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**Pearson Edexcel Level 3 GCE**

Centre Number

Candidate Number

# Mathematics

## Advanced
## Paper 1: Pure Mathematics 1

Wednesday 6 June 2018 – Morning
**Time: 2 hours**

Paper Reference
**9MA0/01**

**You must have:**
Mathematical Formulae and Statistical Tables, calculator

Total Marks

**Candidates may use any calculator allowed by the regulations of the Joint Council for Qualifications. Calculators must not have the facility for symbolic algebra manipulation, differentiation and integration, or have retrievable mathematical formulae stored in them.**

## Instructions

- Use **black** ink or ball-point pen.
- If pencil is used for diagrams/sketches/graphs it must be dark (HB or B).
- **Fill in the boxes** at the top of this page with your name, centre number and candidate number.
- Answer **all** questions and ensure that your answers to parts of questions are clearly labelled.
- Answer the questions in the spaces provided – *there may be more space than you need.*
- You should show sufficient working to make your methods clear. Answers without working may not gain full credit.
- Answers should be given to three significant figures unless otherwise stated.

## Information

- A booklet 'Mathematical Formulae and Statistical Tables' is provided.
- There are 14 questions in this question paper. The total mark for this paper is 100.
- The marks for each question are shown in brackets – *use this as a guide as to how much time to spend on each question.*

## Advice

- Read each question carefully before you start to answer it.
- Try to answer every question.
- Check your answers if you have time at the end

*Turn over*

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Pearson

**Answer ALL questions. Write your answers in the spaces provided.**

1. Given that $\theta$ is small and is measured in radians, use the small angle approximations to find an approximate value of

$$\frac{1 - \cos 4\theta}{2\theta \sin 3\theta}$$

**(3)**

Assume that $3\theta$ and $4\theta$ $< 1$.

$$\begin{aligned} 1 - \cos 4\theta &= 1 - \left(1 - \tfrac{1}{2}(4\theta)^2\right) \\ &= 1 - (1 - 8\theta^2) \\ &= 1 - 1 + 8\theta^2 \\ &= 8\theta^2. \end{aligned}$$

$$\begin{aligned} 2\theta \sin 3\theta &= 2\theta(3\theta) \\ &= 6\theta^2, \end{aligned}$$

$$\Rightarrow \frac{1 - \cos 4\theta}{2\theta \sin 3\theta} = \frac{8\theta^2}{6\theta^2} = \frac{8}{6} = 4/3$$

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2. A curve $C$ has equation

$$y = x^2 - 2x - 24\sqrt{x}, \qquad x > 0$$

(a) Find (i) $\dfrac{dy}{dx}$

(ii) $\dfrac{d^2y}{dx^2}$

**(3)**

(b) Verify that $C$ has a stationary point when $x = 4$

**(2)**

(c) Determine the nature of this stationary point, giving a reason for your answer.

**(2)**

(a)(i) $y = x^2 - 2x - 24\sqrt{x} = x^2 - 2x - 24x^{\frac{1}{2}}$

$\frac{dy}{dx} = 2 \cdot x - 2 - (24 \cdot \frac{1}{2})x^{-\frac{1}{2}}$

$= 2x - 2 - \frac{12}{\sqrt{x}}$

(ii) $\frac{dy}{dx} = 2x - 2 - \frac{12}{\sqrt{x}}$

$= 2x - 2 - 12x^{-\frac{1}{2}}$

$\Rightarrow \frac{d^2y}{dx^2} = 2 - 0 - (12 \cdot -\frac{1}{2})x^{-\frac{3}{2}}$

$= 2 + \frac{6}{x\sqrt{x}}$

(b) When $x = 4$, $\frac{dy}{dx} = (2 \cdot 4) - 2 - \frac{12}{\sqrt{4}}$

$\Rightarrow \frac{dy}{dx} = 8 - 2 - \frac{12}{2}$

$= 8 - 2 - 6$

$= 0.$

If $\frac{dy}{dx} = 0$, this point is stationary.

