Write your name here

Surname	Other names
MODEL SOLUTION	

Pearson Edexcel Level 3 GCE

Centre Number | Candidate Number

Mathematics

Advanced
Paper 2: Pure Mathematics 2

Sample Assessment Material for first teaching September 2017
Time: 2 hours

Paper Reference
9MA0/02

You must have:
Mathematical Formulae and Statistical Tables, calculator

Total Marks

Candidates may use any calculator permitted by Pearson regulations. Calculators must not have the facility for algebraic manipulation, differentiation and integration, or have retrievable mathematical formulae stored in them.

Instructions
- Use **black** ink or ball-point pen.
- If pencil is used for diagrams/sketches/graphs it must be dark (HB or B).
- **Fill in the boxes** at the top of this page with your name, centre number and candidate number.
- Answer **all** questions and ensure that your answers to parts of questions are clearly labelled.
- Answer the questions in the spaces provided – *there may be more space than you need.*
- You should show sufficient working to make your methods clear. Answers without working may not gain full credit.
- Answers should be given to three significant figures unless otherwise stated.

Information
- A booklet 'Mathematical Formulae and Statistical Tables' is provided.
- There are 16 questions in this question paper. The total mark for this paper is 100.
- The marks for **each** question are shown in brackets – *use this as a guide as to how much time to spend on each question.*

Advice
- Read each question carefully before you start to answer it.
- Try to answer every question.
- Check your answers if you have time at the end.
- If you change your mind about an answer cross it out and put your new answer and any working out underneath.

Turn over

S54260A
1/1/1/1/1/1/

Pearson

Answer ALL questions. Write your answers in the spaces provided.

1.

$$f(x) = 2x^3 - 5x^2 + ax + a$$

Given that $(x + 2)$ is a factor of $f(x)$, find the value of the constant a.

(3)

$x+2$ is a factor of $f(x)$.

By factor theorem, $f(-2) = 0$.

$2 \times (-2)^3 - 5 \times (-2)^2 + a \times (-2) + a = 0$.

$-16 - 20 - 2a + a = 0$.

$-16 - 20 - a = 0$

$-36 - a = 0$

$a = -36$.

(Total for Question 1 is 3 marks)

DO NOT WRITE IN THIS AREA
DO NOT WRITE IN THIS AREA

2. Some A level students were given the following question.

Solve, for $-90° < \theta < 90°$, the equation

$$\cos\theta = 2\sin\theta$$

The attempts of two of the students are shown below.

Student *A*	Student *B*
$\cos\theta = 2\sin\theta$ $\tan\theta = 2$ $\theta = 63.4°$	$\cos\theta = 2\sin\theta$ $\cos^2\theta = 4\sin^2\theta$ $1 - \sin^2\theta = 4\sin^2\theta$ $\sin^2\theta = \frac{1}{5}$ $\sin\theta = \pm\frac{1}{\sqrt{5}}$ $\theta = \pm 26.6°$

(a) Identify an error made by student *A*.

(1)

Student *B* gives $\theta = -26.6°$ as one of the answers to $\cos\theta = 2\sin\theta$.

(b) (i) Explain why this answer is incorrect.

(ii) Explain how this incorrect answer arose.

(2)

a) They have taken $\frac{\cos\theta}{\sin\theta} = \tan\theta$ when actually $\frac{\sin\theta}{\cos\theta} = \tan\theta$

b) i) $\cos(-26.6) = 0.894$.
$2\sin(-26.6) = 0.896$.

ii) Squaring introduces another solution, which might not pertain to the original equation.

(Total for Question 2 is 3 marks)

3. Given $y = x(2x + 1)^4$, show that

$$\frac{dy}{dx} = (2x + 1)^n(Ax + B)$$

where n, A and B are constants to be found.

(4)

$y = x(2x+1)^4$.

Product rule :

$\frac{dy}{dx} = x \underbrace{\frac{d}{dx}((2x+1)^4)}_{\text{Chain rule } = 8(2x+1)^3} + \frac{d}{dx}(x)(2x+1)^4$.

$\frac{dy}{dx} = 8x(2x+1)^3 + (2x+1)^4$.

$\frac{dy}{dx} = (8x+2x+1)(2x+1)^3$.

$\frac{dy}{dx} = (10x+1)(2x+1)^3$.

$\Rightarrow n = 3, \quad A = 10, \quad B = 1$.

