AQA

Please write clearly in block capitals.

Centre number | | | | | | Candidate number | | | | |

Surname ______________________________

Forename(s) ______________________________

Candidate signature ______________________________

AS
MATHEMATICS
Paper 1

Wednesday 16 May 2018 Morning Time allowed: 1 hour 30 minutes

Materials
- You must have the AQA Formulae for A-level Mathematics booklet.
- You should have a graphical or scientific calculator that meets the requirements of the specification.

Instructions
- Use black ink or black ball-point pen. Pencil should only be used for drawing.
- Fill in the boxes at the top of this page.
- Answer **all** questions.
- You must answer each question in the space provided for that question. If you require extra space, use an AQA supplementary answer book; do **not** use the space provided for a different question.
- Show all necessary working; otherwise marks for method may be lost.
- Do all rough work in this book. Cross through any work that you do not want to be marked.

Information
- The marks for questions are shown in brackets.
- The maximum mark for this paper is 80.

Advice
- Unless stated otherwise, you may quote formulae, without proof, from the booklet.
- You do not necessarily need to use all the space provided.

For Examiner's Use	
Question	Mark
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Section A

Answer **all** questions in the spaces provided.

1 Three of the following points lie on the same straight line.

Which point does **not** lie on this line?

Tick **one** box. **[1 mark]**

- [ ] $(-2, 14)$
- [x] $(-1, 8)$
- [ ] $(1, -1)$
- [ ] $(2, -6)$

2 A circle has equation $(x-2)^2 + (y+3)^2 = 13$

Find the gradient of the tangent to this circle at the origin.

Circle your answer. **[1 mark]**

$-\frac{3}{2}$ $\quad$ $-\frac{2}{3}$ $\quad$ $\frac{2}{3}$ $\quad$ $\frac{3}{2}$

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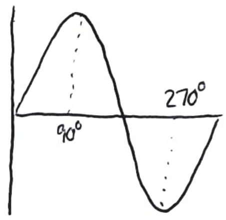
3 State the interval for which $\sin x$ is a decreasing function for $0° \le x \le 360°$

[2 marks]

Between 90° and 270°.

$\Rightarrow 90° < x < 270°$.



Turn over for the next question

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4 (a) Find the first three terms in the expansion of $(1 - 3x)^4$ in ascending powers of x.

[3 marks]

$(1-3x)^4 \approx 1 + 4(-3x) + 6(-3x)^2 + \dots$

$= 1 - 12x + 54x^2 + \dots$

~~For small values of 3x~~

4 (b) Using your expansion, approximate $(0.994)^4$ to six decimal places.

[2 marks]

Let $1 - 3x = 0.994$.

$\Rightarrow x = 0.002$

$\Rightarrow (0.994)^4 = 1 - 12(0.002) + 54(0.002)^2$

$= 0.976216$

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5 Point C has coordinates $(c, 2)$ and point D has coordinates $(6, d)$.

The line $y + 4x = 11$ is the perpendicular bisector of CD.

Find c and d.

[5 marks]

$m_{perp} = -4.$

$\Rightarrow \quad m_{CD} = \frac{1}{4}$

$dy = \quad d - 2$

$dx = \quad 6 - c$

$\frac{dy}{dx} = m_{CD} = \frac{d-2}{6-c} = \frac{1}{4}$

$\Rightarrow \quad 4d - 8 = 6 - c$

$\Rightarrow \quad c + 4d = 14$

Since $y+4x=11$ is the bisector,

we must find the midpoint of C and D:

