AQA

SPECIMEN MATERIAL

Please write clearly, in block capitals.

Centre number | | | | | | Candidate number | | | | |

Surname MODEL SOLUTIONS

Forename(s)

Candidate signature

AS MATHEMATICS

Paper 2

Exam Date Morning Time allowed: 1 hour 30 minutes

Materials
For this paper you must have:
- The AQA booklet of formulae and statistical tables.
- You may use a graphics calculator.

Instructions
- Use black ink or black ball-point pen. Pencil should be used for drawing.
- Answer **all** questions.
- You must answer each question in the space provided for that question. If you require extra space, use an AQA supplementary answer book; do **not** use the space provided for a different question.
- Do not write outside the box around each page.
- Show all necessary working; otherwise marks for method may be lost.
- Do all rough work in this book. Cross through any work that you do not want to be marked.

Information
- The marks for questions are shown in brackets.
- The maximum mark for this paper is 80.

Advice
Unless stated otherwise, you may quote formulae, without proof, from the booklet.
You do not necessarily need to use all the space provided.

Section A

Answer **all** questions in the spaces provided.

1 $p(x) = x^3 - 5x^2 + 3x + a$, where a is a constant.

Given that $x - 3$ is a factor of $p(x)$, find the value of a

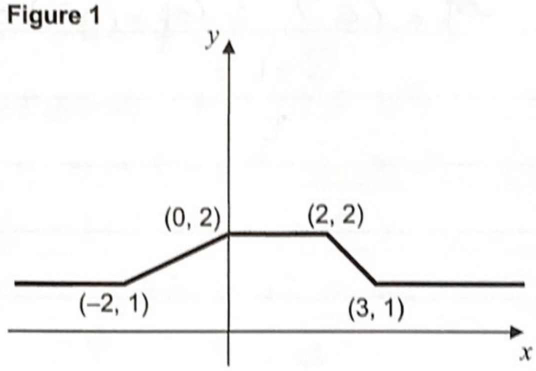
Circle your answer.

[1 mark]

-9 $\quad$ -3 $\quad$ 3 $\quad$ 9

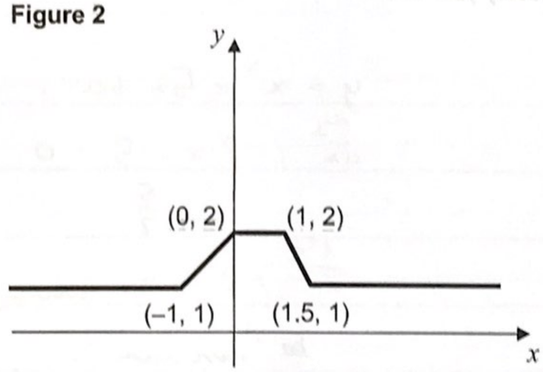
2 The graph of $y = \mathrm{f}(x)$ is shown in **Figure 1**.

Figure 1



State the equation of the graph shown in **Figure 2**.

Figure 2



Circle your answer.

[1 mark]

$y = \mathrm{f}(2x)$ $\qquad y = \mathrm{f}\left(\dfrac{x}{2}\right)$ $\qquad y = 2\mathrm{f}(x)$ $\qquad y = \dfrac{1}{2}\mathrm{f}(x)$

3 Find the value of $\log_a\left(a^3\right)+\log_a\left(\frac{1}{a}\right)$

[2 marks]

$\log_a(a^3) + \log_a(\frac{1}{a}) =$

$\log_a(a^3) + \log_a(a^{-1}) =$

$3 - 1 =$

2

4 Find the coordinates, in terms of a, of the minimum point on the curve $y = x^2 - 5x + a$, where a is a constant.

Fully justify your answer.

[3 marks]

$y = x^2 - 5x + a$.

$\frac{dy}{dx} = 2x - 5 = 0$ at minimum.

$x = \frac{5}{2}$

$\frac{d^2y}{dx^2} = 2 > 0$ always so this must be minimum.

$y = (\frac{5}{2})^2 - 5(\frac{5}{2}) + a$

$y = \frac{25}{4} - \frac{25}{2} + a$

$y = a - \frac{25}{4}$.

Coords: $(\frac{5}{2}, a - \frac{25}{4})$.

5 The quadratic equation $3x^2 + 4x + (2k-1) = 0$ has real and distinct roots.

Find the possible values of the constant k

Fully justify your answer.

[4 marks]

$3x^2 + 4x + (2k-1) = 0$.

Discriminant $b^2 - 4ac > 0$.

$4^2 - 4 \times 3 \times (2k-1) > 0$

$16 - 12(2k-1) > 0$.