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(c) When $x = 4$,

$$\frac{d^2y}{dx^2} = 2 + \frac{6}{(4 \cdot \sqrt{4})}$$

$$= 2 + \frac{6}{8}$$

$$= 2.75 \quad > 0$$

If $\frac{d^2y}{dx^2} > 0$, the stationary point is a minimum.

**(Total for Question 2 is 7 marks)**

**3.**

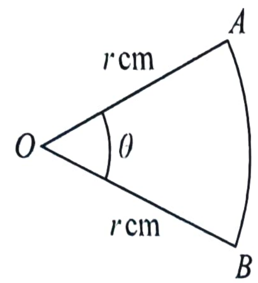


**Figure 1**

Figure 1 shows a sector $AOB$ of a circle with centre $O$ and radius $r$ cm.

The angle $AOB$ is $\theta$ radians.
The area of the sector $AOB$ is $11\text{ cm}^2$

Given that the perimeter of the sector is 4 times the length of the arc $AB$, find the exact value of $r$.

**(4)**

Length of $AB = r\theta$

Area of $AOB = \frac{1}{2}r^2\theta$.

"Length of $OA + AB + OB$" $=$ 4 · "Length of $AB$"

$\Rightarrow \quad r + r + r\theta = 4r\theta$

$\Rightarrow \quad r(2+\theta) = 4r\theta$

$\Rightarrow \quad 2+\theta = 4\theta$

$\Rightarrow \quad 2 = 3\theta$

$\Rightarrow \quad \theta = \frac{2}{3}^c$.

Use the formula for area to find $r$:

$\frac{1}{2}r^2\left(\frac{2}{3}\right) = \frac{1}{3}r^2 = 11\text{ cm}^2$

$\Rightarrow \quad r^2 = 33$

$\Rightarrow \quad r = \sqrt{33}$ cm

**4.** The curve with equation $y = 2\ln(8 - x)$ meets the line $y = x$ at a single point, $x = \alpha$.

(a) Show that $3 < \alpha < 4$ **(2)**

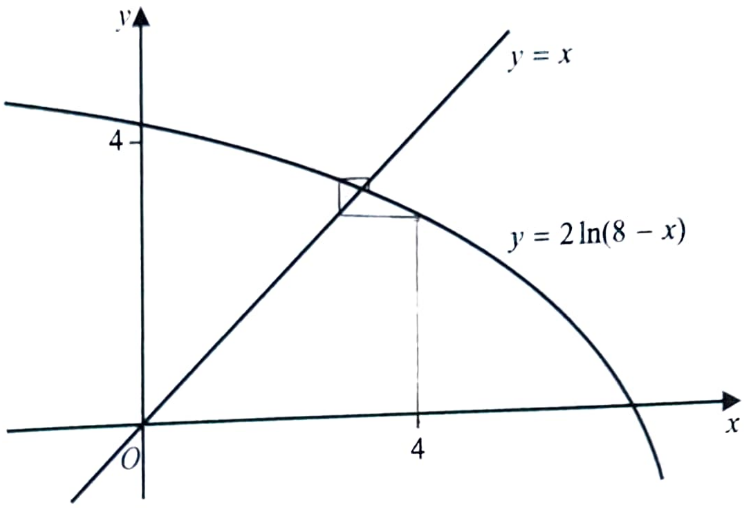


**Figure 2**

Figure 2 shows the graph of $y = 2\ln(8 - x)$ and the graph of $y = x$.

A student uses the iteration formula

$$x_{n+1} = 2\ln(8 - x_n), \quad n \in \mathbb{N}$$

in an attempt to find an approximation for $\alpha$.

Using the graph and starting with $x_1 = 4$

(b) determine whether or not this iteration formula can be used to find an approximation for $\alpha$, justifying your answer. **(2)**

(a) If $y = 2\ln(8-x) = x$, then

$2\ln(8-x) - x = y - y = 0$ at $x = \alpha$.

Let $f(x) = 2\ln(8-x) - x$.

$f(3) = (2\ln 5) - 3 = +0.219$

$f(4) = 2\ln 4 - 4 = -1.227$

The function is continuous, and the change in sign between $f(3)$ and $f(4)$ shows that $3 < \alpha < 4$.

