SPECIMEN MATERIAL

Please write clearly, in block capitals.

Centre number | | | | | | Candidate number | | | | |

Surname ____________________

Forename(s) ____________________

Candidate signature ____________________

A-level
MATHEMATICS
Paper 3

Exam Date Morning Time allowed: 2 hours

Materials
For this paper you must have:
- The AQA booklet of formulae and statistical tables.
- You may use a graphics calculator.

Instructions
- Use black ink or black ball-point pen. Pencil should be used for drawing.
- Answer **all** questions.
- You must answer each question in the space provided for that question. If you require extra space, use an AQA supplementary answer book; do **not** use the space provided for a different question.
- Do not write outside the box around each page.
- Show all necessary working; otherwise marks for method may be lost.
- Do all rough work in this book. Cross through any work that you do not want to be marked.

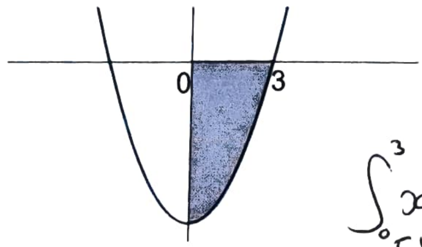
Information
- The marks for questions are shown in brackets.
- The maximum mark for this paper is 100.

Advice
Unless stated otherwise, you may quote formulae, without proof, from the booklet.
You do not necessarily need to use all the space provided.

Section A

Answer **all** questions in the spaces provided.

1 The graph of $y = x^2 - 9$ is shown below.



$\int_0^3 x^2 - 9 \, dx$

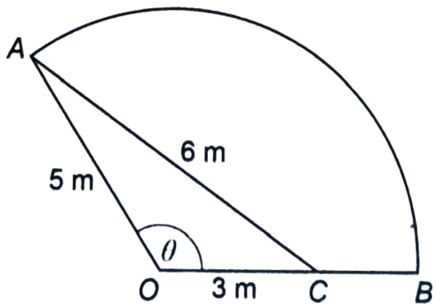
$= \left[\frac{1}{3}x^3 - 9x\right]_0^3$

Find the area of the shaded region.
Circle your answer.

[1 mark]

–18 –6 6 18

2 A wooden frame is to be made to support some garden decking. The frame is to be in the shape of a sector of a circle. The sector *OAB* is shown in the diagram, with a wooden plank *AC* added to the frame for strength. *OA* makes an angle of θ with *OB*.



2 (a) Show that the exact value of $\sin\theta$ is $\dfrac{4\sqrt{14}}{15}$

[3 marks]

$AC^2 = (OC^2 + OA^2) - (2 \times OC \times OA \cos\theta)$

$6^2 = (3^2 + 5^2) - (2 \times 3 \times 5 \cos\theta)$

$\cancel{\cos\theta = \cos\left(\sin^{-1}\left(\frac{4\sqrt{14}}{15}\right)\right) = \frac{1}{15}}$

$\Rightarrow \cos\theta = -\frac{(36 - 9 - 25)}{30} = -\frac{1}{15}$

$\Rightarrow \sin\theta = \sqrt{1-\cos^2\theta} = \sqrt{1-\left(\frac{-1}{15}\right)^2} = \frac{4\sqrt{14}}{15}$.

2 (b) Write down the value of θ in radians to 3 significant figures.

[1 mark]

1.64^c.

2 (c) Find the area of the garden that will be covered by the decking.

[2 marks]

$A = \frac{1}{2} r^2 \theta$.

$A = \frac{1}{2} \times 5^2 \times 1.64$

$= 20.5 m^2$

3 A circular ornamental garden pond, of radius 2 metres, has weed starting to grow and cover its surface.

As the weed grows, it covers an area of A square metres. A simple model assumes that the weed grows so that the rate of increase of its area is proportional to A.

3 (a) Show that the area covered by the weed can be modelled by

$$A = Be^{kt}$$

where B and k are constants and t is time in days since the weed was first noticed.