$16 - 24k + 12 > 0$

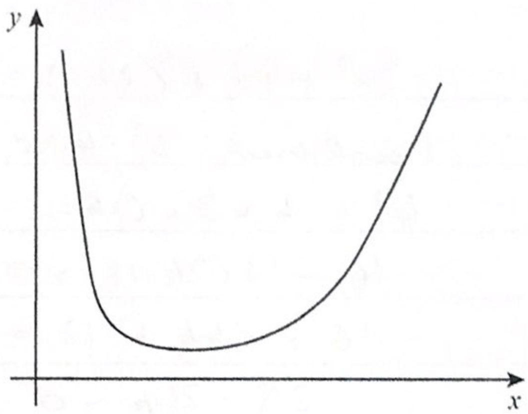
$28 - 24k > 0$.

$28 > 24k$

$k < \frac{28}{24}$

$k < \frac{7}{6}$.

6 A curve has equation $y = 6x^2 + \frac{8}{x^2}$ and is sketched below for $x > 0$



Find the area of the region bounded by the curve, the x-axis and the lines $x = a$ and $x = 2a$, where $a > 0$, giving your answer in terms of a

[4 marks]

$\text{Area} = \int_a^{2a} 6x^2 + \frac{8}{x^2}\, dx$

$= \int_a^{2a} 6x^2 + 8x^{-2}\, dx$

$= \left[2x^3 - 8x^{-1} \right]_a^{2a}$

$= 2(2a)^3 - 8(2a)^{-1} - 2a^3 + 8a^{-1}$

$= 16a^3 - 4a^{-1} - 2a^3 + 8a^{-1}$

$= 14a^3 + 4a^{-1}$

$= 14a^3 + \frac{4}{a}$.

7 Solve the equation

$$\sin\theta\tan\theta + 2\sin\theta = 3\cos\theta \quad \text{where} \quad \cos\theta \neq 0$$

Give **all** values of θ to the nearest degree in the interval $0° < \theta < 180°$

Fully justify your answer.

[5 marks]

$\sin\theta \tan\theta + 2\sin\theta = 3\cos\theta$

$\frac{\sin\theta}{\cos\theta} \tan\theta + 2\frac{\sin\theta}{\cos\theta} = 3\frac{\cos\theta}{\cos\theta}$

$\tan^2\theta + 2\tan\theta = 3$

$\tan^2\theta + 2\tan\theta - 3 = 0$.

$(\tan\theta + 3)(\tan\theta - 1) = 0$

$\tan\theta = 1 \quad \tan\theta = -3$

$\theta = 45°, 108°$.

8 Prove that the function $f(x) = x^3 - 3x^2 + 15x - 1$ is an increasing function.

[6 marks]

$f(x)$ increasing $\Leftrightarrow f'(x) > 0$

$f'(x) = 3x^2 - 6x + 15$.

$f'(x) = 3(x^2 - 2x) + 15$

$f'(x) = 3(x-1)^2 + 15 - 3$

$f'(x) = 3(x-1)^2 + 12$

So the minimum value of $f'(x)$ is 12.

Hence $f'(x) > 0$ always.

Hence $f(x)$ is an increasing function.

9 A curve has equation $y = e^{2x}$

Find the coordinates of the point on the curve where the gradient of the curve is $\frac{1}{2}$
Give your answer in an exact form.

[5 marks]

$y = e^{2x}$.

$\frac{dy}{dx} = 2e^{2x}$

Want $\frac{dy}{dx} = \frac{1}{2}$.

$2e^{2x} = \frac{1}{2}$.

$e^{2x} = \frac{1}{4}$

$2x = \ln(\frac{1}{4})$

$x = \frac{1}{2}\ln(\frac{1}{4})$

$x = \ln(\frac{1}{4}^{\frac{1}{2}})$

$x = \ln(\frac{1}{2})$

$x = -\ln(2)$.

$y = e^{-2\ln(2)}$

$y = e^{-\ln(4)}$

$y = e^{\ln(\frac{1}{4})}$

$y = \frac{1}{4}$.

Coordinates are $(-\ln(2), \frac{1}{4})$.

10 David has been investigating the population of rabbits on an island during a three-year period.

Based on data that he has collected, David decides to model the population of rabbits, R, by the formula

$$R = 50e^{0.5t}$$

where t is the time in years after 1 January 2016.

10 (a) Using David's model:

10 (a) (i) state the population of rabbits on the island on 1 January 2016;

[1 mark]

50

10 (a) (ii) predict the population of rabbits on 1 January 2021.

[1 mark]

$t = 5 \quad 50e^{0.5\times5} = 609$

10 (b) Use David's model to find the value of t when $R = 150$, giving your answer to three significant figures.

[2 marks]

$150 = 50e^{0.5t}$

$e^{0.5t} = 3$

$0.5t = \ln(3)$

$t = 2\ln(3)$

$t = 2.20$

10 (c) Give **one** reason why David's model may **not** be appropriate.

