AQA

Please write clearly in block capitals.

Centre number	
Candidate number	
Surname	
Forename(s)	
Candidate signature	

A-level
MATHEMATICS
Paper 2

Wednesday 13 June 2018 Morning Time allowed: 2 hours

Materials
- You must have the AQA Formulae for A-level Mathematics booklet.
- You should have a graphical or scientific calculator that meets the requirements of the specification.

Instructions
- Use black ink or black ball-point pen. Pencil should only be used for drawing.
- Fill in the boxes at the top of this page.
- Answer **all** questions.
- You must answer each question in the space provided for that question. If you require extra space, use an AQA supplementary answer book; do **not** use the space provided for a different question.
- Show all necessary working; otherwise marks for method may be lost.
- Do all rough work in this book. Cross through any work that you do not want to be marked.

Information
- The marks for questions are shown in brackets.
- The maximum mark for this paper is 100.

Advice
- Unless stated otherwise, you may quote formulae, without proof, from the booklet.
- You do not necessarily need to use all the space provided.

For Examiner's Use	
Question	Mark
1	
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TOTAL	

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Section A

Answer **all** questions in the spaces provided.

1 Which of these statements is correct?

Tick **one** box.

[1 mark]

$x = 2 \Rightarrow x^2 = 4$ ☑

$x^2 = 4 \Rightarrow x = 2$ ☐

$x^2 = 4 \Leftrightarrow x = 2$ ☐

$x^2 = 4 \Rightarrow x = -2$ ☐

2 Find the coefficient of x^2 in the expansion of $(1 + 2x)^7$

Circle your answer.

[1 mark]

42 4 21 84

$\binom{7}{2} 1^{7-2} (2x)^2 = 21 \times 4x^2 = 84x^2$

Jun18/7357/2

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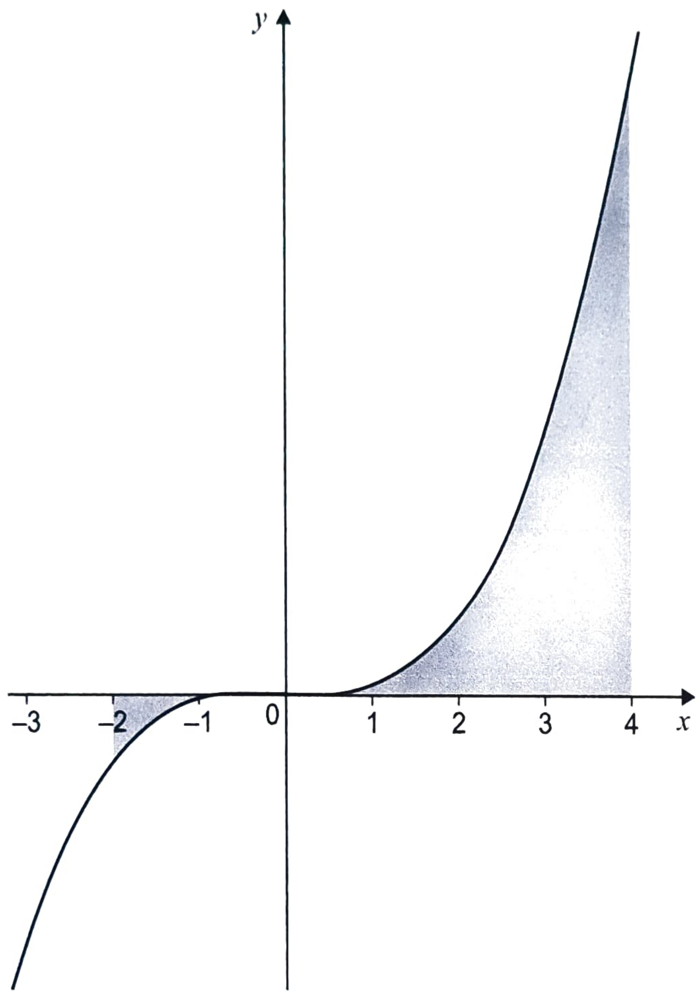
3 The graph of $y = x^3$ is shown.



