5) = 0.1487... = \underline{0.149}$

(c) - n is large
- $p \approx 0.5$

(d) $X \sim N(132, 59.4)$

$P(X \geqslant 149.5) = P\left(Z \geqslant \frac{149.5 - 132}{\sqrt{59.4}}\right)$

$= 0.01158... = \underline{0.0116}$

(e) The probability is very small, so

DO NOT WRITE IN THIS AREA

DO NOT WRITE IN THIS AREA

DO NOT WRITE IN THIS AREA

SECTION B: MECHANICS

Answer ALL questions. Write your answers in the spaces provided.

Unless otherwise indicated, whenever a numerical value of g is required, take $g = 9.8$ m s^{-2} and give your answer to either 2 significant figures or 3 significant figures.

6. At time t seconds, where $t \geqslant 0$, a particle P moves so that its acceleration $\mathbf{a}$ m s^{-2} is given by

$$\mathbf{a} = 5t\mathbf{i} - 15t^{\frac{1}{2}}\mathbf{j}$$

When $t = 0$, the velocity of P is $20\mathbf{i}$ m s^{-1}

Find the speed of P when $t = 4$

(6)

$\int a = v = \int 5t\underline{\hat{i}} - 15t^{\frac{1}{2}}\underline{\hat{j}} \, dt$

$= \frac{5}{2}t^2\underline{\hat{i}} - 10t^{\frac{3}{2}}\underline{\hat{j}} + \underline{c}$

when $t=0$, $v = 20\underline{\hat{i}}$, so $\underline{c} = 20\underline{\hat{i}}$.

$v = \frac{5}{2}t^2\underline{\hat{i}} + 20\underline{\hat{i}} - 10t^{\frac{3}{2}}\underline{\hat{j}}$ ms^{-1}

$t = 4$:

$v = 40\underline{\hat{i}} + 20\underline{\hat{i}} - 80\underline{\hat{j}}$ ms^{-1}

$= 60\underline{\hat{i}} - 80\underline{\hat{j}}$ ms^{-1}

$\Rightarrow$ speed $= |v| = \sqrt{60^2 + 80^2} = 100$ ms^{-1}

DO NOT WRITE IN THIS AREA

DO NOT WRITE IN THIS AREA

DO NOT WRITE IN THIS AREA

7. A rough plane is inclined to the horizontal at an angle α, where $\tan \alpha = \frac{3}{4}$.

A particle of mass m is placed on the plane and then projected up a line of greatest slope of the plane.

The coefficient of friction between the particle and the plane is μ.

The particle moves up the plane with a constant deceleration of $\frac{4}{5}g$.

(a) Find the value of μ.

(6)

The particle comes to rest at the point A on the plane.

(b) Determine whether the particle will remain at A, carefully justifying your answer.

(2)

(a) • $R = mg\cos\alpha$

• $-F - mg\sin\alpha = \cancel{-0.8g}\ -0.8\,mg$

• $F = \mu R.$

$\Rightarrow -\mu R - mg\sin\alpha = -0.8mg$

$\Rightarrow -\mu \cancel{mg}\cos\alpha = -0.8\cancel{mg} + \cancel{mg}\sin\alpha$

$-\mu = \dfrac{-0.8 + \sin\alpha}{\cos\alpha} \Rightarrow \mu = 1/4.$

(b) Limiting friction is less than the weight component down the plane, so the particle will not rest.

8. [*In this question* **i** *and* **j** *are horizontal unit vectors due east and due north respectively*]

A radio controlled model boat is placed on the surface of a large pond.

The boat is modelled as a particle.

At time $t = 0$, the boat is at the fixed point O and is moving due north with speed $0.6\ \text{m s}^{-1}$.

Relative to O, the position vector of the boat at time t seconds is $\mathbf{r}$ metres.

At time $t = 15$, the velocity of the boat is $(10.5\mathbf{i} - 0.9\mathbf{j})\ \text{m s}^{-1}$.

The acceleration of the boat is constant.

(a) Show that the acceleration of the boat is $(0.7\mathbf{i} - 0.1\mathbf{j})\ \text{m s}^{-2}$.
(2)

(b) Find $\mathbf{r}$ in terms of t.
(2)

(c) Find the value of t when the boat is north-east of O.
(3)

(d) Find the value of t when the boat is moving in a north-east direction.
(3)

(a) $\underline{v} = \underline{u} + \underline{a}t$

$$10.5\underline{i} - 0.9\underline{j} = 0.6\underline{j} + 15\underline{a}$$

$$\Rightarrow \underline{a} = 0.7\underline{i} - 0.1\underline{j} \quad ms^{-2}.$$

(b) $\underline{r} = \underline{u}t + \frac{1}{2}\underline{a}t^2$

$$\underline{r} = 0.6\underline{j}t + \frac{1}{2}(0.7\underline{i} - 0.1\underline{j})t^2$$

(c) $0.6t - 0.05t^2 = 0.35t^2$

$\Rightarrow t = 1.5s.$

(d) $\underline{v} = \underline{u} + \underline{a}t$: $\underline{v} = 0.6\underline{j} + (0.7\underline{i} - 0.1\underline{j})t$

$0.6 - 0.1t = 0.7t \Rightarrow t = 0.75s.$

9.

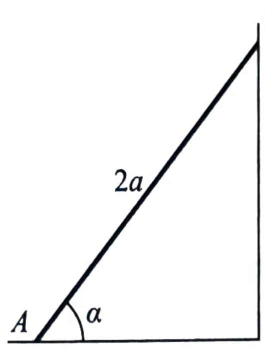


Figure 1

A uniform ladder AB, of length $2a$ and weight W, has its end A on rough horizontal ground.