[4 marks]

$\frac{dA}{dt} \propto A.$

$\Rightarrow \frac{dA}{dt} = kA.$

$\Rightarrow \int \frac{1}{A} dA = \int k \, dt$

$\Rightarrow \ln A = kt + c$

$\Rightarrow A = Be^{kt}$ where $B = e^{c}$.

3 (b) When it was first noticed, the weed covered an area of 0.25 m^2. Twenty days later the weed covered an area of 0.5 m^2

3 (b) (i) State the value of B.

[1 mark]

$B = 0.25.$

3 (b) (ii) Show that the model for the area covered by the weed can be written as

$$A = 2^{\frac{t}{20} - 2}$$

[4 marks]

$A = 0.25 e^{kt}$

$t = 20 \Rightarrow A = 0.5$

$0.5 = 0.25 e^{20k}$

$\Rightarrow e^{20k} = 2$

$\Rightarrow 20k = \ln 2$

$\Rightarrow k = \frac{1}{20} \ln 2$

$\Rightarrow A = \frac{1}{4}(e^{\ln 2})^{\frac{t}{20}}$

$= 2^{-2} \, 2^{\frac{t}{20}}$

$= 2^{\frac{t}{20} - 2}$

Question 3 continues on the next page

3 (b) (iii) How many days does it take for the weed to cover half of the surface of the pond?

[2 marks]

Area of pond $= 2^2\pi = 4\pi$.

$2\pi = 2^{\frac{t}{20}-2}$

$\Rightarrow 2^{\frac{t}{20}} = 8\pi$

$\Rightarrow t = 20\log_2 8\pi \approx 93.03$.

3 (c) State one limitation of the model.

[1 mark]

Model suggests that A will increase without a limit. This is false.

3 (d) Suggest one refinement that could be made to improve the model.

[1 mark]

Introduce a limiting factor, i.e. if weed is eaten by fish.

4 $\int_1^2 x^3 \ln(2x)\,\mathrm{d}x$ can be written in the form $p\ln 2 + q$, where p and q are rational numbers.

Find p and q.

[5 marks]

Let $u = \ln 2x$, $v' = x^3$

$u' = \frac{1}{x}$, $v = \frac{1}{4}x^4$.

$$\int_1^2 uv' = [uv]_1^2 - \int_1^2 u'v$$

$$= \left[\frac{1}{4}x^4 \ln 2x\right]_1^2 - \frac{1}{4}\int_1^2 x^3 \, dx$$

$$= \left(\frac{1}{4}\cdot 16 \cdot \ln 4 - \cancel{\tfrac{16}{16}}\ 1\right) - \left(\frac{1}{4}\ln 2 - \frac{1}{16}\right)$$

$$= (4\ln 4 - 1) - \left(\frac{1}{4}\ln 2 - \frac{1}{16}\right)$$

$$= (8\ln 2 - \tfrac{1}{4}\ln 2) - \left(1 - \frac{1}{16}\right)$$

$$= \frac{31}{4}\ln 2 - \frac{15}{16}$$

$\Rightarrow p = \frac{31}{4}$, $q = \frac{-15}{16}$.

5 (a) Find the first three terms, in ascending powers of x, in the binomial expansion of $(1+6x)^{\frac{1}{3}}$

[2 marks]

$(1+6x)^{\frac{1}{3}} \approx 1 + \left(\frac{1}{3}\cdot 6x\right) + \left(\frac{1}{3}\cdot\frac{-2}{3}\cdot\frac{(6x)^2}{2}\right)$

$= 1 + 2x - 4x^2$

5 (b) Use the result from part **(a)** to obtain an approximation to $\sqrt[3]{1.18}$ giving your answer to 4 decimal places.

[2 marks]

Let $1 + 6x = 1.18 \Rightarrow x = 0.03$.