(Total for Question 3 is 4 marks)

DO NOT WRITE IN THIS AREA DO NOT WRITE IN THIS AREA DO NOT WRITE IN THIS AREA

4. Given

$$f(x) = e^x, \quad x \in \mathbb{R}$$

$$g(x) = 3\ln x, \quad x > 0,\ x \in \mathbb{R}$$

(a) find an expression for gf(x), simplifying your answer.

(2)

(b) Show that there is only one real value of x for which gf(x) = fg(x)

(3)

a) $gf(x) = 3\ln(e^x) = 3x$.

b) $fg(x) = e^{3\ln x} = e^{\ln x^3} = x^3$.

$x^3 = 3x$.

$x^3 - 3x = 0$.

$x(x^2 - 3) = 0$.

$x(x - \sqrt{3})(x + \sqrt{3}) = 0$.

$x = 0 \quad x = \sqrt{3} \quad x = -\sqrt{3}$.

$\ln x$ not defined at $x = 0, -\sqrt{3}$.

$x = \sqrt{3}$ only.

(Total for Question 4 is 5 marks)

DO NOT WRITE IN THIS AREA

5. The mass, m grams, of a radioactive substance, t years after first being observed, is modelled by the equation

$$m = 25e^{-0.05t}$$

According to the model,

(a) find the mass of the radioactive substance six months after it was first observed,

(2)

(b) show that $\frac{dm}{dt} = km$, where k is a constant to be found.

(2)

a) $t = 0.5$

$m = 25e^{-0.05\times0.5}$

$m = 25e^{-0.025}$

$m = 24.4g$.

b) $m = 25e^{-0.05t}$

$\frac{dm}{dt} = -0.05\times25e^{-0.05t}$

$\frac{dm}{dt} = -0.05m$.

$k = -0.05$.

DO NOT WRITE IN THIS AREA DO NOT WRITE IN THIS AREA DO NOT WRITE IN THIS AREA

(Total for Question 5 is 4 marks)

6. Complete the table below. The first one has been done for you.

For each statement you must state if it is always true, sometimes true or never true, giving a reason in each case.

Statement	Always True	Sometimes True	Never True	Reason
The quadratic equation $ax^2 + bx + c = 0, \quad (a \neq 0)$ has 2 real roots.		✓		It only has 2 real roots when $b^2 - 4ac > 0$. When $b^2 - 4ac = 0$ it has 1 real root and when $b^2 - 4ac < 0$ it has 0 real roots.
(i) When a real value of x is substituted into $x^2 - 6x + 10$ the result is positive. (2)	✓			$(x-3)^2 > 0$ always. $(x-3)^2 + 1 > 0$ always $x^2 - 6x + 9 + 1 > 0$ always $x^2 - 6x + 10 > 0$ always.
(ii) If $ax > b$ then $x > \frac{b}{a}$ (2)		✓		If $a > 0$ then $ax > b \Rightarrow x > \frac{b}{a}$. If $a < 0$ then $ax > b \Rightarrow x < \frac{b}{a}$.
(iii) The difference between consecutive square numbers is odd. (2)	✓			$(n+1)^2 - n^2 = n^2 + 2n + 1 - n^2$ $= 2n + 1$ odd for all n.

(Total for Question 6 is 6 marks)

7. (a) Use the binomial expansion, in ascending powers of x, to show that

$$\sqrt{(4-x)} = 2 - \frac{1}{4}x + kx^2 + \ldots$$

where k is a rational constant to be found.

(4)

A student attempts to substitute $x = 1$ into both sides of this equation to find an approximate value for $\sqrt{3}$.

(b) State, giving a reason, if the expansion is valid for this value of x.

(1)

a) $\sqrt{(4-x)} = (4-x)^{\frac{1}{2}}$

$= 2(1-\frac{1}{4}x)^{\frac{1}{2}}$

$= 2\left(1 + \frac{1}{2}(-\frac{1}{4}x) + \frac{(\frac{1}{2})(-\frac{1}{2})}{2!}(-\frac{1}{4}x)^2\right)$

$= 2(1 - \frac{1}{8}x - \frac{1}{128}x^2 \ldots)$

$= 2 - \frac{1}{4}x - \frac{1}{64}x^2 \ldots$

$k = -\frac{1}{64}$.

b) This is valid for $|\frac{1}{4}x| < 1$

ie $|x| < 4$,

so $x = 1$ can be used.

8.

