AQA

Please write clearly in block capitals.

Centre number | | | | | | Candidate number | | | | |

Surname MODEL SOLUTIONS

Forename(s)

Candidate signature

AS MATHEMATICS

Paper 2

Please note that question 17 uses the original Large Data Set "Family Food". This was replaced by the data set "Transport Stock Vehicle Database" in AS exams from June 2019. If you'd like to see the original data set, please contact maths@aqa.org.uk.

Wednesday 23 May 2018   Morning   Time allowed: 1 hour 30 minutes

Materials
- You must have the AQA Formulae for A-level Mathematics booklet.
- You should have a graphical or scientific calculator that meets the requirements of the specification.

Instructions
- Use black ink or black ball-point pen. Pencil should only be used for drawing.
- Fill in the boxes at the top of this page.
- Answer **all** questions.
- You must answer each question in the space provided for that question. If you require extra space, use an AQA supplementary answer book; do **not** use the space provided for a different question.
- Show all necessary working; otherwise marks for method may be lost.
- Do all rough work in this book. Cross through any work that you do not want to be marked.

Information
- The marks for questions are shown in brackets.
- The maximum mark for this paper is 80.

Advice
- Unless stated otherwise, you may quote formulae, without proof, from the booklet.
- You do not necessarily need to use all the space provided.

For Examiner's Use	
Question	Mark
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TOTAL	

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Section A

Answer **all** questions in the spaces provided.

1 Given that $\frac{dy}{dx} = \frac{1}{6x^2}$ find y.

Circle your answer.

[1 mark]

$\frac{-1}{3x^3} + c$ $\quad$ $\frac{1}{2x^3} + c$ $\quad$ $\frac{-1}{6x} + c$ $\quad$ $\frac{-1}{3x} + c$

$$\frac{dy}{dx} = \frac{1}{6x^2}$$

$$\frac{dy}{dx} = \frac{1}{6}x^{-2}$$

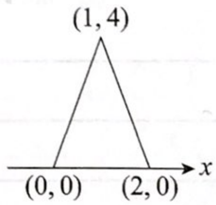
$$y = \int \frac{1}{6}x^{-2}\,dx$$

$$y = -\frac{1}{6}x^{-1} + c$$

$$y = \frac{-1}{6x} + c.$$

2 **Figure 1** shows $y = \mathrm{f}(x)$.

Figure 1



Which figure below shows $y = \mathrm{f}(2x)$?

Tick **one** box.

[1 mark]

Figure 2

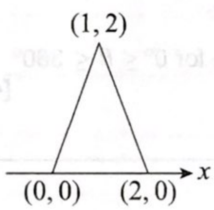


Figure 3

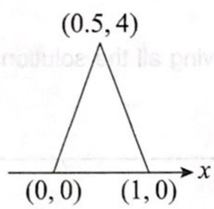


Figure 4

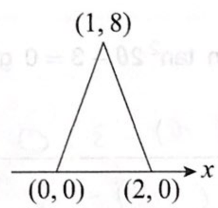


Figure 5

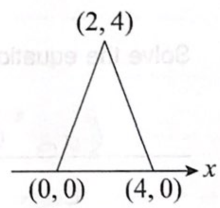


Figure 2 ☐

Figure 3 ☑

Figure 4 ☐

Figure 5 ☐

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3 Express as a single logarithm

$$2\log_a 6 - \log_a 3$$

[2 marks]

$2\log_a 6 - \log_a 3 =$

$\log_a(6^2) - \log_a 3 =$

$\log_a(36) - \log_a 3 =$

$\log_a(36 \div 3) =$

$\log_a(12)$

4 Solve the equation $\tan^2 2\theta - 3 = 0$ giving all the solutions for $0° \le \theta \le 360°$

[4 marks]

$\tan^2(2\theta) - 3 = 0$.

