AQA

SPECIMEN MATERIAL

Please write clearly, in block capitals.

Centre number						Candidate number				

Surname MODEL SOLUTIONS

Forename(s)

Candidate signature

AS MATHEMATICS

Paper 1

Exam Date Morning Time allowed: 1 hour 30 minutes

Materials

For this paper you must have:

- The AQA booklet of formulae and statistical tables.
- You may use a graphics calculator.

Instructions

- Use black ink or black ball-point pen. Pencil should be used for drawing.
- Answer **all** questions.
- You must answer each question in the space provided for that question. If you require extra space, use an AQA supplementary answer book; do **not** use the space provided for a different question.
- Do not write outside the box around each page.
- Show all necessary working; otherwise marks for method may be lost.
- Do all rough work in this book. Cross through any work that you do not want to be marked.

Information

- The marks for questions are shown in brackets.
- The maximum mark for this paper is 80.

Advice

Unless stated otherwise, you may quote formulae, without proof, from the booklet.
You do not necessarily need to use all the space provided.

Section A

Answer **all** questions in the spaces provided.

1 The curve $y = \sqrt{x}$ is translated onto the curve $y = \sqrt{x+4}$

The translation is described by a vector.

Find this vector.

Circle your answer.

[1 mark]

$\begin{bmatrix} 4 \\ 0 \end{bmatrix}$ $\begin{bmatrix} -4 \\ 0 \end{bmatrix}$ $\begin{bmatrix} 0 \\ 4 \end{bmatrix}$ $\begin{bmatrix} 0 \\ -4 \end{bmatrix}$

2 Consider the two statements, A and B, below.

A: $x^2 - 6x + 8 > 0$

B: $x > 4$

Choose the most appropriate option below.

Circle your answer.

[1 mark]

$A \Rightarrow B$ $A \Leftarrow B$ $A \Leftrightarrow B$ There is no connection between A and B

3 (a) Write down the value of p and the value of q given that:

3 (a) (i) $\sqrt{3} = 3^p$

[1 mark]

$p = \frac{1}{2}$

3 (a) (ii) $\frac{1}{9} = 3^q$

[1 mark]

$q = -2$

3 (b) Find the value of x for which $\sqrt{3} \times 3^x = \frac{1}{9}$

[2 marks]

$\sqrt{3} \times 3^x = \frac{1}{9}$

$3^{\frac{1}{2}+x} = 3^{-2}$

$\frac{1}{2} + x = -2$

$x = -2 - \frac{1}{2}$

$x = -\frac{5}{2}$

4 Show that $\dfrac{5\sqrt{2}+2}{3\sqrt{2}+4}$ can be expressed in the form $m+n\sqrt{2}$, where m and n are integers.

[3 marks]

$$\frac{5\sqrt{2}+2}{3\sqrt{2}+4} = \frac{(5\sqrt{2}+2)(3\sqrt{2}-4)}{(3\sqrt{2}+4)(3\sqrt{2}-4)}$$

$$= \frac{15\times 2+6\sqrt{2}-20\sqrt{2}-8}{9\times 2+12\sqrt{2}-12\sqrt{2}-16}$$

$$= \frac{30-14\sqrt{2}-8}{18-16}$$

$$= \frac{22-14\sqrt{2}}{2}$$

$$= 11-7\sqrt{2}$$

5 Jessica, a maths student, is asked by her teacher to solve the equation $\tan x = \sin x$, giving all solutions in the range $0° \leq x \leq 360°$

The steps of Jessica's working are shown below.

$$\tan x = \sin x$$

Step 1	$\Rightarrow$	$\dfrac{\sin x}{\cos x} = \sin x$	Write $\tan x$ as $\dfrac{\sin x}{\cos x}$
Step 2	$\Rightarrow$	$\sin x = \sin x \cos x$	Multiply by $\cos x$
Step 3	$\Rightarrow$	$1 = \cos x$	Cancel $\sin x$
	$\Rightarrow$	$x = 0°$ or $360°$	

The teacher tells Jessica that she has not found all the solutions because of a mistake.

Explain why Jessica's method is not correct.