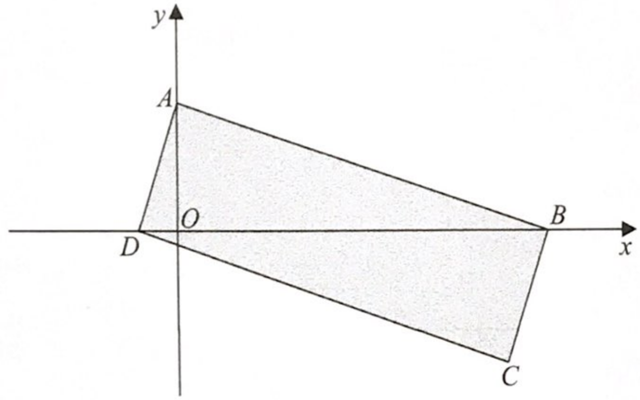

Figure 1

Figure 1 shows a rectangle $ABCD$.

The point A lies on the y-axis and the points B and D lie on the x-axis as shown in Figure 1.

Given that the straight line through the points A and B has equation $5y + 2x = 10$

(a) show that the straight line through the points A and D has equation $2y - 5x = 4$

(4)

(b) find the area of the rectangle $ABCD$.

(3)

a) At A, $x = 0$

$5y = 10$

$y = 2$

A is the point $(0, 2)$

$5y + 2x = 10$ has gradient $-\frac{2}{5}$

So AD has gradient $\frac{5}{2}$, so is of the form $2y - 5x = k$ for some k.

Sub in $(0, 2)$: $2\times 2 - 5\times 0 = k$ so $k = 4$; $2y - 5x = 4$.

b) At B, D, $y = 0$.

$2x = 10$

$x = 5$

$B = (5, 0)$

$5x = 4$

$x = \frac{4}{5}$

$D = (\frac{4}{5}, 0)$

$|AD| = \sqrt{2^2 + (\frac{4}{5})^2}$

$|AD| = \sqrt{4 + \frac{16}{25}}$

$|AD| = \sqrt{\frac{100}{25} + \frac{16}{25}}$

$|AD| = \sqrt{\frac{116}{25}}$

$|AB| = \sqrt{5^2 + 2^2}$

$|AB| = \sqrt{25 + 4}$

$|AB| = \sqrt{29}$

Area $= \sqrt{\frac{116}{25}} \times \sqrt{29}$

Area $= \sqrt{\frac{116 \times 29}{25}}$

Area $= 11.6$.

9. Given that A is constant and

$$\int_1^4 \left(3\sqrt{x} + A\right)\mathrm{d}x = 2A^2$$

show that there are exactly two possible values for A.

(5)

$\int_1^4 3\sqrt{x} + A \, dx = 2A^2$.

$\left[3 \times \frac{2}{3} x^{\frac{3}{2}} + Ax\right]_1^4 = 2A^2$.

$\left[2x^{\frac{3}{2}} + Ax\right]_1^4 = 2A^2$.

$2 \times 8 + 4A - 2 \times 1 - A = 2A^2$.

$16 - 2 + 3A = 2A^2$.

$14 + 3A = 2A^2$.

$2A^2 - 3A - 14 = 0$.

$(2A - 7)(A + 2)$

$A = \frac{7}{2} \quad A = 2$. two roots.

Two possible values

(Total for Question 9 is 5 marks)

10. In a geometric series the common ratio is r and sum to n terms is S_n

Given

$$S_\infty = \frac{8}{7} \times S_6$$

show that $r = \pm\frac{1}{\sqrt{k}}$, where k is an integer to be found.

(4)

$S_\infty = \frac{8}{7} \times S_6$

$\frac{a}{1-r} = \frac{8}{7} \times \frac{a(1-r^6)}{1-r}$

$1 = \frac{8}{7} \times (1-r^6)$

$1 = \frac{8}{7} - \frac{8}{7}r^6$

$\frac{8}{7}r^6 = \frac{8}{7} - 1$

$\frac{8}{7}r^6 = \frac{1}{7}$

$r^6 = \frac{1}{8}$

$r = \pm\frac{1}{\sqrt{2}}$

Hence, $k = 2$

(Total for Question 10 is 4 marks)

DO NOT WRITE IN THIS AREA

11.