$\tan^2(2\theta) = 3$

$\tan(2\theta) = \pm\sqrt{3}$

+ve root

$\tan(2\theta) = \sqrt{3}$

$2\theta = 60°, 240°, 420°, 600°$

$\theta = 30°, 120°, 210°, 300°$

-ve root

$\tan(2\theta) = -\sqrt{3}$

$2\theta = 120°, 300°, 480°, 660°$

$\theta = 60°, 150°, 240°, 330°$.

$\theta = 30°, 60°, 120°, 150°, 210°, 240°, 300°, 330°$.

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5 $f'(x) = \left(2x - \frac{3}{x}\right)^2$ and $f(3) = 2$

Find $f(x)$.

[4 marks]

$f'(x) = (2x - \frac{3}{x})^2$.

$f(x) = \int (2x - \frac{3}{x})^2 dx$.

$f(x) = \int 4x^2 - 12 + \frac{9}{x^2} dx$.

$f(x) = \int 4x^2 - 12 + 9x^{-2} dx$.

$f(x) = \frac{4}{3}x^3 - 12x - 9x^{-1} + c$.

$f(x) = \frac{4x^3}{3} - 12x - \frac{9}{x} + c$.

Find c:

$f(3) = 2$.

$\frac{4(3)^3}{3} - 12(3) - \frac{9}{3} + c = 2$.

$\frac{4 \times 27}{3} - 36 - 3 + c = 2$

$\frac{108}{3} - 36 - 3 + c = 2$

$36 - 36 - 3 + c = 2$

$-3 + c = 2$

$c = 5$.

$f(x) = \frac{4x^3}{3} - 12x - \frac{9}{x} + 5$.

Turn over ▶

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6 Points A $(-7, -7)$, B $(8, -1)$, C $(4, 9)$ and D $(-11, 3)$ are the vertices of a quadrilateral $ABCD$.

6 (a) Prove that $ABCD$ is a rectangle.

[4 marks]

Gradient $AB = \frac{-1-(-7)}{8-(-7)} = \frac{6}{15} = \frac{2}{5}$

Gradient $DC = \frac{3-9}{-11-4} = \frac{-6}{-15} = \frac{2}{5}$

Gradient $BC = \frac{9-(-1)}{4-8} = \frac{10}{-4} = -\frac{5}{2}$.

Gradient $DA = \frac{-7-3}{-7-(-11)} = \frac{-10}{4} = -\frac{5}{2}$.

Gradient AB = Gradient DC

and Gradient DA = Gradient AB

So the points form a parallelogram.

Gradient AB × Gradient BC $= \frac{2}{5} \times -\frac{5}{2} = -1$.

So AB and BC are perpendicular to each other.

So $ABC = 90°$.

Hence, we have a rectangle.

6 (b) Find the area of $ABCD$.

[2 marks]

Length $(AB) = \sqrt{(8-(-7))^2 + (-1-(-7))^2}$

$= \sqrt{15^2 + 6^2}$

$= \sqrt{225 + 36}$

$= \sqrt{261} = 3\sqrt{29}$

Length $(BC) = \sqrt{(9-(-1))^2 + (4-8)^2}$

$= \sqrt{10^2 + 4^2}$

$= \sqrt{100 + 16}$

$= \sqrt{116} = 2\sqrt{29}$

Area $= 3\sqrt{29} \times 2\sqrt{29} = 3 \times 2 \times 29 = 174$.

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7 (a) Express $2x^2 - 5x + k$ in the form $a(x-b)^2 + c$

[3 marks]

$2x^2 - 5x + k =$

$2(x^2 - \frac{5}{2}x) + k =$

$2(x - \frac{5}{4})^2 + k - 2(\frac{5}{4})^2 =$

$2(x - \frac{5}{4})^2 + k - 2 \times \frac{25}{16} =$

$2(x - \frac{5}{4})^2 + k - \frac{25}{8}$.

$a = 2$, $b = \frac{5}{4}$, $c = k - \frac{25}{8}$.