[2 marks]

She divided through by $\sin x$, removing solutions corresponding to $\sin x = 0$. Such cancelling is only allowed for terms that can never be 0.

6 A parallelogram has sides of length 6 cm and 4.5 cm.

The larger interior angles of the parallelogram have size α

Given that the area of the parallelogram is 24 cm^2, find the exact value of $\tan\alpha$

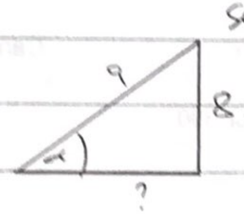
[4 marks]

Area $= 6 \times 4.5 \times \sin\alpha = 24$.

$27 \sin\alpha = 24$

$\sin\alpha = \frac{24}{27}$.

$\sin\alpha = \frac{8}{9}$



$? = \sqrt{9^2 - 8^2} = \sqrt{81-64} = \sqrt{17}$

Hence, $\tan\alpha = \pm\frac{8}{\sqrt{17}}$

α is larger angle so must be obtuse.

$\tan\alpha = \frac{-8}{\sqrt{17}}$

7 Determine whether the line with equation $2x + 3y + 4 = 0$ is parallel to the line through the points with coordinates (9, 4) and (3, 8).

[4 marks]

$2x + 3y + 4 = 0$

$3y = -2x - 4$

$y = -\frac{2}{3}x - \frac{4}{3}$

gradient is $-\frac{2}{3}$.

Gradient from points: $\frac{8-4}{3-9} = \frac{4}{-6} = -\frac{2}{3}$

which is the same.

Hence, the lines are parallel.

Turn over for the next question

8 (a) Find the first **three** terms, in ascending powers of x, of the expansion of $(1-2x)^{10}$

[3 marks]

$(1-2x)^{10} = \binom{10}{0} 1^{10}(-2x)^0 + \binom{10}{1} 1^9(-2x)^1 + \binom{10}{2} 1^8(-2x)^2$

$= 1\times1\times1 + 10\times1\times(-2x) + 45\times1\times(4x^2)$

$= 1 - 20x + 180x^2$

8 (b) Carly has lost her calculator. She uses the first three terms, in ascending powers of x, of the expansion of $(1-2x)^{10}$ to evaluate 0.998^{10}

Find Carly's value for 0.998^{10} and show that it is correct to **five** decimal places.

[3 marks]

$1-2x = 0.998$

$2x = 1-0.998$

$2x = 0.002$

$x = 0.001$

$1-20\times0.001 + 180\times0.001^2 =$

$1- 0.02 + 180\times0.00001 =$

$1- 0.02 + 0.0018 =$

$0.098018.$

Actual value is 0.980179 which is 0.98018 to 5dp.

9 (a) Given that $f(x) = x^2 - 4x + 2$, find $f(3+h)$

Express your answer in the form $h^2 + bh + c$, where b and $c \in \mathbb{Z}$.

[2 marks]

$f(3+h) = (3+h)^2 - 4(3+h) + 2$

$= h^2 + 6h + 9 - 4h - 12 + 2$

$= h^2 + 2h - 1$

9 (b) The curve with equation $y = x^2 - 4x + 2$ passes through the point $P(3, -1)$ and the point Q where $x = 3 + h$.

Using differentiation from first principles, find the gradient of the tangent to the curve at the point P.

[3 marks]

Gradient is $\dfrac{f(3+h) - f(3)}{h} =$

$\dfrac{h^2+2h-1+1}{h} =$

$\dfrac{h^2+2h}{h} =$

$h + 2$

As $h \to 0$, $h+2 \to 2$

Gradient of tangent $= 2$.