$x = \frac{6+c}{2}, \quad y = \frac{2+d}{2}.$

$\Rightarrow \quad \frac{2+d}{2} + 4\left(\frac{6+c}{2}\right) = 11$

$\Rightarrow \quad 2+d + 4(6+c) = 22$

$\Rightarrow \quad d + 4c = -4$

$d+4c = -4, \quad c+4d = 14$

$\Rightarrow \quad 4d + 16c = -16, \quad c + 4d = 14$

$\Rightarrow \quad 15c = -30$

$\Rightarrow \quad c = -2 \quad \Rightarrow \quad d = 4.$

Turn over ▶

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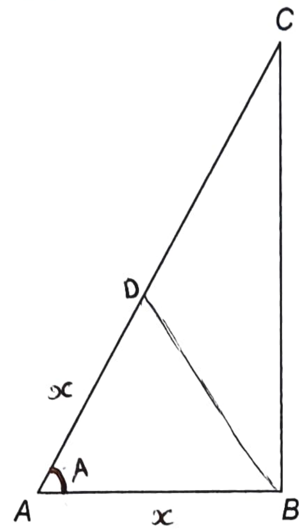
6 ABC is a right-angled triangle.



D is the point on hypotenuse AC such that $AD = AB$.

The area of ΔABD is equal to half that of ΔABC.

6 (a) Show that $\tan A = 2\sin A$

[4 marks]

Let $AD = AB = x$

If the area of $\Delta ABD = \frac{1}{2}\Delta ABC$,

the area of $\Delta ABD = \Delta CBD$.

$\Rightarrow \frac{1}{2}AC = AD = x. \Rightarrow AC = 2x$

$\cos A = \frac{x}{2x} = \frac{1}{2}$

$\Rightarrow A = \cos^{-1}(\frac{1}{2}) = 60°.$

$\Rightarrow \sin A = \frac{\sqrt{3}}{2}, \tan A = \sqrt{3}.$

$\Rightarrow 2\sin A = \sqrt{3} = \tan A.$

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6 (b) (i) Show that the equation given in part **(a)** has two solutions for $0° \le A \le 90°$

[2 marks]

$\tan A = \frac{\sin A}{\cos A} = 2\sin A$

$\Rightarrow \sin A = 2\sin A \cos A.$

which is true for $A = 0°, 60°$.

6 (b) (ii) State the solution which is appropriate in this context.

[1 mark]

$A = 60°$. ($A = 0°$ isn't appropriate as we would not have a triangle).

Turn over for the next question

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7 Prove that

$$n \text{ is a prime number greater than } 5 \Rightarrow n^4 \text{ has final digit } 1$$

[5 marks]

If last digit of $n = 1$: $(10k+1)^4$

$= \ldots + 1^4 = \ldots + 1$

$\Rightarrow$ last digit is 1.

If last digit of $n = 3$: $(10k+3)^4$

$= \ldots + 3^4 = \ldots + 81$

$\Rightarrow$ last digit is 1.

If last digit of $n = 7$: $(10k+7)^4$

$= \ldots + 7^4 = \ldots + 2401$

$\Rightarrow$ last digit is 1.

If last digit of $n = 9$: $(10k+9)^4$

$= \ldots + 9^4 = \ldots + 6561.$

$\Rightarrow$ last digit is 1.

Last digit of $n = 5$ can be ignored as n will be divisible by 5 (i.e. not prime).