$\Rightarrow \sqrt[3]{1.18} \approx 1 + 2(0.03) - 4(0.03)^2$

$= 1 + 0.06 - 0.0036$

$= 1.0564$.

5 (c) Explain why substituting $x = \frac{1}{2}$ into your answer to part **(a)** does not lead to a valid approximation for $\sqrt[3]{4}$.

[1 mark]

The expression is only valid for $|x| < \frac{1}{6}$.

6 Find the value of $\int_1^2 \frac{6x+1}{6x^2-7x+2}\,dx$, expressing your answer in the form

$m\ln 2 + n\ln 3$, where m and n are integers.

[8 marks]

$$\frac{6x+1}{6x^2-7x+2} = \frac{6x+1}{(3x-2)(2x-1)} = \frac{A}{3x-2} + \frac{B}{2x-1}$$

$$\Rightarrow A(2x-1) + B(3x-2) = 6x+1$$

$$\Rightarrow 2A + 3B = 6, \quad -A - 2B = 1$$

$$\Rightarrow A = 15, \quad B = -8$$

$$\Rightarrow \int_1^2 \frac{6x+1}{6x^2-7x+2}\,dx = \int_1^2 \frac{15}{3x-2} - \frac{8}{2x-1}\,dx$$

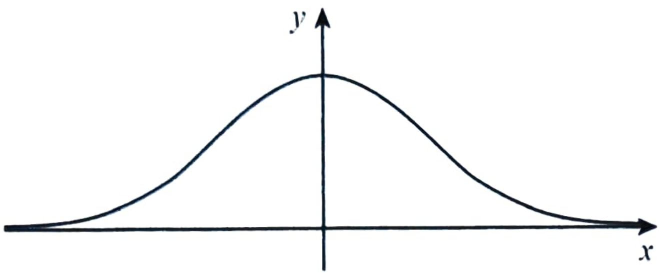
$$= \Big[5\ln(3x-2) - 4\ln(2x-1)\Big]_1^2$$

$$= (5\ln 4 - 4\ln 3) - (5\ln 1 - 4\ln 1)$$

$$= 5\ln 4 - 4\ln 3$$

$$= 10\ln 2 - 4\ln 3.$$

7 The diagram shows part of the graph of $y = e^{-x^2}$



The graph is formed from two convex sections, where the gradient is increasing, and one concave section, where the gradient is decreasing.

7 (a) Find the values of x for which the graph is concave.

[4 marks]

$\frac{dy}{dx} = -2xe^{-x^2}$

$\Rightarrow \frac{d^2y}{dx^2} = -2e^{-x^2} + 4x^2e^{-x^2}$ (by product rule).

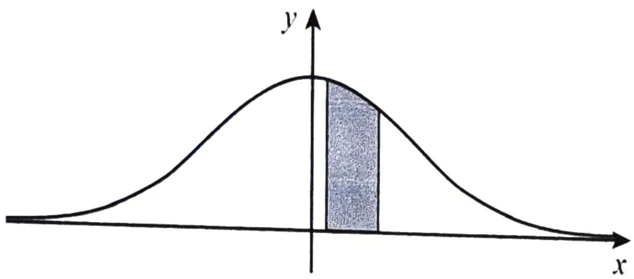
For concave, we have $\frac{d^2y}{dx^2} < 0$.

$\Rightarrow -2e^{-x^2} + 4x^2e^{-x^2} < 0$.

$\Rightarrow 4x^2 - 2 < 0$

$\Rightarrow -\frac{1}{\sqrt{2}} < x < \frac{1}{\sqrt{2}}$.

7 (b) The finite region bounded by the x-axis and the lines $x = 0.1$ and $x = 0.5$ is shaded.



Use the trapezium rule, with 4 strips, to find an estimate for $\int_{0.1}^{0.5} e^{-x^2}\,dx$

Give your estimate to four decimal places.