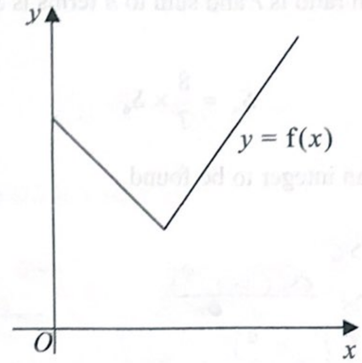

Figure 2

Figure 2 shows a sketch of part of the graph $y = f(x)$, where

$$f(x) = 2|3 - x| + 5, \quad x \geqslant 0$$

(a) State the range of f

(1)

(b) Solve the equation

$$f(x) = \frac{1}{2}x + 30$$

(3)

Given that the equation $f(x) = k$, where k is a constant, has two distinct roots,

(c) state the set of possible values for k.

(2)

a) Range of f is $f(x) \geq 5$

b) $f(x) = \frac{1}{2}x + 30$.

$2|3-x| + 5 = \frac{1}{2}x + 30$.

$2|3-x| = \frac{1}{2}x + 25$

$|3-x| = \frac{1}{4}x + \frac{25}{2}$.

$3 - x = \frac{1}{4}x + \frac{25}{2}$.

$3 - \frac{25}{2} = \frac{5}{4}x$

$-\frac{19}{2} = \frac{5}{4}x$.

$x = -\frac{38}{5} < 0$ so no.

$x - 3 = \frac{1}{4}x + \frac{25}{2}$.

$\frac{3}{4}x = 3 + \frac{25}{2}$

$\frac{3}{4}x = \frac{31}{2}$.

$\boxed{x = \frac{62}{3}}$.

c) $k > 5$ as 5 is minimum of $f(x)$.

$f(0) = 2 \times 3 + 5 = 11$.

$\Rightarrow k \leq 11$.

$5 < k \leq 11$.

12. (a) Solve, for $-180° \leqslant x < 180°$, the equation

$$3\sin^2 x + \sin x + 8 = 9\cos^2 x$$

giving your answers to 2 decimal places.

(6)

(b) Hence find the smallest positive solution of the equation

$$3\sin^2(2\theta - 30°) + \sin(2\theta - 30°) + 8 = 9\cos^2(2\theta - 30°)$$

giving your answer to 2 decimal places.

(2)

a) $3\sin^2 x + \sin x + 8 = 9\cos^2 x$.

$3\sin^2 x + \sin x + 8 = 9(1 - \sin^2 x)$

$3\sin^2 x + \sin x + 8 = 9 - 9\sin^2 x$.

$12\sin^2 x + \sin x - 1 = 0$.

$(4\sin x - 1)(3\sin x + 1) = 0$.

$\sin x = \frac{1}{4}$ $\quad \sin x = -\frac{1}{3}$.

$x = 14.48°, 165.52°, -19.47°, -160.53°$

b) $2\theta - 30 = 14.48, 165.52, -19.47, -160.53$.

Clearly, $2\theta - 30 = -19.47$ is smallest +ve θ.

$2\theta = 10.53$

$\theta = 5.26°$.

DO NOT WRITE IN THIS AREA

13. (a) Express $10\cos\theta - 3\sin\theta$ in the form $R\cos(\theta + \alpha)$, where $R > 0$ and $0 < \alpha < 90°$
Give the exact value of R and give the value of α, in degrees, to 2 decimal places.
(3)

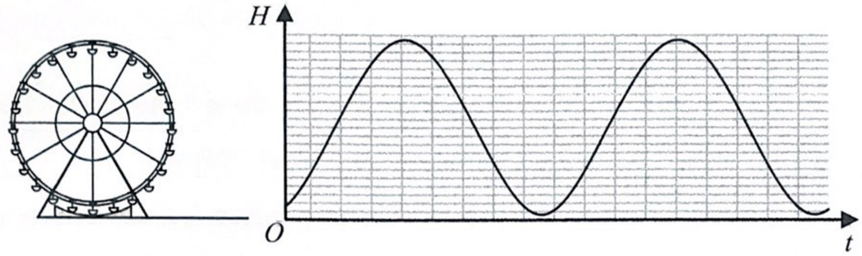

Figure 3

The height above the ground, H metres, of a passenger on a Ferris wheel t minutes after the wheel starts turning, is modelled by the equation

$$H = a - 10\cos(80t)° + 3\sin(80t)°$$

where a is a constant.

Figure 3 shows the graph of H against t for two complete cycles of the wheel.