Turn over for the next question

Turn over ▶

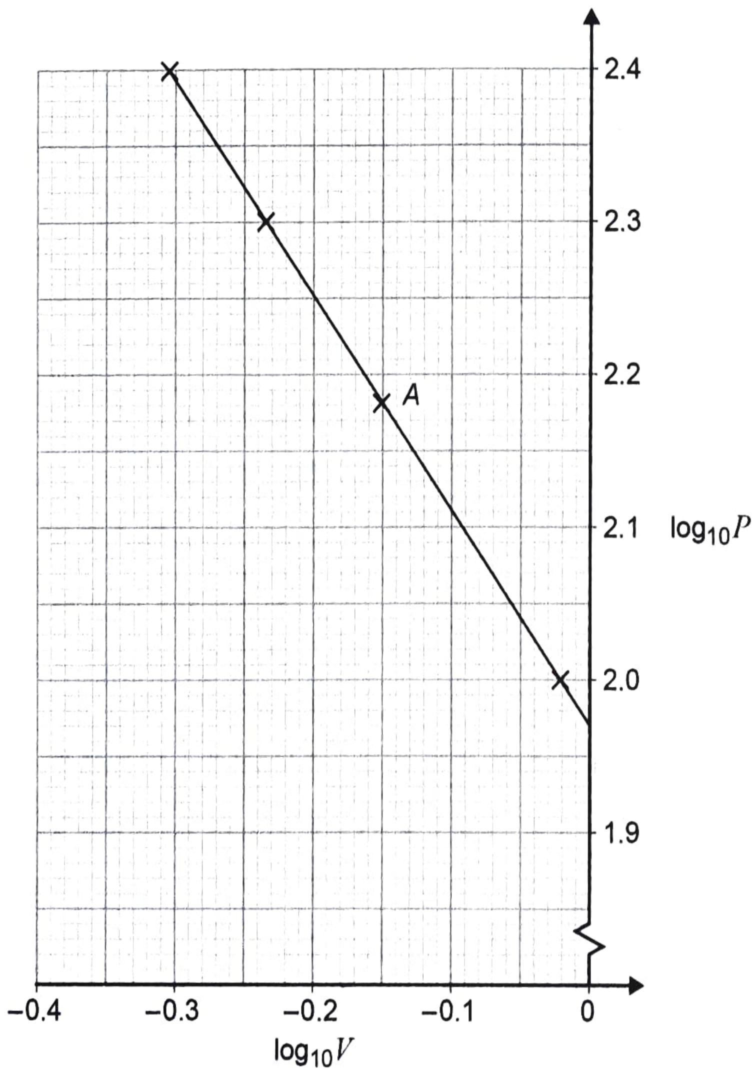
8 Maxine measures the pressure, P kilopascals, and the volume, V litres, in a fixed quantity of gas.

Maxine believes that the pressure and volume are connected by the equation

$$P = cV^d$$

where c and d are constants.

Using four experimental results, Maxine plots $\log_{10}P$ against $\log_{10}V$, as shown in the graph below.



8 (a) Find the value of P and the value of V for the data point labelled A on the graph.
[2 marks]

$\log_{10} P_A = 2.18 \Rightarrow P_A = 10^{2.18} = 151.$

$\log_{10} P_V = -0.15 \Rightarrow V_A = 10^{-0.15} = 0.708.$

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8 (b) Calculate the value of each of the constants c and d.

[4 marks]

$P = cV^d \Rightarrow \log_{10} P = \log_{10} cV^d$

$= \log_{10} c + d\log_{10} V$

$d =$ gradient $= -1.4$

$\log_{10} c = \log_{10} P$ intercept $=$ [redacted] 1.97

$\Rightarrow c = 93.3$.

8 (c) Estimate the pressure of the gas when the volume is 2 litres.

[2 marks]

$P = 93.3V^{-1.4}$.

Let $V = 2$

$\Rightarrow P = 93.3(2)^{-1.4} = 35.35$

$= 35.4$ kilo pascals.

Turn over ▶

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9 Craig is investigating the gradient of chords of the curve with equation $f(x) = x - x^2$

Each chord joins the point (3, –6) to the point $(3+h, f(3+h))$

The table shows some of Craig's results.

x	$f(x)$	h	$x+h$	$f(x+h)$	Gradient
3	–6	1	4	–12	–6
3	–6	0.1	3.1	–6.51	–5.1
3	–6	0.01	3.01	–6.0501	–5.01
3	–6	0.001			
3	–6	0.0001			

9 (a) Show how the value –5.1 has been calculated.

[1 mark]

$$\frac{((-6.51) - (-6))}{(3.1 - 3)} = -5.1$$

9 (b) Complete the third row of the table above.