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**Question 4 continued**

(b) The cobweb on the graph spirals inwards, therefore the iteration formula <u>can</u> be used to approximate $\alpha$.

**(Total for Question 4 is 4 marks)**

5. Given that

$$y = \frac{3\sin\theta}{2\sin\theta + 2\cos\theta} \qquad -\frac{\pi}{4} < \theta < \frac{3\pi}{4}$$

show that

$$\frac{dy}{d\theta} = \frac{A}{1+\sin 2\theta} \qquad -\frac{\pi}{4} < \theta < \frac{3\pi}{4}$$

where $A$ is a rational constant to be found.

**(5)**

Use the quotient rule: $\frac{dy}{dx} = \frac{f'(\theta)\,g(\theta) - f(\theta)\,g'(\theta)}{g(\theta)^2}$

$$y = \frac{3\sin\theta}{2\sin\theta + 2\cos\theta}.$$

$f(\theta) = 3\sin\theta$, $\quad g(\theta) = 2\sin\theta + 2\cos\theta$.

$\Rightarrow f'(\theta) = 3\cos\theta$, $\quad g'(\theta) = 2\cos\theta - 2\sin\theta$

$$\Rightarrow \frac{dy}{dx} = \frac{3\cos\theta(2\sin\theta + 2\cos\theta) - 3\sin\theta(2\cos\theta - 2\sin\theta)}{(2\sin\theta + 2\cos\theta)^2}$$

$$= \frac{(6\sin\theta\cos\theta + 6\cos^2\theta) - (6\sin\theta\cos\theta - 6\sin^2\theta)}{4\sin^2\theta + 8\sin\theta\cos\theta + 4\cos^2\theta}$$

$$= \frac{6\cos^2\theta + 6\sin^2\theta)}{(4\sin^2\theta + 4\cos^2\theta) + (8\sin\theta\cos\theta)}$$

$$= \frac{6}{4 + 4\sin 2\theta} = \frac{\left(\frac{3}{2}\right)}{1 + \sin 2\theta}$$

which is in the required form.

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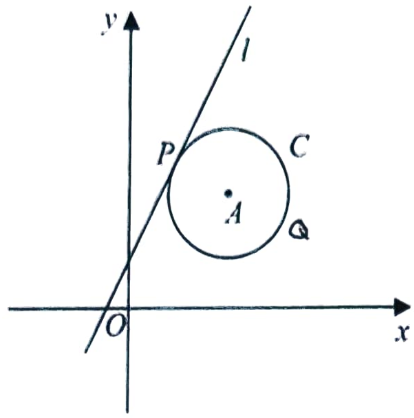
6.




Not to scale

**Figure 3**

The circle $C$ has centre $A$ with coordinates $(7, 5)$.

The line $l$, with equation $y = 2x + 1$, is the tangent to $C$ at the point $P$, as shown in Figure 3.

(a) Show that an equation of the line $PA$ is $2y + x = 17$

**(3)**

(b) Find an equation for $C$.

**(4)**

The line with equation $y = 2x + k$, $k \neq 1$ is also a tangent to $C$.

(c) Find the value of the constant $k$.

**(3)**

(a) Since $l$ is a tangent to $C$, the line $PA$ is perpendicular to $l$.

That is, $m_l m_{PA} = -1$.

$m_l = 2$, as per the equation $y = 2x+1$.

Therefore, $m_{PA} = -\frac{1}{2}$.

$\Rightarrow$ PA is represented by

$y = \frac{-1}{2}x + c$. Use $(7,5)$, which is on the line:

$5 = \left(-\frac{1}{2} \cdot 7\right) + c \Rightarrow c = \frac{17}{2}$.

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**Question 6 continued**

$y = -\frac{1}{2}x + \frac{17}{2}$

$\Rightarrow \quad 2y = -x + 17$

$\Rightarrow \quad x + 2y = 17$, as required.

(b) Find point $P$.