Given that the initial height of the passenger above the ground is 1 metre,

(b) (i) find a complete equation for the model,

(ii) hence find the maximum height of the passenger above the ground.
(2)

(c) Find the time taken, to the nearest second, for the passenger to reach the maximum height on the second cycle.

(*Solutions based entirely on graphical or numerical methods are not acceptable.*)
(3)

It is decided that, to increase profits, the speed of the wheel is to be increased.

(d) How would you adapt the equation of the model to reflect this increase in speed?
(1)

a) $10\cos\theta - 3\sin\theta$

$R\cos(\theta+\alpha) =$

$R(\cos\theta \sin\alpha \; \cancel{\theta} \; \sin\theta\cos\alpha)$.

$R\sin\alpha = 10, \quad R\cos\alpha = 3$.

$R^2\sin^2\alpha + R^2\cos^2\alpha = 10^2 + 3^2$

$R^2 = 100 + 9$

$R = \sqrt{109}$.

$\tan\alpha = \frac{3}{10}$.

$\alpha = 16.70°$.

$10\cos\theta - 3\sin\theta = \sqrt{109}\cos(\theta + 16.70)$

DO NOT WRITE IN THIS AREA

Question 13 continued

b) i) $H = a - 10\cos(80t) + 3\sin(80t)$.

$H(0) = a - 10$

$a - 10 = 1$

$a = 11$.

$H = 11 - 10\cos(80t) + 3\sin(80t)$.

$H = 11 - \sqrt{109}\cos(80t + 16.70)$.

ii) $\text{Max} = 11 + \sqrt{109} = 21.44\text{m}$.

c) Happens at $540°$

$80t + 16.70 = 540$.

$80t = 523.30$.

$t = 6.54$ mins.

d) 80 represents speed, so increase this 80, eg $H = 11 - \sqrt{109}\cos(100t + 16.7)$.

(Total for Question 13 is 9 marks)

14. A company decides to manufacture a soft drinks can with a capacity of 500 ml.

The company models the can in the shape of a right circular cylinder with radius r cm and height h cm.

In the model they assume that the can is made from a metal of negligible thickness.

(a) Prove that the total surface area, S cm^2, of the can is given by

$$S = 2\pi r^2 + \frac{1000}{r}$$

(3)

Given that r can vary,

(b) find the dimensions of a can that has minimum surface area.

(5)

(c) With reference to the shape of the can, suggest a reason why the company may choose not to manufacture a can with minimum surface area.

(1)

a) Volume $= \pi r^2 h = 500$

Surface area $=$ ~~[illegible]~~ $2\pi r^2 + 2\pi r h$.

$h = \frac{500}{\pi r^2}$.

$S = 2\pi r^2 +$ ~~[illegible]~~ $2\pi r \times \frac{500}{\pi r^2}$.

$S = 2\pi r^2 + \frac{1000}{r}$.

b) Minimum at $\frac{dS}{dr} = 0$.

$\frac{dS}{dr} = 4\pi r$ ~~[illegible]~~ $- \frac{1000}{r^2} = 0$.

$4\pi r^3 - 1000 = 0$.

$r^3 = \frac{1000}{4\pi}$

$r = \left(\frac{1000}{4\pi}\right)^{\frac{1}{3}}$

$r = 4.30$ cm.

$h = \frac{500}{\pi \times 4.30^2}$

$h = 8.60$ cm.

c) This can would have ~~[illegible]~~ almost same diameter and height, but most drinks cans are taller than they are wide.

15.

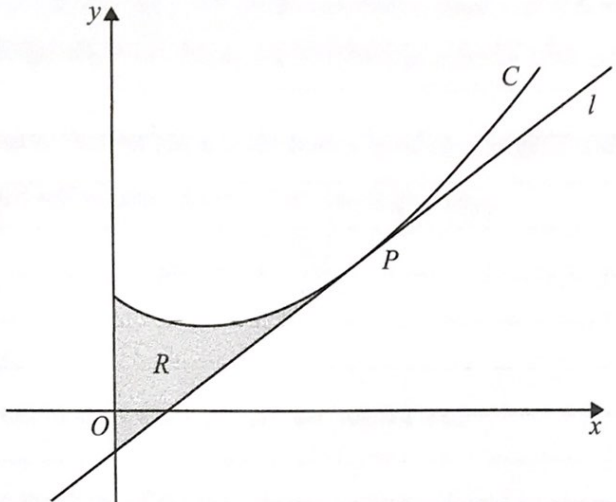

Figure 4

Figure 4 shows a sketch of the curve C with equation

$$y = 5x^{\frac{3}{2}} - 9x + 11, x \geqslant 0$$

The point P with coordinates (4, 15) lies on C.