[2 marks]

$x+h = 3 + 0.01 = 3.01$

$f(x+h) = 3.01 - (3.01)^2 = -6.0501$

$$\text{Gradient} = \frac{((-6.0501) - (-6))}{(3.01 - 3)} = -5.01$$

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9 (c) State the limit suggested by Craig's investigation for the gradient of these chords as h tends to 0

[1 mark]

As $h \to 0$, $m \to -5$.

9 (d) Using differentiation from first principles, verify that your result in part **(c)** is correct.

[4 marks]

$$\text{Gradient} = \frac{f(x+h) - f(x)}{x+h-x}.$$

$$= \frac{(3+h) - (3+h)^2 - (3-3^2)}{h}$$

$$= \frac{3+h-9-6h-h^2-(-6)}{h}$$

$$= \frac{-5h-h^2}{h}$$

$$= -5-h.$$

As $h \to 0$, $m \to -5$.

Therefore, $x = 3 \Rightarrow m = -5$.

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10 A curve has equation $y = 2x^2 - 8x\sqrt{x} + 8x + 1$ for $x \geq 0$

10 (a) Prove that the curve has a maximum point at (1, 3)

Fully justify your answer.

[9 marks]

When $x=1$, $y = 2(1)^2 - 8(1)(\sqrt{1}) + 8(1) + 1$
$= 2 - 8 + 8 + 1$
$= 3$.

$x\sqrt{x} = x^{3/2}$

$\frac{dy}{dx} = (2 \cdot 2)x - (8 \cdot \frac{3}{2})x^{1/2} + 8$
$= 4x - 12\sqrt{x} + 8$

Stationary point occurs when $\frac{dy}{dx} = 0$.

$4x - 12\sqrt{x} + 8 = 0 \Rightarrow x - 3\sqrt{x} + 2 = 0$
$\Rightarrow x = 1, 4$.

Stationary point at $x=1$ verified.

$\frac{d^2y}{dx^2} = 4 - 6x^{-1/2}$
$x=1 \Rightarrow \frac{d^2y}{dx^2} = -2 < 0$
If $\frac{d^2y}{dx^2} < 0$, this is a maximum point.

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10 (b) Find the coordinates of the other stationary point of the curve and state its nature.
[2 marks]

$x = 4 . \Rightarrow \quad y = 2(4)^2 - 8(4)(\sqrt{4}) + 8(4) + 1 = 1$

$\frac{d^2y}{dx^2} = 4 - 6x^{-\frac{1}{2}} = 1 > 0$

when $x = 4$.

$\frac{d^2y}{dx^2} > 0$ suggests $(4, 1)$ is a minimum point.

Turn over for Section B

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Section B

Answer **all** questions in the spaces provided.

11 **In this question use $g = 9.8\,\mathrm{m\,s^{-2}}$**

A ball, initially at rest, is dropped from a height of 40 m above the ground.

Calculate the speed of the ball when it reaches the ground.

Circle your answer.

[1 mark]

$-28\,\mathrm{m\,s^{-1}}$ $\quad$ $28\,\mathrm{m\,s^{-1}}$ $\quad$ $-780\,\mathrm{m\,s^{-1}}$ $\quad$ $780\,\mathrm{m\,s^{-1}}$

$v^2 = u^2 + 2as \Rightarrow v^2 = 0^2 + (2 \times 9.8 \times 40)$

12 An object of mass 5 kg is moving in a straight line.

As a result of experiencing a forward force of F newtons and a resistant force of R newtons it accelerates at $0.6\,\mathrm{m\,s^{-2}}$

Which one of the following equations is correct?

Circle your answer.

[1 mark]

$F - R = 0$ $\quad$ $F - R = 5$ $\quad$ $F - R = 3$ $\quad$ $F - R = 0.6$

$F_x = ma$ $\qquad$ $a = 0.6\,\mathrm{ms^{-2}}$, $m = 5\,\mathrm{kg}$

$\Rightarrow F_x = 3N$.