$x + 2y = 17, \qquad y = 2x + 1.$

$\Rightarrow x + 2(2x+1) = 17$

$\Rightarrow 5x + 2 = 17$

$\Rightarrow x_p = 3$

$\Rightarrow y_p = 7$

Radius of $C = |AP|$

$= \sqrt{(7-3)^2 + (5-7)^2} = \sqrt{20}$

Therefore, the equation of $C$ is

$(x-7)^2 + (y-5)^2 = 20.$

(c) ~~Let $y = 2x + k$~~

The tangent meets at $\vec{OA} + \vec{PA}$, the opposite point to $P$ on $C$, denote this point $Q$.

$Q$'s co-ordinates are $\underset{\vec{OA}}{(7, 5)} + \underset{\vec{PA}}{(4, -2)}$,

$(11, 3)$. Substitute this into $y = 2x + k$:

$3 = (11 \cdot 2) + k$

$\Rightarrow \underline{k = -19}$

7. Given that $k \in \mathbb{Z}^+$

(a) show that $\int_k^{3k} \frac{2}{(3x-k)}\,dx$ is independent of $k$,

**(4)**

(b) show that $\int_k^{2k} \frac{2}{(2x-k)^2}\,dx$ is inversely proportional to $k$.

**(3)**

(a) $\int \frac{2}{(3x-k)}\,dx = \frac{2}{3}\ln(3x-k)$.

$\Rightarrow \int_k^{3k} \frac{2}{3x-k}\,dx = \left(\frac{2}{3}\ln(9k-k)\right) - \left(\frac{2}{3}\ln(3k-k)\right)$

$= \frac{2}{3}\ln(8k) - \frac{2}{3}\ln(2k)$

$= \frac{2}{3}\ln\frac{8k}{2k} = \frac{2}{3}\ln 4.$

This term does not contain k, so it is independent of k.

(b) $\int \frac{2}{(2x-k)^2}\,dx = 2\int(2x-k)^{-2}\,dx = \frac{-1}{(2x-k)}$

$\int_k^{2k} \frac{2}{(2x-k)^2} = \frac{-1}{(4k-k)} - \frac{-1}{(2k-k)} = \frac{-1}{3k} + \frac{1}{k}$

$= \frac{2}{3k}$. $\left[= \left(\frac{2}{3}\right)\left(\frac{1}{k}\right)\right]$

The term $\frac{1}{k}$, shows this integral is inversely proportional to k.

**8.** The depth of water, $D$ metres, in a harbour on a particular day is modelled by the formula

$$D = 5 + 2\sin(30t)^\circ \qquad 0 \leqslant t < 24$$

where $t$ is the number of hours after midnight.

A boat enters the harbour at 6:30 am and it takes 2 hours to load its cargo.
The boat requires the depth of water to be at least 3.8 metres before it can leave the harbour.

(a) Find the depth of the water in the harbour when the boat enters the harbour.
**(1)**

(b) Find, to the nearest minute, the earliest time the boat can leave the harbour.
(*Solutions based entirely on graphical or numerical methods are not acceptable.*)
**(4)**

(a) $D = 5 + 2\sin(30^\circ \cdot 6.5)$
$= 4.48\,m$

(b) $3.8 = 5 + 2\sin(30t)$

$\Rightarrow \sin(30t) = -0.6$

$\Rightarrow t = \dfrac{\sin^{-1}(0.6)}{30}$. Remember, $t > 8$.

The lowest value of $t$ beyond $t > 8$ is 10.77. This translates as

10:46 am.

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**9.**

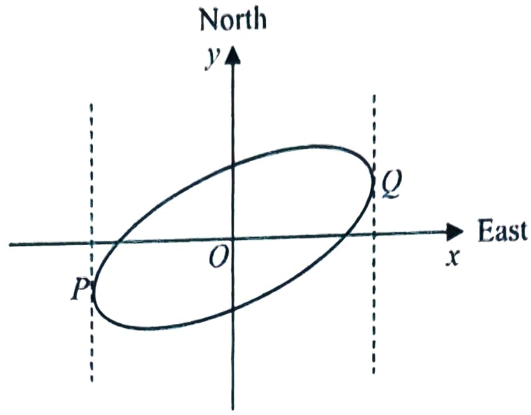


**Figure 4**

Figure 4 shows a sketch of the curve with equation $x^2 - 2xy + 3y^2 = 50$

(a) Show that $\dfrac{dy}{dx} = \dfrac{y - x}{3y - x}$ **(4)**

The curve is used to model the shape of a cycle track with both $x$ and $y$ measured in km.