7 (b) Find the values of k for which the curve $y = 2x^2 - 5x + k$ does **not** intersect the line $y = 3$

[3 marks]

$y = 2x^2 - 5x + k$

$y = \underbrace{2(x - \frac{5}{4})^2}_{0 \text{ at } x = 0} + k - \frac{25}{8}$.

So min ~~of~~ y is $k - \frac{25}{8}$.

Need $k - \frac{25}{8} > 3$.

$k > 3 + \frac{25}{8}$

$k > \frac{49}{8}$.

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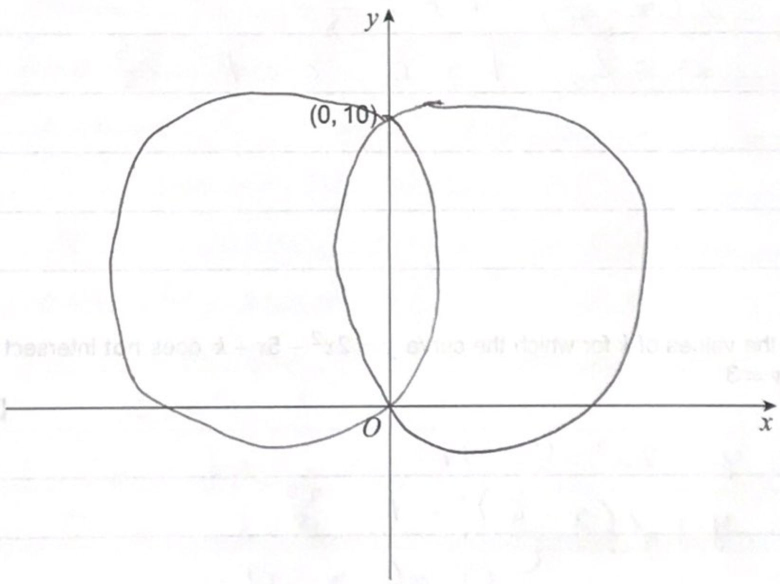
8 A circle of radius 6 passes through the points (0, 0) and (0, 10).

8 (a) Sketch the two possible positions of the circle.

[1 mark]



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8 (b) Find the equations of the two circles.

[3 marks]

Same distance from (0,0) and (0,10) so centre must lie on line $y = 5$.

Pythagoras give $\sqrt{(5-0)^2 + (x-0)^2} = 6$.

$\sqrt{5^2 + x^2} = 6$

$25 + x^2 = 36$

$x^2 = 11$.

$x = \pm\sqrt{11}$

Circles are:

$(x - \sqrt{11})^2 + (y - 5)^2 = 36$

$(x + \sqrt{11})^2 + (y - 5)^2 = 36$.

Turn over for the next question

Do not write outside the box

9 It is given that $\cos 15° = \frac{1}{2}\sqrt{2+\sqrt{3}}$ and $\sin 15° = \frac{1}{2}\sqrt{2-\sqrt{3}}$

Show that $\tan^2 15°$ can be written in the form $a + b\sqrt{3}$, where a and b are integers.

Fully justify your answer.

[3 marks]

$\tan 15 = \frac{\sin 15}{\cos 15}$

$\tan 15 = \frac{\frac{1}{2}\sqrt{2+\sqrt{3}}}{\frac{1}{2}\sqrt{2-\sqrt{3}}}$

$\tan 15 = \frac{\sqrt{2+\sqrt{3}}}{\sqrt{2-\sqrt{3}}}$

$\tan 15 = \sqrt{\frac{2+\sqrt{3}}{2-\sqrt{3}}}$

$\tan^2 15 = \frac{2+\sqrt{3}}{2-\sqrt{3}}$

$\tan^2 15 = \frac{(2+\sqrt{3})^2}{(2-\sqrt{3})(2+\sqrt{3})}$

$\tan^2 15 = \frac{4+4\sqrt{3}+3}{4-2\sqrt{3}+2\sqrt{3}-3}$

$\tan^2 15 = \frac{7+4\sqrt{3}}{1}$

$\tan^2 15 = 7+4\sqrt{3}$

Do not write outside the box

10 In the binomial expansion of $(1+x)^n$, where $n \geq 4$, the coefficient of x^4 is $1\frac{1}{2}$ times the sum of the coefficients of x^2 and x^3

Find the value of n.