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13 A vehicle, which begins at rest at point P, is travelling in a straight line.

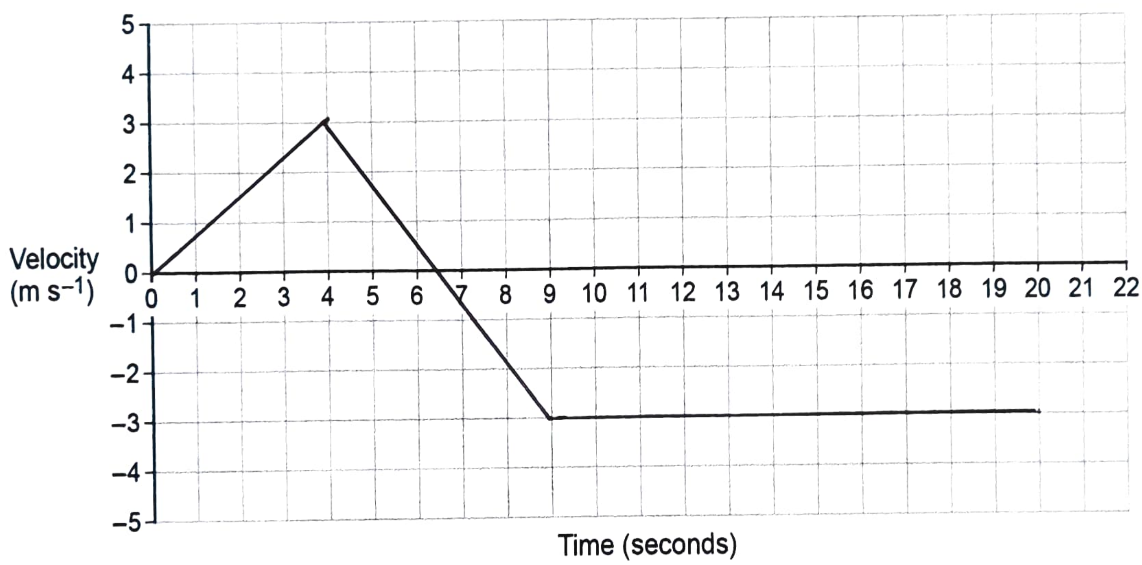
For the first 4 seconds the vehicle moves with a constant acceleration of $0.75\,\text{m s}^{-2}$

For the next 5 seconds the vehicle moves with a constant acceleration of $-1.2\,\text{m s}^{-2}$

The vehicle then immediately stops accelerating, and travels a further 33 m at constant speed.

13 (a) Draw a velocity–time graph for this journey on the grid below.

[3 marks]



13 (b) Find the distance of the car from P after 20 seconds.

[3 marks]

For $0 \le t < 4$: $\frac{1}{2} \times 3 \times 4 = +6\text{m}$.

For $4 \le t < 6.5$: $\frac{1}{2} \times 3 \times 2.5 = +3.75\text{m}$

For $6.5 \le t < 9$: $\frac{1}{2} \times -3 \times 2.5 = -3.75\text{m}$

For $9 \le t \le 20$: -33m.

Total $= -27\text{m}$, distance from P is 27m.

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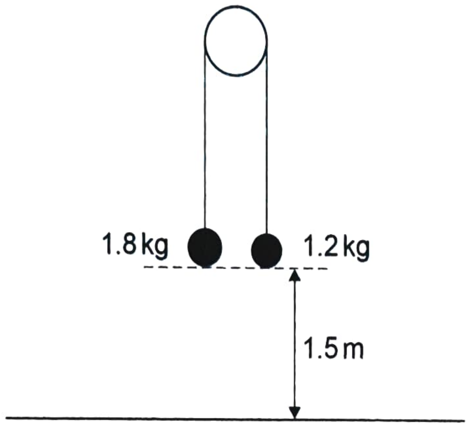
14 **In this question use $g = 9.81\,\text{m s}^{-2}$**

Two particles, of mass 1.8 kg and 1.2 kg, are connected by a light, inextensible string over a smooth peg.