[3 marks]

$$\int_{0.1}^{0.5} e^{-x^2}\,dx = \frac{0.1}{2}\left(e^{-0.01} + e^{-0.25} + 2\left(e^{-0.04} + e^{-0.09} + e^{-0.16}\right)\right)$$

$$\approx 0.3611.$$

Question 7 continues on the next page

7 (c) Explain with reference to your answer in part **(a)**, why the answer you found in part **(b)** is an underestimate.

[2 marks]

The region exists within the concave region in part (a), so all trapezia lie below the curve, giving an underestimate.

7 (d) By considering the area of a rectangle, and using your answer to part (b), prove that the shaded area is 0.4 correct to 1 decimal place.

[3 marks]

Let rectangle be $0.5 - 0.1$ wide and $e^{-0.01}$ high.

Its area, then, is 0.396 units, which is an overestimate as this overlaps over the curve.

$\Rightarrow 0.361 < A < 0.396$

$\Rightarrow A = 0.4$ to 1d.p.

END OF SECTION A
TURN OVER FOR SECTION B

Section B

Answer **all** questions in the spaces provided.

8 Edna wishes to investigate the energy intake from eating out at restaurants for the households in her village.

She wants a sample of 100 households.

She has a list of all 2065 households in the village.

Ralph suggests this selection method.

> "Number the households 0000 to 2064. Obtain 100 different four-digit random numbers between 0000 and 2064 and select the corresponding households for inclusion in the investigation."

8 (a) What is the population for this investigation?

Circle your answer.

[1 mark]

Edna and Ralph	The 2065 households in the village	The energy intake for the village from eating out	The 100 households selected

8 (b) What is the sampling method suggested by Ralph?

Circle your answer.

[1 mark]

Opportunity	Random number	Continuous random variable	Simple random

9 A survey has found that, of the 2400 households in Growmore, 1680 eat home-grown fruit and vegetables.

9 (a) Using the binomial distribution, find the probability that, out of a random sample of 25 households in Growmore, exactly 22 eat home-grown fruit and vegetables.

[2 marks]

$p = \frac{1680}{2400} = 0.7$

Let $X \sim B(25, 0.7)$.

Then $P(X=22) = \binom{25}{22} \cdot 0.7^{22} \cdot 0.3^{3}$

$= 0.0243$

9 (b) Give a reason why you would **not** expect your calculation in part **(a)** to be valid for the 25 households in Gifford Terrace, a residential road in Growmore.

[1 mark]

It is likely that the 25 households in Gifford Terrace are similar, so they may not be independent of each other in growing fruit and vegetables.

10 Shona calculated four correlation coefficients using data from the Large Data Set.

In each case she calculated the correlation coefficient between the masses of the cars and the CO_2 emissions for varying sample sizes.

A summary of these calculations, labelled A to D, are listed in the table below.

	Sample size	Correlation coefficient
A	3827	0.088
B	3735	0.246
C	24	0.400
D	1250	- 1.183

Shona would like to use calculation A to test whether there is evidence of positive correlation between mass and CO2 emissions.

She finds the critical value for a one-tailed test at the 5% level for a sample of size 3827 is 0.027

10 (a) (i) State appropriate hypotheses for Shona to use in her test.

[1 mark]

$H_0: \rho = 0$, $H_1: \rho > 0$.

Question 10 continues on the next page

10 (a) (ii) Determine if there is sufficient evidence to reject the null hypothesis.

Fully justify your answer.

[1 mark]

$0.088 > 0.027$, so we can reject H_0.

10 (b) Shona's teacher tells her to remove calculation D from the table as it is incorrect.

Explain how the teacher knew it was incorrect.

[1 mark]

We must have $-1 \leq \rho \leq 1$.

$-1.183 < -1$, so this value cannot be true.

10 (c) Before performing calculation B, Shona cleaned the data. She removed all cars from the Large Data Set that had incorrect masses.

Using your knowledge of the large data set, explain what was incorrect about the masses which were removed from the calculation.