[5 marks]

$\binom{n}{4} = 1\frac{1}{2}\left(\binom{n}{2}+\binom{n}{3}\right)$

$\frac{n!}{4!(n-4)!} = \frac{3}{2}\left(\frac{n!}{2!(n-2)!} + \frac{n!}{3!(n-3)!}\right)$

$\frac{n(n-1)(n-2)(n-3)}{24} = \frac{3}{2}\left(\frac{n(n-1)}{2} + \frac{n(n-1)(n-2)}{6}\right)$

$\frac{(n-2)(n-3)}{24} = \frac{3}{2}\left(\frac{1}{2} + \frac{n-2}{6}\right)$

$\frac{(n-2)(n-3)}{24} = \frac{3}{2}\left(\frac{1}{2} + \frac{n}{6} - \frac{1}{3}\right)$

$\frac{n^2-5n+6}{24} = \frac{3}{2}\left(\frac{1}{6} + \frac{n}{6}\right)$

$\frac{n^2-5n+6}{24} = \frac{1}{4} + \frac{n}{4}$

$n^2-5n+6 = 6+6n$.

$n^2-11n=0$

$n(n-11)=0$

$n=0$ $\qquad$ $n=11$.

Would not have x^2, x^3, x^4 coefficients

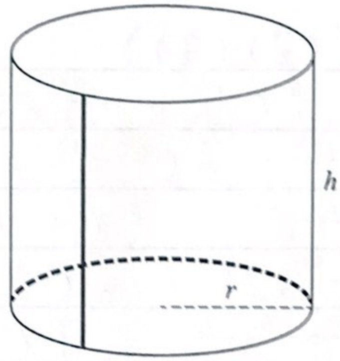
$n=11$ only.

Turn over for the next question

Turn over ▶

11 Rakti makes open-topped cylindrical planters out of thin sheets of galvanised steel.

She bends a rectangle of steel to make an open cylinder and welds the joint. She then welds this cylinder to the circumference of a circular base.



The planter must have a capacity of $8000\,\text{cm}^3$

Welding is time consuming, so Rakti wants the total length of weld to be a minimum.

Calculate the radius, r, and height, h, of a planter which requires the minimum total length of weld.

Fully justify your answers, giving them to an appropriate degree of accuracy.

[9 marks]

Weld $= 2\pi r + h$.

Volume: $\pi r^2 h = 8000$

$h = \frac{8000}{\pi r^2}$.

Weld $= 2\pi r + \frac{8000}{\pi r^2}$

Need $\frac{d\text{Weld}}{dr} = 0$ for minimum.

$\frac{d\text{Weld}}{dr} = 2\pi - \frac{16000}{\pi r^3} = 0$.

$2\pi = \frac{16000}{\pi r^3}$

$r^3 = \frac{16000}{2\pi^2}$

$r^3 = \frac{8000}{\pi^2}$.

$r = \left(\frac{8000}{\pi^2}\right)^{\frac{1}{3}}$

$r = 9.32$ cm.

$h = \frac{8000}{\pi\left(\left(\frac{8000}{\pi^2}\right)^{\frac{1}{3}}\right)^2}$

$h = 29.3$ cm

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Check it's a minimum:

$\frac{d^2 weld}{dr^2} = \frac{48000}{\pi r^4} > 0$ always,

so minimum.

Turn over ▶

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12 Trees in a forest may be affected by one of two types of fungal disease, but not by both.

The number of trees affected by disease A, n_A, can be modelled by the formula

$$n_A = ae^{0.1t}$$

where t is the time in years after 1 January 2017.