[1 mark]

The model predicts unlimited growth, which is not plausible for rabbit populations.

10 (d) On the same island, the population of crickets, C, can be modelled by the formula

$$C = 1000e^{0.1t}$$

where t is the time in years after 1 January 2016.

Using the two models, find the year during which the population of rabbits first exceeds the population of crickets.

[3 marks]

$1000e^{0.1t} = 50e^{0.5t}$

$20 = e^{0.4t}$

$0.4t = \ln(20)$.

$t = \frac{5}{2}\ln(20)$

$t = 7.49$

So in year 2023.

11 The circle with equation $(x-7)^2+(y+2)^2=5$ has centre C.

11 (a) (i) Write down the radius of the circle.

[1 mark]

$\sqrt{5}$

11 (a) (ii) Write down the coordinates of C.

[1 mark]

$(7, -2)$

11 (b) The point P (5, −1) lies on the circle.

Find the equation of the tangent to the circle at P, giving your answer in the form $y = mx + c$

[4 marks]

Gradient of radius $= \dfrac{-1-(-2)}{5-7} = -\dfrac{1}{2}$.

Gradient of tangent $= 2$.

$y - y_1 = m(x - x_1)$

$y + 1 = 2(x - 5)$

$y + 1 = 2x - 10$

$y = 2x - 11$.

11 (c) The point Q (3, 3) lies outside the circle and the point T lies on the circle such that QT is a tangent to the circle. Find the length of QT.

[4 marks]

QTC is a right angled triangle.

Use Pythagoras:

$QC^2 = (7-3)^2 + (-2-3)^2$

$QC^2 = 4^2 + 5^2$

Hence $4^2 + 5^2 = \sqrt{5}^2 + QT^2$

$QT^2 = 16 + 25 - 5$

$QT^2 = 36$

$QT = 6$.

12 (a) Given that n is an even number, prove that $9n^2 + 6n$ has a factor of 12

[3 marks]

$9n^2 + 6n =$

$3n(3n+2)$.

If n is even then n is even and $3n+2$ is even, so $n(3n+2)$ has a factor of 4.

Hence, $3n(3n+2)$ has a factor of 12.

Can set $n = 2m$ to see

$6m(6m+4) =$

$\underbrace{12}_{\text{Factor of 12.}}m(3m+2)$

12 (b) Determine if $9n^2 + 6n$ has a factor of 12 for any integer n .

[1 mark]

$n = 1$

$9 \times 1^2 + 6 \times 1 = 15$

Does not have a factor of 12 .

END OF SECTION A

Section B

Answer **all** questions in the spaces provided.

13 The number of pots of yoghurt, X, consumed per week by adults in Milton is a discrete random variable with probability distribution given by

x	0	1	2	3	4	5	6	7 or more
$\mathrm{P}(X = x)$	0.30	0.10	0.05	0.07	0.03	0.16	0.09	0.20

Find $\mathrm{P}(3 \leq X < 6)$

Circle the correct answer.

[1 mark]

0.26   0.31   0.35   0.40

14 In the Large Data Set, the emissions of carbon dioxide are measured in what units?

Circle your answer.

[1 mark]

mg/litre   g/litre   g/km   mg/km

15 A school took 225 children on a trip to a theme park.

After the trip the children had to write about their favourite ride at the park from a choice of three.

The table shows the number of children who wrote about each ride.

		Ride written about			
		The Drop	The Beanstalk	The Giant	**Total**
Year group	Year 7	24	45	23	**92**
	Year 8	36	17	22	**75**
	Year 9	20	13	25	**58**
	Total	**80**	**75**	**70**	**225**

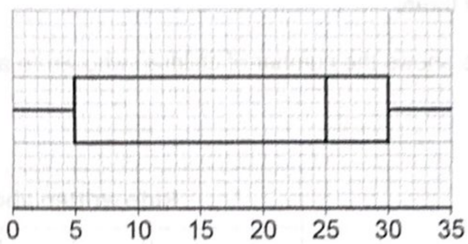
Three children were randomly selected from those who went on the trip.

Calculate the probability that one wrote about 'The Drop', one wrote about 'The Beanstalk' and one wrote about The Giant'.

[2 marks]

$\frac{80}{225} \times \frac{75}{224} \times \frac{70}{223} \times 6 = 0.224.$

16 The boxplot below represents the time spent in hours by students revising for a history exam.



16 (a) Use the information in the boxplot to state the value of a measure of central tendency of the revision times, stating clearly which measure you are using.

[1 mark]

Median is 25 hours.

16 (b) Use the information in the boxplot to explain why the distribution of revision times is negatively skewed.