[1 mark]

The masses were zero.

10 (d) Apart from CO2 and CO emissions, state one other type of emission that Shona could investigate using the Large Data Set.

[1 mark]

NO_x

10 (e) Wesley claims that calculation C shows that a heavier car causes higher CO2 emissions.

Give **two** reasons why Wesley's claim may be incorrect.

[2 marks]

- The sample size is very ~~low~~ small.

- The correlation may be due to a different error.

11 Terence owns a local shop. His shop has three checkouts, at least one of which is always staffed.

A regular customer observed that the probability distribution for N, the number of checkouts that are staffed at any given time during the spring, is

$$P(N = n) = \begin{cases} \dfrac{3}{4}\left(\dfrac{1}{4}\right)^{n-1} & \text{for } n = 1, 2 \\ k & \text{for } n = 3 \end{cases}$$

11 (a) Find the value of k.

[1 mark]

$1 - \left(\frac{3}{4} + \frac{3}{16}\right) = \frac{1}{16}$

11 (b) Find the probability that a customer, visiting Terence's shop during the spring, will find at least 2 checkouts staffed.

[2 marks]

$\frac{3}{16} + \frac{1}{16} = \frac{1}{4}.$

12 During the 2006 Christmas holiday, John, a maths teacher, realised that he had fallen ill during 65% of the Christmas holidays since he had started teaching.

In January 2007, he increased his weekly exercise to try to improve his health.

For the next 7 years, he only fell ill during 2 Christmas holidays.

12 (a) Using a binomial distribution, investigate, at the 5% level of significance, whether there is evidence that John's rate of illness during the Christmas holidays had decreased since increasing his weekly exercise.

[6 marks]

X = no. of Xmas holidays without illness since Jan 2007.

$X \sim B(7, p)$.

$H_0: p = 0.65$, $H_1: p < 0.65$

$P(X \leq 2) = P(x=0) + P(x=1) + P(x=2)$

$= 0.0556$

$0.0556 > 0.05$,

we cannot reject H_0, so there is not sufficient evidence to suggest John's rate of illness has decreased.

12 (b) State **two** assumptions, regarding illness during the Christmas holidays, that are necessary for the distribution you have used in part **(a)** to be valid.

For **each** assumption, comment, in context, on whether it is likely to be correct.

[4 marks]

- Probability of illness is constant, year on year. False, age is a factor.

- Annual results are independent of each other. True, it's highly unlikely that an illness will span two years.

Turn over for the next question

13 In the South West region of England, 100 households were randomly selected and, for each household, the weekly expenditure, £X, per person on food and drink was recorded.

The maximum amount recorded was £40.48 and the minimum amount recorded was £22.00

The results are summarised below, where $\bar{x}$ denotes the sample mean.

$$\sum x = 3046.14 \qquad \sum (x-\bar{x})^2 = 1746.29$$

13 (a) (i) Find the mean of X

Find the standard deviation of X

[2 marks]

$\bar{X} = \frac{\Sigma x}{n} = \frac{3046.14}{100} = 30.46$

$S = \sqrt{\frac{\Sigma(x-\bar{x})^2}{n-1}} = \sqrt{\frac{1746.29}{99}} = 4.20$

13 (a) (ii) Using your results from part **(a)(i)** and other information given, explain why the normal distribution can be used to model X.

[2 marks]

$\bar{X} + 3s \approx 43$

$\bar{X} - 3s \approx 18$

The range of amounts exist between $\bar{X} \pm 3s$, so the normal distribution is fine.

13 (a) (iii) Find the probability that a household in the South West spends less than £25.00 on food and drink per person per week.

[1 mark]

$P(X < 25) = 0.0967$

13 (b) For households in the North West of England, the weekly expenditure, £Y, per person on food and drink can be modelled by a normal distribution with mean £29.55

It is known that $\mathrm{P}(Y < 30) = 0.55$

Find the standard deviation of Y, giving your answer to one decimal place.