The number of trees affected by disease B, n_B, can be modelled by the formula

$$n_B = be^{0.2t}$$

On 1 January 2017 a **total** of 290 trees were affected by a fungal disease.

On 1 January 2018 a **total** of 331 trees were affected by a fungal disease.

12 (a) Show that $b = 90$, to the nearest integer, and find the value of a.

[3 marks]

$a + b = 290$

$ae^{0.1} + be^{0.2} = 331.$

$a = 290 - b.$

$(290-b)e^{0.1} + be^{0.2} = 331.$

$b(e^{0.2} - e^{0.1}) = 331 - 290e^{0.1}$

$b = \dfrac{331 - 290e^{0.1}}{e^{0.2} - e^{0.1}}.$

$b = 90$ (nearest integer).

$a = 290 - 90$

$a = 200$ (nearest integer).

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12 (b) Estimate the total number of trees that will be affected by a fungal disease on 1 January 2020.

[1 mark]

$n = 200e^{0.1\times 3} + 90e^{0.2\times 3}$

$n = 200e^{0.3} + 90e^{0.6}$

$n = 434$

12 (c) Find the year in which the number of trees affected by disease B will first exceed the number affected by disease A.

[3 marks]

$90e^{0.2t} > 200e^{0.1t}$

$e^{0.1t} > \frac{200}{90}$

$0.1t > \ln\left(\frac{200}{90}\right)$

$t > 10\ln\left(\frac{200}{90}\right)$

$t > 7.985$

So in 8th year

In 2024.

12 (d) Comment on the long-term accuracy of the model.

[1 mark]

The model predicts unlimited disease growth, however there are only finitely many trees, so model will eventually break down.

Turn over for Section B

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Section B

Answer **all** questions in the spaces provided.

13 The table below shows the probability distribution for a discrete random variable X.

x	0	1	2	3	4 or more
$\mathrm{P}(X = x)$	0.35	0.25	k	0.14	0.1

Find the value of k.

Circle your answer.

[1 mark]

0.14 0.16 0.18 1

14 Given that $\sum x = 364$, $\sum x^2 = 19412$, $n = 10$, find σ, the standard deviation of X.

Circle your answer.

[1 mark]

24.8 44.1 616.2 1941.2

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15 Nicola, a darts player, is practising hitting the bullseye. She knows from previous experience that she has a probability of 0.3 of hitting the bullseye with each dart.

Nicola throws eight practice darts.

15 (a) Using a binomial distribution, calculate the probability that she will hit the bullseye three or more times.

[2 marks]

$N \sim B(8, 0.3)$.

$P(N \geq 3) = 1 - P(N \leq 2)$

$= 1 - 0.55177$

$= 0.448$.

15 (b) Nicola throws eight practice darts on three different occasions. Calculate the probability that she will hit the bullseye three or more times on all three occasions.

[2 marks]

$Prob = 0.448^3$

$= 0.090$

15 (c) State two assumptions that are necessary for the distribution you have used in part **(a)** to be valid.

[2 marks]

The probability of hitting the bullseye stays at 0.3

Each dart throw is independent of all others.

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16 Kevin is the Principal of a college.

He wishes to investigate types of transport used by students to travel to college.

There are 3200 students in the college and Kevin decides to survey 60 of them.

Describe how he could obtain a simple random sample of size 60 from the 3200 students.

[4 marks]

Assign a number from 1 to 3200 to each student (without repeats).
Generate 60 random numbers (without repeats) between 1 and 3200.
Select the students corresponding to those random numbers to be surveyed.

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17 The table below is an extract from the Large Data Set, showing the purchased quantities of fats and oils for the South East of England in 2014.

Description	Purchased quantity
Butter	42
Soft margarine	16
Olive oil	17
Other vegetable and salad oils	28

Kim claims that more olive oil was purchased in the South East than soft margarine.

Explain why Kim may be incorrect.