The points $P$ and $Q$ represent points that are furthest west and furthest east of the origin $O$, as shown in Figure 4.

Using part (a),

(b) find the exact coordinates of the point $P$. **(5)**

(c) Explain briefly how to find the coordinates of the point that is furthest north of the origin $O$. (You **do not** need to carry out this calculation). **(1)**

(a) $\dfrac{d}{dx}x^2 - \underbrace{\dfrac{d}{dx}2xy}_{\text{product rule}} + \dfrac{d}{dx}3y^2 = \dfrac{d}{dx}50.$

product rule ⟵ $-2y + 6y\dfrac{dy}{dx}$

$\cancel{2x - \dfrac{dy}{dx}2x}$

$\Rightarrow 2x - \dfrac{dy}{dx}2x - 2y + 6y\dfrac{dy}{dx} = 0$

$2y - 2x = \dfrac{dy}{dx}(6y - 2x)$

$\Rightarrow \dfrac{dy}{dx} = \dfrac{2y - 2x}{6y - 2x} = \dfrac{y - x}{3y - x}$

Question 9 continued

(b) For P and Q, $\frac{dy}{dx} = m = \infty$,

meaning $3y - x = 0$.
Set $x = 3y$. Substitute into curve equation.

$(3y)^2 - 2(3y)y + 3y^2 = 50$

$\Rightarrow 9y^2 - 6y^2 + 3y^2 = 50$

$\Rightarrow 6y^2 = 50$

$\Rightarrow y^2 = \frac{25}{3} \Rightarrow y = \pm\frac{5}{\sqrt{3}}$

$\Rightarrow x = -5\sqrt{3}$ when $y = -\frac{5}{\sqrt{3}}$

and

$x = +5\sqrt{3}$ when $y = +\frac{5}{\sqrt{3}}$

P is furthest west, so the co-ordinates are $\left(-5\sqrt{3}, -\frac{5}{\sqrt{3}}\right)$.

(c) The furthest north point occurs when $\frac{dy}{dx} = 0$, or, $y = x$.

Solve the simultaneous equations for $y = x$ and $x^2 - 2xy + 3y^2 = 50$, and choose the positive solution.

**10.** The height above ground, $H$ metres, of a passenger on a roller coaster can be modelled by the differential equation

$$\frac{\mathrm{d}H}{\mathrm{d}t} = \frac{H\cos(0.25t)}{40}$$

where $t$ is the time, in seconds, from the start of the ride.

Given that the passenger is 5 m above the ground at the start of the ride,

(a) show that $H = 5e^{0.1\sin(0.25t)}$

**(5)**

(b) State the maximum height of the passenger above the ground.

**(1)**

The passenger reaches the maximum height, for the second time, $T$ seconds after the start of the ride.

(c) Find the value of $T$.

**(2)**

(a) $\frac{dH}{dt} = \frac{H\cos(0.25t)}{40}$

$\Rightarrow \frac{1}{H}\,dH = \frac{\cos(0.25t)}{40}\,dt$

$\Rightarrow \int \frac{1}{H}\,dH = \frac{1}{40}\int \cos(0.25t)\,dt$

$\Rightarrow \ln H = \frac{1}{10}\sin(0.25t) + c$

$\Rightarrow H = e^{0.1\sin(0.25t)+c}$

When $t=0$, $H=5$, so we have

$5 = e^{0+c} \quad \Rightarrow \quad c = \ln 5.$

$\Rightarrow H = 5e^{0.1\sin(0.25t)}$

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**Question 10 continued**

(b) $\max\left(5e^{0.1\sin(0.25t)}\right)$ occurs when $\sin 0.25t = 1$.

max height $= 5e^{0.1} \approx 5.53\text{m}$.