[1 mark]

The median is closer to the upper quartile than the lower quartile so it is negatively skewed.

Turn over for the next question

DO NOT WRITE ON THIS PAGE
ANSWER IN THE SPACES PROVIDED

17 The table below is an extract from the Large Data Set.

Make	Region	Engine size	Mass	CO2	CO
VAUXHALL	South West	1398	1163	118	0.463
VOLKSWAGEN	London	999	1055	106	0.407
VAUXHALL	South West	1248	1225	85	0.141
BMW	South West	2979	1635	194	0.139
TOYOTA	South West	1995	1650	123	0.274
BMW	South West	2979	0	244	0.447
FORD	South West	1596	0	165	0.518
TOYOTA	South West	1299	1050	144	
VAUXHALL	London	1398	1361	140	0.695
FORD	North West	4951	1799	299	0.621

17 (a) (i) Calculate the standard deviation of the engine sizes in the table.

[1 mark]

$$\frac{1398^2+999^2+1248^2+2979^2+1995^2+2979^2+1596^2+1299^2+1398^2+4951^2}{10}$$

$$-\left(\frac{1398+999+1248+2979+1995+2979+1596+1299+1398+4951}{10}\right)^2$$

$= 1161.95$.

17 (a) (ii) The mean of the engine sizes is 2084

Any value more than 2 standard deviations from the mean can be identified as an outlier.

Using this definition of an outlier, show that the sample of engine sizes has **exactly one** outlier.

Fully justify your answer.

[3 marks]

$2084 + 2\times 1161.95 = 4408$.

Only 4951 is greater so that is one outlier.

$2084 - 2\times 1161.95 = -239.7$

No values smaller than -239.7.

Hence one outlier.

17 (b) Rajan calculates the mean of the masses of the cars in this extract and states that it is 1094 kg.

Use your knowledge of the Large Data Set to suggest what error Rajan is likely to have made in his calculation.

[1 mark]

Rajan has included the masses of 0 in his calculation, even though this data is clearly wrong.

17 (c) Rajan claims there is an error in the data recorded in the table for one of the Toyotas from the South West, because there is no value for its carbon monoxide emissions.

Use your knowledge of the Large Data Set to comment on Rajan's claim.

[1 mark]

Not all CO emissions data is known by the LDS, so the omission on the table is not an error.

18 Neesha wants to open an Indian restaurant in her town.

Her cousin, Ranji, has an Indian restaurant in a neighbouring town. To help Neesha plan her menu, she wants to investigate the dishes chosen by a sample of Ranji's customers.

Ranji has a list of the 750 customers who dined at his restaurant during the past month and the dish that each customer chose.

Describe how Neesha could obtain a simple random sample of size 50 from Ranji's customers.

[4 marks]

Give each customer a unique number from 1 to 750.
Generate 50 random numbers between 1 and 750, ignoring repeats.
Use these 50 unique random numbers to to select the corresponding customers.

19 Ellie, a statistics student, read a newspaper article that stated that 20 per cent of students eat at least five portions of fruit and vegetables every day.

Ellie suggests that the number of people who eat at least five portions of fruit and vegetables every day, in a sample of size n, can be modelled by the binomial distribution $B(n, 0.20)$.

19 (a) There are 10 students in Ellie's statistics class.

Using the distributional model suggested by Ellie, find the probability that, of the students in her class:

19 (a) (i) two or fewer eat at least five portions of fruit and vegetables every day;

[1 mark]

$X \sim B(10, 20)$ $\quad P(X \leq 2) = 0.678$.

19 (a) (ii) at least one but fewer than four eat at least five portions of fruit and vegetables every day;

[2 marks]

$P(1 \leq X \leq 3) = P(X \leq 3) - P(X = 0)$

$= 0.8791 - 0.1074$

$= 0.772$.

19 (b) Ellie's teacher, Declan, believes that more than 20 per cent of students eat at least five portions of fruit and vegetables every day. Declan asks the 25 students in his other statistics classes and 8 of them say that they eat at least five portions of fruit and vegetables every day.

19 (b) (i) Name the sampling method used by Declan.

[1 mark]

Opportunity sampling.

19 (b) (ii) Describe **one** weakness of this sampling method.

[1 mark]

Unrepresentative as all students come from the same college.

19 (b) (iii) Assuming that these 25 students may be considered to be a random sample, carry out a hypothesis test at the 5% significance level to investigate whether Declan's belief is supported by this evidence.

[6 marks]

$H_0: p = 0.2$ $\quad H_1: p > 0.2$.

Under H_0, $X \sim B(25, 0.2)$

$P(X \geq 8) = 0.109$

$0.109 > 0.05$

Hence, accept H_0.

No significant evidence to suggest that more than 20% of students eat at least five a day.