14 (a) Initially the particles are held at rest 1.5 m above horizontal ground and the string between them is taut.

The particles are released from rest.

Find the time taken for the 1.8 kg particle to reach the ground.

[5 marks]

$(1.8\text{kg} \times 9.81\text{ms}^{-2}) - T = 1.8 \times a$

$T - (1.2\text{kg} \times 9.81\text{ms}^{-2}) = 1.2 \times a$

$17.658 - T = 1.8a$

$T - 11.772 = 1.2a$

$\Rightarrow a = 1.962\text{ms}^{-2}$

$S = ut + \frac{1}{2}at^2$

$1.5 = 0t + (\frac{1}{2} \times 1.962 \times t^2)$

$\Rightarrow t^2 = \frac{3}{1.962} \Rightarrow t = 1.2365\text{s}$

$= 1.24\text{s}.$

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14 (b) State one assumption you have made in answering part **(a)**.

[1 mark]

Air resistance is ignored.

Turn over for the next question

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15 A cyclist, Laura, is travelling in a straight line on a horizontal road at a constant speed of 25 km h^{-1}

A second cyclist, Jason, is riding closely and directly behind Laura. He is also moving with a constant speed of 25 km h^{-1}

15 (a) The driving force applied by Jason is likely to be less than the driving force applied by Laura.

Explain why.

[1 mark]

He faces less air resistance.

15 (b) Jason has a problem and stops, but Laura continues at the same constant speed.

Laura sees an accident 40 m ahead, so she stops pedalling and applies the brakes.

She experiences a total resistance force of 40 N

Laura and her cycle have a combined mass of 64 kg

15 (b) (i) Determine whether Laura stops before reaching the accident.

Fully justify your answer.

[4 marks]

$F = ma$. $F = 40N$, $m = 64kg$ $\Rightarrow$ $a = -0.625 ms^{-2}$

$u = 25kmh^{-1} = 6.944ms^{-1}$.

$v = 0 ms^{-1}$

$v^2 = u^2 + 2as$

$0 = 6.944^2 + (2 \times -0.625\, s)$.

$\Rightarrow S = 38.575\, m < 40m$

Laura stops before the accident.

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15 (b) (ii) State one assumption you have made that could affect your answer to part **(b)(i)**.
[1 mark]

Resistive force diminishes as speed decreases.

Turn over for the next question

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16 A remote-controlled toy car is moving over a horizontal surface. It moves in a straight line through a point A.

The toy is initially at the point with displacement 3 metres from A. Its velocity, $v\ \mathrm{m\,s^{-1}}$, at time t seconds is defined by

$$v = 0.06(2 + t - t^2)$$

16 (a) Find an expression for the displacement, r metres, of the toy from A at time t seconds.

[4 marks]

$r = \int v\,dt = \int 0.06(2+t-t^2)\,dt$

$= 0.12t + 0.03t^2 - 0.02t^3 + c$

$t=0 \Rightarrow s = 3. \Rightarrow c = 3.$

$\Rightarrow r = 0.12t + 0.03t^2 - 0.02t^3 + 3$

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16 (b) **In this question use $g = 9.8\,\text{m}\,\text{s}^{-2}$**

At time $t = 2$ seconds, the toy launches a ball which travels directly upwards with initial speed $3.43\,\text{m}\,\text{s}^{-1}$

Find the time taken for the ball to reach its highest point.

[3 marks]

$V = u + at_{max}$

$V = 0, \quad u = 3.43, \quad a = -9.8$ ~~t_{max}~~

$\Rightarrow t_{max} = \frac{-3.43}{-9.8} = 0.35s$

Since $t_{init} = 2$,

t_{max} is actually $2.35s$.

END OF QUESTIONS

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