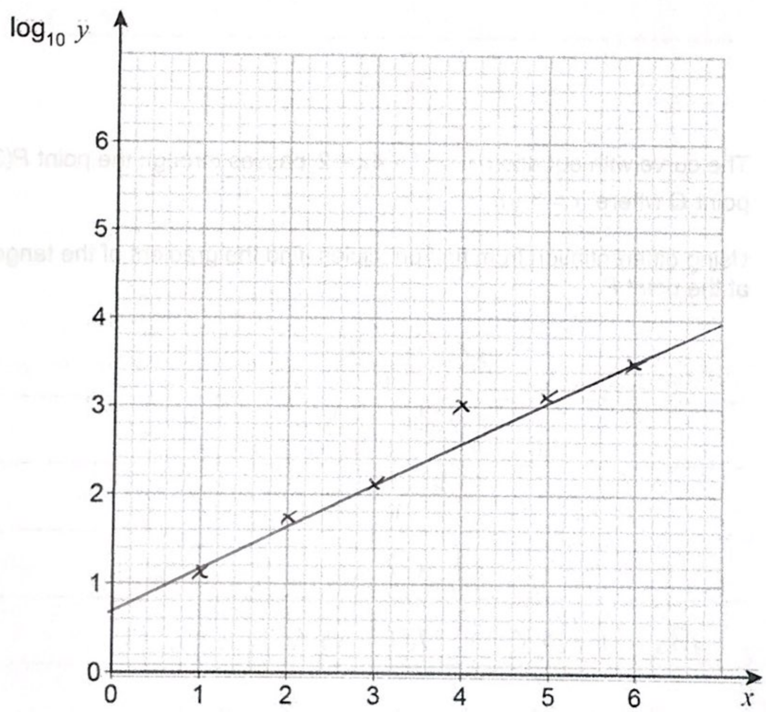
10 A student conducts an experiment and records the following data for two variables, x and y.

x	1	2	3	4	5	6
y	14	45	130	1100	1300	3400
$\log_{10} y$	1.1	1.7	2.1	3.0	3.1	3.5

The student is told that the relationship between x and y can be modelled by an equation of the form $y = kb^x$

10 (a) Plot values of $\log_{10} y$ against x on the grid below.

[2 marks]



10 (b) State, with a reason, which value of y is likely to have been recorded incorrectly.

[1 mark]

(4, 1100). All other points are (almost) on a line, while this point is somewhat away from the line.

10 (c) By drawing an appropriate straight line, find the values of k and b.

[4 marks]

$y = k b^x$

$\log_{10} y = x \log_{10} b + \log_{10} k$

Intercept $= 0.7 = \log_{10} k$.

$k = 10^{0.7}$

$k = 5.01$

Line passes through $(0, 0.7)$ and $(7, 4.0)$

Gradient $= \frac{4 - 0.7}{7 - 0} = \log_{10} b$

$\frac{3.3}{7} = \log_{10} b$

$b = 10^{\frac{3.3}{7}}$

$b = 2.96$

Turn over for the next question

11 Chris claims that, "for any given value of x, the gradient of the curve $y = 2x^3 + 6x^2 - 12x + 3$ is always greater than the gradient of the curve $y = 1 + 60x - 6x^2$ ".

Show that Chris is wrong by finding all the values of x for which his claim is **not** true.

[7 marks]

$y = 2x^3 + 6x^2 - 12x + 3$

$\frac{dy}{dx} = 6x^2 + 12x - 12$

$y = 1 + 60x - 6x^2$

$\frac{dy}{dx} = 60 - 12x$.

Chris incorrect when:

$6x^2 + 12x - 12 \leq 60 - 12x$

$6x^2 + 24x - 72 \leq 0$.

$x^2 + 4x - 12 \leq 0$

$(x+6)(x-2) \leq 0$

$x = -6$ $x = 2$ critical values.

Hence, Chris is incorrect for

$-6 \leq x \leq 2$, so he is wrong.

12 A curve has equation $y = 6x\sqrt{x} + \frac{32}{x}$ for $x > 0$

12 (a) Find $\frac{dy}{dx}$

[4 marks]

$y = 6x\sqrt{x} + \frac{32}{x}$

$y = 6x^{\frac{3}{2}} + 32x^{-1}$

$\frac{dy}{dx} = 9x^{\frac{1}{2}} - 32x^{-2}$

$\frac{dy}{dx} = 9\sqrt{x} - \frac{32}{x^2}$.

12 (b) The point *A* lies on the curve and has x-coordinate 4

Find the coordinates of the point where the tangent to the curve at *A* crosses the x-axis.