(c) $\sin 0.25t = 1$ when ~~t~~ $0.25t = \frac{5\pi}{2}$

$\Rightarrow T = 10\pi \approx 31.4\text{s}$



Turn over

**11.** (a) Use binomial expansions to show that $\sqrt{\dfrac{1+4x}{1-x}} \approx 1 + \dfrac{5}{2}x - \dfrac{5}{8}x^2$ **(6)**

A student substitutes $x = \dfrac{1}{2}$ into both sides of the approximation shown in part (a) in an attempt to find an approximation to $\sqrt{6}$

(b) Give a reason why the student **should not** use $x = \dfrac{1}{2}$ **(1)**

(c) Substitute $x = \dfrac{1}{11}$ into

$$\sqrt{\frac{1+4x}{1-x}} = 1 + \frac{5}{2}x - \frac{5}{8}x^2$$

to obtain an approximation to $\sqrt{6}$. Give your answer as a fraction in its simplest form. **(3)**

(a) $\sqrt{\dfrac{1+4x}{1-x}} = (1+4x)^{\frac{1}{2}} \cdot (1-x)^{-\frac{1}{2}}$

$(1+4x)^{\frac{1}{2}} = 1 + \left(\frac{1}{2} \cdot 4x\right) + \left(\left(\dfrac{\frac{1}{2} \cdot -\frac{1}{2}}{2}\right) \cdot (4x)^2\right)$

$(1-x)^{-\frac{1}{2}} = 1 + \left(-\frac{1}{2}(-x)\right) + \left(\left(\dfrac{-\frac{1}{2} \times -\frac{3}{2}}{2}\right) \cdot (-x)^2\right)$

$(1+4x)^{\frac{1}{2}} \cdot (1-x)^{-\frac{1}{2}}$

$= 1 + \left(2x + \frac{1}{2}x\right) + \left(-2x^2 + x^2 + \frac{3}{8}x^2\right) + \ldots$

$= 1 + \frac{5}{2}x - \frac{5}{8}x^2 \ldots$

(b) The expression is only valid for $|x| < \frac{1}{4}$

(c) $x = \frac{1}{11} \Rightarrow \sqrt{\frac{3}{2}} \approx \frac{1183}{968}$.

$\sqrt{6} = 2\sqrt{\frac{3}{2}} \approx \frac{1183}{484}$

**12.** The value, £$V$, of a vintage car $t$ years after it was first valued on 1st January 2001, is modelled by the equation

$$V = Ap^t \qquad \text{where } A \text{ and } p \text{ are constants}$$

Given that the value of the car was £32 000 on 1st January 2005 and £50 000 on 1st January 2012

(a) (i) find $p$ to 4 decimal places,

(ii) show that $A$ is approximately 24 800

**(4)**

(b) With reference to the model, interpret

(i) the value of the constant $A$,

(ii) the value of the constant $p$.

**(2)**

Using the model,

(c) find the year during which the value of the car first exceeds £100 000

**(4)**

(a) (i) $32000 = Ap^4$ , $50000 = Ap^{11}$

$p^7 = \frac{50000}{32000} \Rightarrow p \approx 1.0658$

(ii) $Ap^4 = 32000 \Rightarrow A = \frac{32000}{1.0658^4}$

$= £24799 \approx £24800$

(b) (i) The value of the car ~~at~~ on 01/01/2001

(ii) The yearly rate of increase of the car's value.

(c) $100000 = 24800(1.0688)^t$

$t = \log_{1.0688}\left(\frac{100000}{24800}\right) = 21.8$

The car's value will exceed £100000 in 2022.

**13.** Show that

$$\int_0^2 2x\sqrt{x+2}\,dx = \frac{32}{15}\left(2+\sqrt{2}\right)$$

**(7)**

Let $u = \sqrt{x+2}$.

$\Rightarrow \quad x = u^2 - 2.$

Then we have $\frac{dx}{du} = 2u$

$x=2 \rightarrow u = 2.$
$x=0 \rightarrow u = \sqrt{2}$

$$\int_{\sqrt{2}}^{2} 2(u^2-2)\,u \cdot (2u\,du).$$

$$= \int_{\sqrt{2}}^{2} 4(u^4 - 2u^2)\,du$$

$$= \left[\frac{4}{5}u^5 - \frac{8}{3}u^3\right]_{\sqrt{2}}^{2}$$

$$= \left(\frac{128}{5} - \frac{64}{3}\right) - \left(\frac{16}{5}\sqrt{2} - \frac{16}{3}\sqrt{2}\right)$$

$$= \frac{64}{15} + \frac{32}{15}\sqrt{2}$$

$$= \frac{32}{15}\left(2+\sqrt{2}\right).$$

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**14.** A curve $C$ has parametric equations

$$x = 3 + 2\sin t, \qquad y = 4 + 2\cos 2t, \qquad 0 \leqslant t < 2\pi$$

(a) Show that all points on $C$ satisfy $y = 6 - (x - 3)^2$

**(2)**

(b) (i) Sketch the curve $C$.