The line l is the tangent to C at the point P.

The region R, shown shaded in Figure 4, is bounded by the curve C, the line l and the y-axis.

Show that the area of R is 24, making your method clear.

(*Solutions based entirely on graphical or numerical methods are not acceptable.*)

(10)

$y = 5x^{\frac{3}{2}} - 9x + 11$

$\frac{dy}{dx} = \frac{15}{2}x^{\frac{1}{2}} - 9$.

At P, $\frac{dy}{dx} = \frac{15}{2} \times 2 - 9 = 15 - 9 = 6$.

$\Rightarrow$ l: $y = 6x - 9$.

Area $= \int_0^4 5x^{\frac{3}{2}} - 9x + 11 - (6x - 9)\, dx$.

$= \int_0^4 5x^{\frac{3}{2}} - 15x + 20\, dx$.

$= \left[2x^{\frac{5}{2}} - \frac{15}{2}x^2 + 20x\right]_0^4$

$= 2 \times 32 - \frac{15}{2} \times 16 + 20 \times 4$

$= 64 - 120 + 80$

$= 24$.

16. (a) Express $\dfrac{1}{P(11-2P)}$ in partial fractions.

(3)

A population of meerkats is being studied.

The population is modelled by the differential equation

$$\frac{\mathrm{d}P}{\mathrm{d}t} = \frac{1}{22}P(11-2P), \quad t \geqslant 0, \qquad 0 < P < 5.5$$

where P, in thousands, is the population of meerkats and t is the time measured in years since the study began.

Given that there were 1000 meerkats in the population when the study began,

(b) determine the time taken, in years, for this population of meerkats to double,

(6)

(c) show that

$$P = \frac{A}{B + Ce^{-\frac{1}{2}t}}$$

where A, B and C are integers to be found.

(3)

a) $\frac{1}{P(11-2P)} = \frac{A}{P} + \frac{B}{11-2P}$

$A(11-2P) + BP = 1.$

$A = \frac{1}{11}, \quad B = \frac{2}{11}.$

$\frac{1}{P(11-2P)} = \frac{1}{11P} + \frac{2}{11(11-2P)}$

b) $\frac{dP}{dt} = \frac{1}{22}P(11-2P).$

$\frac{22}{P(11-2P)} \frac{dP}{dt} = 1$

$\left(\frac{2}{P} + \frac{4}{11-2P}\right)dP = dt.$

$\int \frac{2}{P} + \frac{4}{11-2P}\, dP = \int dt.$

$2\log P - 2\log(11-2P) = t + C.$

At $t=0$, $P=1$

$-2\log 9 = C.$

$2\log P - 2\log(11-2P) = t - 2\log 9.$

Now take $P=2$ for doubling.

$2\log 2 - 2\log 7 = t - 2\log 9.$

$t = 2\log 2 - 2\log 7 + 2\log 9$

$t = 1.89$ years.

c) $2\log P - 2\log(11-2P) = t - 2\log 9.$

$\log\left(\frac{P}{11-2P}\right) = \frac{1}{2}t - \log 9.$

DO NOT WRITE IN THIS AREA

Question 16 continued

$$\frac{P}{11-2P} = e^{\frac{1}{2}t - \ln 9}$$

$$\frac{P}{11-2P} = \frac{1}{9}e^{\frac{1}{2}t}$$

$$P = \frac{1}{9}e^{\frac{1}{2}t}(11-2P)$$

$$P\left(1 + \frac{2}{9}e^{\frac{1}{2}t}\right) = \frac{11}{9}e^{\frac{1}{2}t}$$

$$P = \frac{\frac{11}{9}e^{\frac{1}{2}t}}{1 + \frac{2}{9}e^{\frac{1}{2}t}}$$

$$P = \frac{\frac{11}{9}}{\frac{2}{9} + e^{-\frac{1}{2}t}}$$

$$P = \frac{11}{2 + 9e^{-\frac{1}{2}t}}$$

$A = 11, \quad B = 2, \quad C = 9$

DO NOT WRITE IN THIS AREA

DO NOT WRITE IN THIS AREA

DO NOT WRITE IN THIS AREA