[5 marks]

Gradient of tangent $= 9\sqrt{4} - \frac{32}{4^2}$

$= 9 \times 2 - \frac{32}{16}$

$= 18 - 2$

$= 16$.

y-coord $= 6 \times 4 \times \sqrt{4} + \frac{32}{4} = 24 \times 2 + 8 = 48 + 8 = 56$.

$y - 56 = 16(x-4)$.

At $y = 0$:

$-56 = 16(x-4)$

$-\frac{56}{16} = x - 4$

$x = 4 - \frac{56}{16} = \frac{64}{16} - \frac{56}{16} = \frac{8}{16} = \frac{1}{2}$.

END OF SECTION A
TURN OVER FOR SECTION B

Section B

Answer **all** questions in the spaces provided.

13 (a) The unit vectors **i** and **j** are perpendicular.

Find the magnitude of the vector $-20\mathbf{i} + 21\mathbf{j}$

Circle your answer.

[1 mark]

-1 $\quad$ 1 $\quad$ $\sqrt{41}$ $\quad$ 29

13 (b) The angle between the vector **i** and the vector $-20\mathbf{i} + 21\mathbf{j}$ is θ

Which statement about θ is true?

Circle your answer.

[1 mark]

$0° < \theta < 45°$ $\quad$ $45° < \theta < 90°$ $\quad$ $90° < \theta < 135°$ $\quad$ $135° < \theta < 180°$

14 **In this question use $g = 10\ \text{m s}^{-2}$.**

A man of mass 80 kg is travelling in a lift.

The lift is rising vertically.

The lift decelerates at a rate of $1.5\ \text{m s}^{-2}$

Find the magnitude of the force exerted on the man by the lift.

[3 marks]

$F = ma$

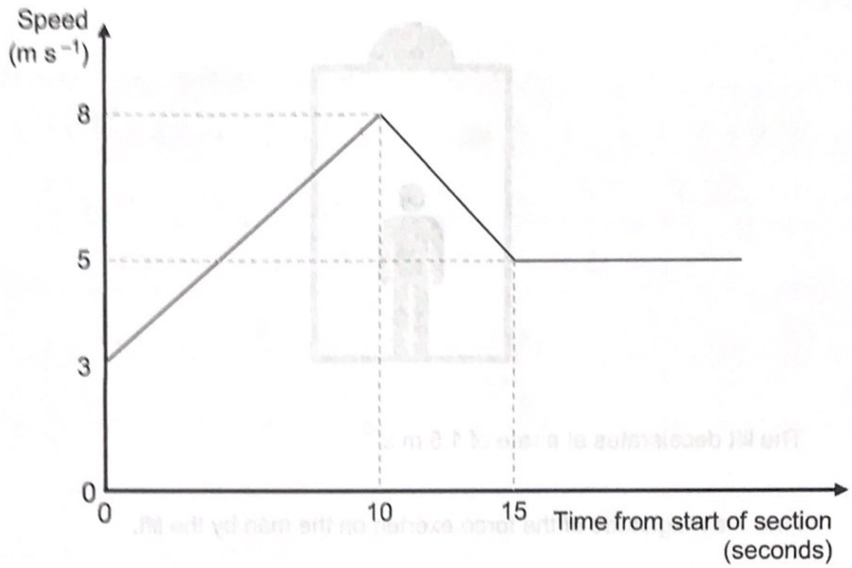
$F - 80 \times 10 = -80 \times 1.5$

$F - 800 = -120$

$F = 680N$.

$= 700N$ (1sf).

15 The graph shows how the speed of a cyclist varies during a timed section of length 120 metres along a straight track.



15 (a) Find the acceleration of the cyclist during the first 10 seconds.

[1 mark]

$a = \frac{v-u}{t} = \frac{8-3}{10} = \frac{5}{10} = \frac{1}{2} = 0.5 ms^{-2}$.

15 (b) After the first 15 seconds, the cyclist travels at a constant speed of 5 m s^{-1} for a further T seconds to complete the 120-metre section.

Calculate the value of T.

[4 marks]

Distance is area under graph.