[2 marks]

There are no units so all units could be different. Furthermore, oil is a liquid so is likely measured in units of capacity, while soft margarine is likely measured in units of weight.

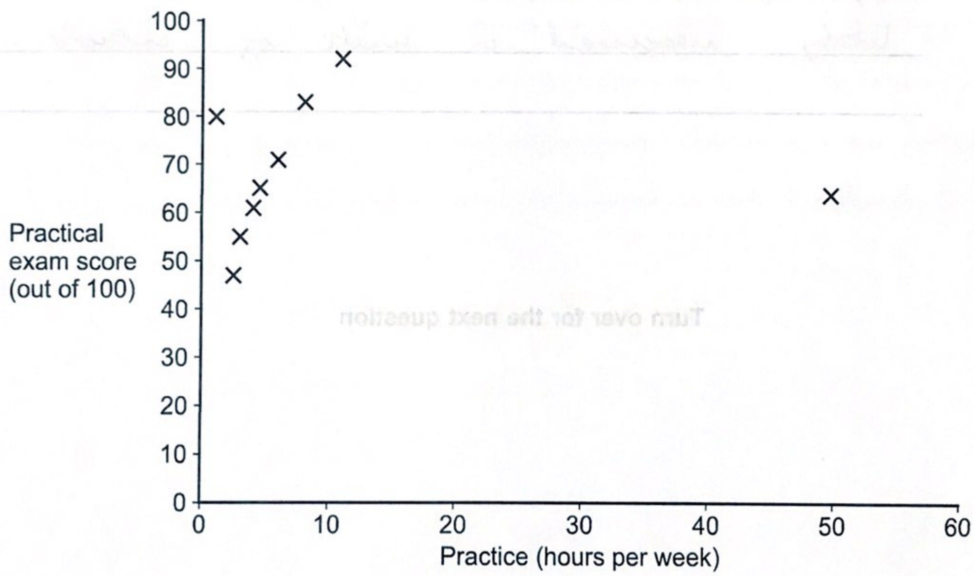
Turn over for the next question

18 Jennie is a piano teacher who teaches nine pupils.

She records how many hours per week they practice the piano along with their most recent practical exam score.

Student	**Practice (hours per week)**	**Practical exam score (out of 100)**
Donovan	50	64
Vazquez	6	71
Higgins	3	55
Begum	2.5	47
Collins	1	80
Coldbridge	4	61
Nedbalek	4.5	65
Carter	8	83
White	11	92

She plots a scatter diagram of this data, as shown below.



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18 (a) Identify two possible outliers by name, giving a possible explanation for the position on the scatter diagram of each outlier.

[4 marks]

First outlier Donovan

Possible reason Error ~~in~~ in data entry (perhaps meant to be 5 not 50).

Second outlier Collins

Possible reason Naturally able student.

18 (b) Jennie discards the two outliers.

18 (b) (i) Describe the correlation shown by the scatter diagram for the remaining points.

[1 mark]

Strong positive correlation.

18 (b) (ii) Interpret this correlation in the context of the question.

[1 mark]

Students who practice more perform better in the exam.

Turn over for the next question

19 Martin grows cucumbers from seed.

In the past, he has found that 70% of all seeds successfully germinate and grow into cucumber plants.

He decides to try out a new brand of seed.

The producer of this brand claims that these seeds are more likely to successfully germinate than other brands of seeds.

Martin sows 20 of this new brand of seed and 18 successfully germinate.

Carry out a hypothesis test at the 5% level of significance to investigate the producer's claim.

[7 marks]

$H_0: p = 0.7$

$H_1: p > 0.7$

Under Null hypothesis:

$X \sim B(20, 0.7)$.

$P(X \geq 18) = 1 - P(X \leq 17) = 1 - 0.965$

$= 0.035 < 0.05$.

So reject H_0.

There is sufficient evidence that the new seeds are more likely to germinate.

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END OF QUESTIONS

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