(ii) Explain briefly why $C$ does not include all points of $y = 6 - (x - 3)^2, \quad x \in \mathbb{R}$

**(3)**

The line with equation $x + y = k$, where $k$ is a constant, intersects $C$ at two distinct points.

(c) State the range of values of $k$, writing your answer in set notation.

**(5)**

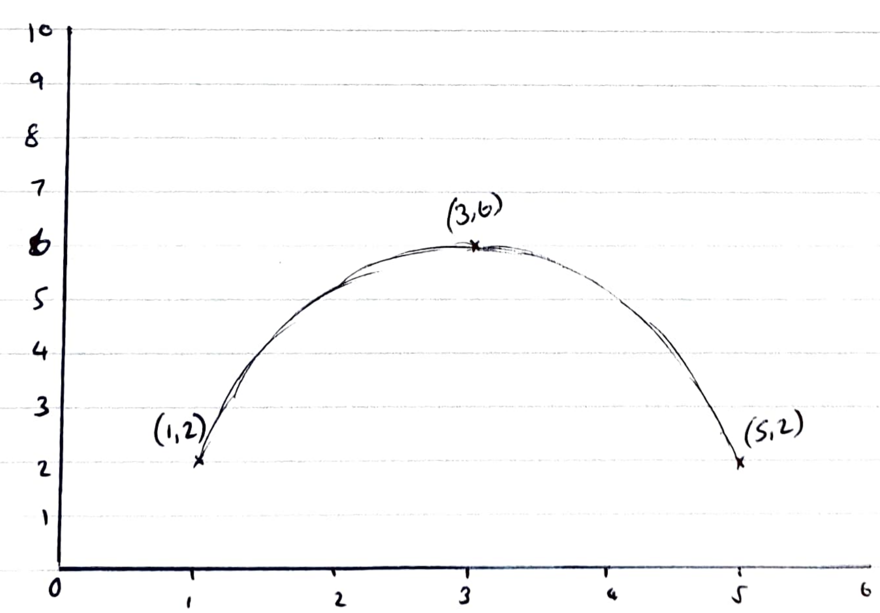
(a) $\cos 2t = 1 - 2\sin^2 t$

$\Rightarrow y = 4 + 2 - 4\sin^2 t = 2 + 4\cos^2 t$

$6 - (x-3)^2 = 6 - (2\sin t)^2$
$= 6 - 4\sin^2 t$
$= 4\cos^2 t + 2$

$\Rightarrow y = 6 - (x-3)^2.$

(b) (i)




(ii) As $0 \le t \le 2\pi$, $1 \le x \le 5$.
(Because $-1 \le \sin t \le +1$).

**Question 14 continued**

(c) $x + y = k$ and $y = 6 - (x-3)^2$

$k - x = 6 - (x-3)^2$

$\Rightarrow k = -x^2 + 7x - 3$

$\Rightarrow x^2 - 7x + (3 + k) = 0.$

Solution only exists when $b^2 - 4ac > 0$

$b^2 - 4ac = 49 - (4 \cdot (3 + k))$

$= 49 - 12 - 4k$

$= 37 - 4k > 0$

$\Rightarrow k < \frac{37}{4}.$

We also have that $k \geqslant 7$, as a lower value of $k$ will not intercept the curve twice. (See the graph).

**Question 14 continued**

Therefore, we have

$$k = \left\{ k : 7 \leq k < \frac{37}{4} \right\}$$

**(Total for Question 14 is 10 marks)**

**TOTAL FOR PAPER IS 100 MARKS**

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