[3 marks]

$P\left(Z < \frac{30 - 29.55}{\sigma}\right) = 0.55$

$\Rightarrow \frac{30 - 29.55}{\sigma} = 0.1257$

$\Rightarrow \sigma = \frac{30 - 29.55}{0.1257} = 3.6$

Turn over for the next question

14 A survey during 2013 investigated mean expenditure on bread and on alcohol.

The 2013 survey obtained information from 12 144 adults.

The survey revealed that the mean expenditure per adult per week on bread was 127p.

14 (a) For 2012, it is known that the expenditure per adult per week on bread had mean 123p, and a standard deviation of 70p.

14 (a) (i) Carry out a hypothesis test, at the 5% significance level, to investigate whether the mean expenditure per adult per week on bread changed from 2012 to 2013.

Assume that the survey data is a random sample taken from a normal distribution.

[5 marks]

$H_0: \mu = 123$, $H_1: \mu \neq 123$

Test Statistic $= \dfrac{127 - 123}{\frac{70}{\sqrt{12144}}} = 6.30$

Critical z values : ± 1.96.

$6.30 > 1.96$ So we have evidence to reject H_0, and suggest the mean expenditure on bread has changed between 2012 and 2013

14 (a) (ii) Calculate the greatest and least values for the sample mean expenditure on bread per adult per week for 2013 that would have resulted in acceptance of the null hypothesis for the test you carried out in part **(a)(i)**.

Give your answers to two decimal places.

[2 marks]

$123 \pm \left(1.96 \times \frac{70}{\sqrt{12144}}\right)$

$\Rightarrow$ min $= 121.75$, max $= 124.25$

14 (b) The 2013 survey revealed that the mean expenditure per adult, per week on alcohol was 324p.

The mean expenditure per adult per week on alcohol for 2009 was 307p.

A test was carried out on the following hypotheses relating to mean expenditure per adult per week on alcohol in 2013.

$H_0 : \mu = 307$

$H_1 : \mu \neq 307$

This test resulted in the null hypothesis, H_0, being rejected.

State, with a reason, whether the test result supports the following statements:

14 (b) (i) the mean UK expenditure on alcohol per adult per week increased by 17p from 2009 to 2013;

[2 marks]

The conclusion implies that the mean changed, not that it increased by a specific amount, so it is not supported.

14 (b) (ii) the mean UK consumption of alcohol per adult per week changed from 2009 to 2013.

[2 marks]

The conclusion implies there is evidence, that the mean changed, but the expenditure increase may be due to price changes. The statement is not supported.

15 A sample of 200 households was obtained from a small town.

Each household was asked to complete a questionnaire about their purchases of takeaway food.

A is the event that a household regularly purchases Indian takeaway food.

B is the event that a household regularly purchases Chinese takeaway food.

It was observed that $\mathrm{P}(B|A) = 0.25$ and $\mathrm{P}(A|B) = 0.1$

Of these households, 122 indicated that they did **not** regularly purchase Indian or Chinese takeaway food.

A household is selected at random from those in the sample.

Find the probability that the household regularly purchases **both** Indian and Chinese takeaway food.

[6 marks]

$$\frac{P(A \cap B)}{P(A)} = \frac{1}{4}$$

$$\Rightarrow 4P(A \cap B) = P(A)$$

$$\frac{P(A \cap B)}{P(B)} = \frac{1}{10}$$

$$\Rightarrow 10P(A \cap B) = P(B)$$

$$\Rightarrow P(A \cup B) = 1 - \frac{122}{200} = \frac{39}{100}$$

$$P(A) + P(B) - P(A \cap B) = \frac{39}{100}$$

$$\Rightarrow 4P(A \cap B) + 10P(A \cap B) - P(A \cap B) = \frac{39}{100}$$

$$\Rightarrow P(A \cap B) = \frac{3}{100}.$$

END OF QUESTIONS