$\frac{1}{2} \times (3+8) \times 10 + \frac{1}{2} \times (5+8) \times 5 + 5T = 120$

$5 \times 11 + \frac{1}{2} \times 13 \times 5 + 5T = 120$

$11 + 6.5 + T = 24$

$17.5 + T = 24$

$T = 6.5\text{s}$

Turn over for the next question

16 A particle, of mass 400 grams, is initially at rest at the point O.

The particle starts to move in a straight line so that its velocity, v m s^{-1}, at time t seconds is given by

$$v = 6t^2 - 12t^3 \text{ for } t > 0$$

16 (a) Find an expression, in terms of t, for the force acting on the particle.

[3 marks]

$a = \frac{dv}{dt}$

$a = 12t - 36t^2.$

$F = ma.$

$F = 0.4 \times (12t - 36t^2)$

$F = 4.8t - 14.4t^2.$

16 (b) Find the time when the particle next passes through O.

[5 marks]

$s = \int v \, dt$

$s = \int 6t^2 - 12t^3 \, dt$

$s = 2t^3 - 3t^4 + c.$

At $t = 0$, $s = 0$ so $c = 0$.

$s = 2t^3 - 3t^4.$

$s = 0$ at O.

$2t^3 - 3t^4 = 0$

$t^3(2 - 3t) = 0$

$t = 0 \quad t = \frac{2}{3}$

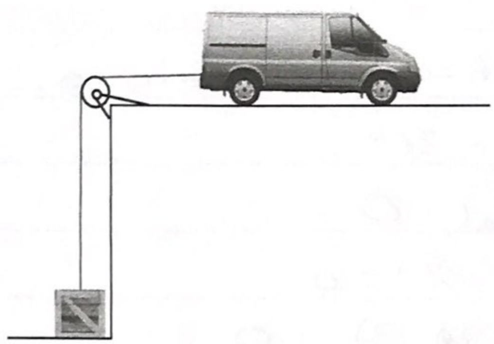
So next at O at $t = \frac{2}{3}$ s.

17 **In this question use $g = 9.8\ \text{m s}^{-2}$.**

A van of mass 1300 kg and a crate of mass 300 kg are connected by a light inextensible rope.

The rope passes over a light smooth pulley, as shown in the diagram.

The rope between the pulley and the van is horizontal.



Initially, the van is at rest and the crate rests on the lower level. The rope is taut.

The van moves away from the pulley to lift the crate from the lower level.

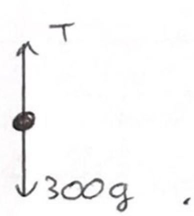
The van's engine produces a constant driving force of 5000 N.

A constant resistance force of magnitude 780 N acts on the van.

Assume there is no resistance force acting on the crate.

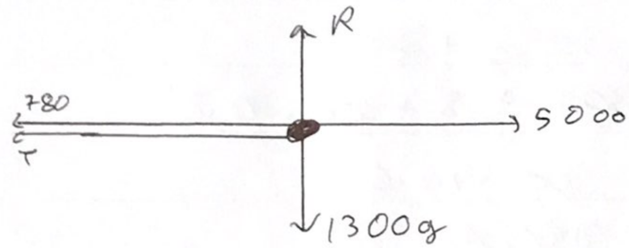
17 (a) (i) Draw a diagram to show the forces acting on the crate while it is being lifted.

[1 mark]



17 (a) (ii) Draw a diagram to show the forces acting on the van while the crate is being lifted.

[1 mark]



17 (b) Show that the acceleration of the van is 0.80 m s^{-2}

[4 marks]

Crate : $T - 300g = 300a$

Van : $5000 - T - 780 = 1300a$

$4220 - T = 1300a$

$4220 - 300g = 1600a$

$a = \dfrac{4220 - 300g}{1600}$

$a = \dfrac{4220 - 300 \times 9.8}{1600}$

$a = 0.80 \text{ ms}^{-2}$

17 (c) Find the tension in the rope.

[2 marks]

$T - 300g = 300a$

$T = 300g + 300a$

$T = 300 \times 9.8 + 300 \times 0.8$

$T = 300 \times 10.6$

$T = 3180\,N$

$= 3200\,N$ (2sf).

17 (d) Suggest how the assumption of a constant resistance force could be refined to produce a better model.

[1 mark]

Include air resistance, which would increase with speed.

END OF QUESTIONS