The coefficient of friction between the ladder and the ground is $\frac{1}{4}$.

The end B of the ladder is resting against a smooth vertical wall, as shown in Figure 1.

A builder of weight $7W$ stands at the top of the ladder.
To stop the ladder from slipping, the builder's assistant applies a horizontal force of magnitude P to the ladder at A, towards the wall.
The force acts in a direction which is perpendicular to the wall.

The ladder rests in equilibrium in a vertical plane perpendicular to the wall and makes an angle α with the horizontal ground, where $\tan\alpha = \frac{5}{2}$.

The builder is modelled as a particle and the ladder is modelled as a uniform rod.

(a) Show that the reaction of the wall on the ladder at B has magnitude $3W$.

(5)

(b) Find, in terms of W, the range of possible values of P for which the ladder remains in equilibrium.

(5)

Often in practice, the builder's assistant will simply stand on the bottom of the ladder.

(c) Explain briefly how this helps to stop the ladder from slipping.

(3)

(a) $Wa\cos\alpha + 7W \cdot 2a\cos\alpha = S \cdot 2a\sin\alpha$

$\Rightarrow 15Wa\cos\alpha = 2Sa\sin\alpha$

$\tan\alpha = 5/2 \Rightarrow 3W.$

(b) $R = 8W$

$F = \frac{1}{4}R \; (= 2W)$

$\Rightarrow P_{max} = 3W + F, \quad P_{min} = 3W - F.$

DO NOT WRITE IN THIS AREA DO NOT WRITE IN THIS AREA DO NOT WRITE IN THIS AREA

Question 9 continued

$\Rightarrow \quad P_{max} = 5W, \qquad P_{min} = W.$

$\Rightarrow \quad W \leq P \leq 5W$

(c) The reaction at B is unchanged.
R increases, which increases the max F.

DO NOT WRITE IN THIS AREA

DO NOT WRITE IN THIS AREA

DO NOT WRITE IN THIS AREA

10.

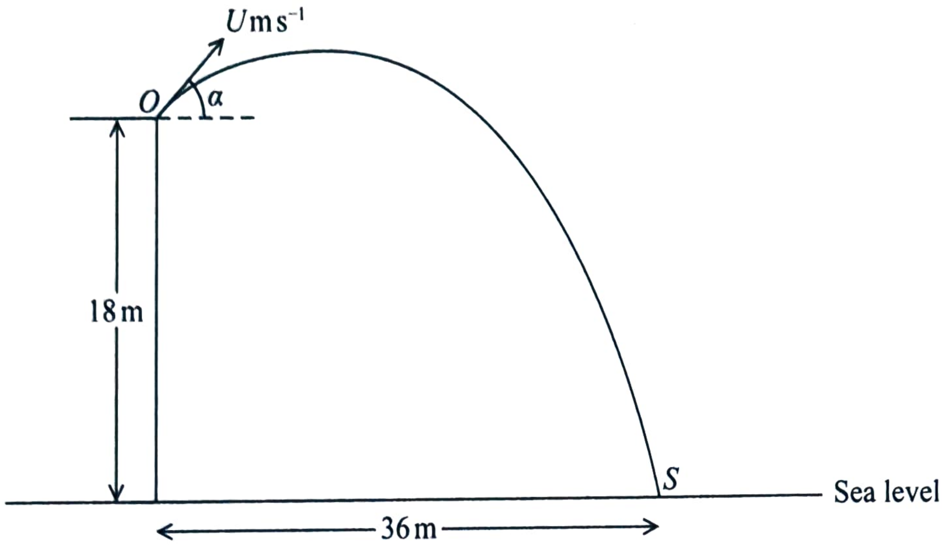


Figure 2

A boy throws a stone with speed $U\,\mathrm{m\,s^{-1}}$ from a point O at the top of a vertical cliff.
The point O is 18 m above sea level.

The stone is thrown at an angle α above the horizontal, where $\tan\alpha = \frac{3}{4}$.

The stone hits the sea at the point S which is at a horizontal distance of 36 m from the foot of the cliff, as shown in Figure 2.
The stone is modelled as a particle moving freely under gravity with $g = 10\,\mathrm{m\,s^{-2}}$

Find

(a) the value of U,
(6)

(b) the speed of the stone when it is 10.8 m above sea level, giving your answer to 2 significant figures.
(5)

(c) Suggest two improvements that could be made to the model.
(2)

(a) $s = ut$, horizontally.

$36 = Ut\cos\alpha$.

$s = ut + \frac{1}{2}at^2$, vertically.

$-18 = Ut\sin\alpha - \frac{1}{2}gt^2$.

$\Rightarrow U = 15\,\mathrm{ms^{-1}}$.

Question 10 continued

(b) $U\cos\alpha = 15 \cdot 0.8 = 12\ \text{ms}^{-1}$ horizontally

$v^2 = u^2 + 2as$

$v^2 = (\underset{15}{U}\sin\underset{0.6}{\alpha})^2 + 2(-10)(-7.2)$.

$\Rightarrow v = 15\ \text{ms}^{-1}$.

$\text{Speed} = \sqrt{V_{vert}^2 + V_{hor}^2} = \sqrt{12^2 + 15^2}$

$= \sqrt{369}$

$\approx 19\ \text{ms}^{-1}$

(c) – Include air resistance in model.

– Include wind effects.