Find the total shaded area.

Circle your answer.

[1 mark]

–68   60   68   128

$\int_{-2}^{0} x^3 = \left[\frac{1}{4}x^4\right]_{-2}^{0} =$ [illegible] $- 4$

$\int_{0}^{4} x^3 = \left[\frac{1}{4}x^4\right]_{0}^{4} = 64.$

$64 + -(-4) = 68.$

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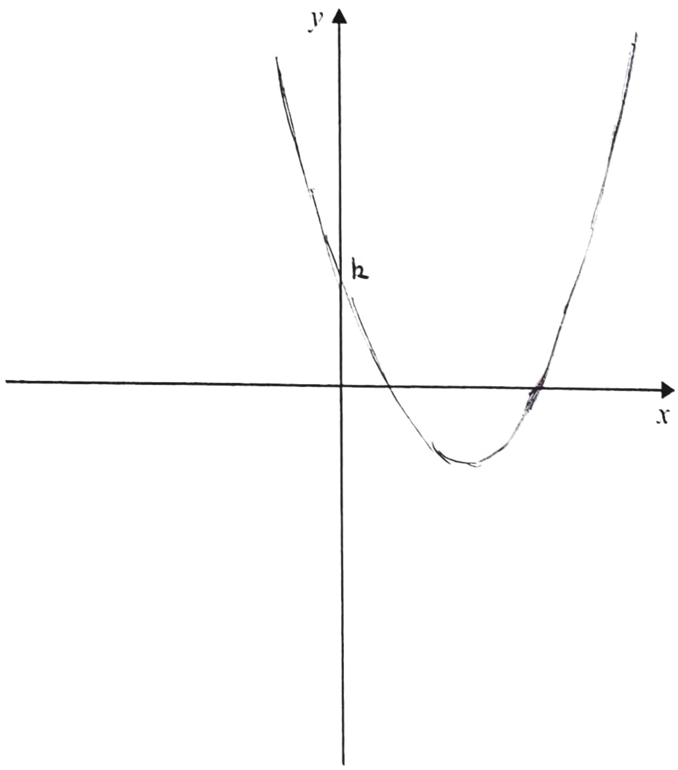
4 A curve, C, has equation $y = x^2 - 6x + k$, where k is a constant.

The equation $x^2 - 6x + k = 0$ has two distinct positive roots.

4 (a) Sketch C on the axes below.

[2 marks]



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4 (b) Find the range of possible values for k.

Fully justify your answer.

[4 marks]

$b^2 - 4ac > 0$ (2 distinct roots).

$36 - 4k > 0 \Rightarrow k < 9$

Since both roots are positive, we have $k > 0$.

$\Rightarrow 0 < k < 9$

Turn over for the next question

0 6

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5 Prove that 23 is a prime number.

[2 marks]

Check for factors under $\sqrt{23} \approx 4.8$.

Check 2, ~~and~~ 3, 4.

$\frac{23}{2} = 11.5$, $\frac{23}{3} = 7.\dot{6}$, $\frac{23}{4} = 5.75$

$\Rightarrow$ 23 is prime.

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6 Find the coordinates of the stationary point of the curve with equation

$$(x + y - 2)^2 = e^y - 1$$

[7 marks]

$\Rightarrow 2(x + y - 2)\left(1 + \frac{dy}{dx}\right) = e^y \frac{dy}{dx}$

Stationary point $\Rightarrow$

$\frac{dy}{dx} = 0$

$\Rightarrow x + y - 2 = 0.$

$\Rightarrow e^y - 1 = 0$

$\Rightarrow e^y = 1$

$\Rightarrow y = 0.$

$\Rightarrow x = 2.$

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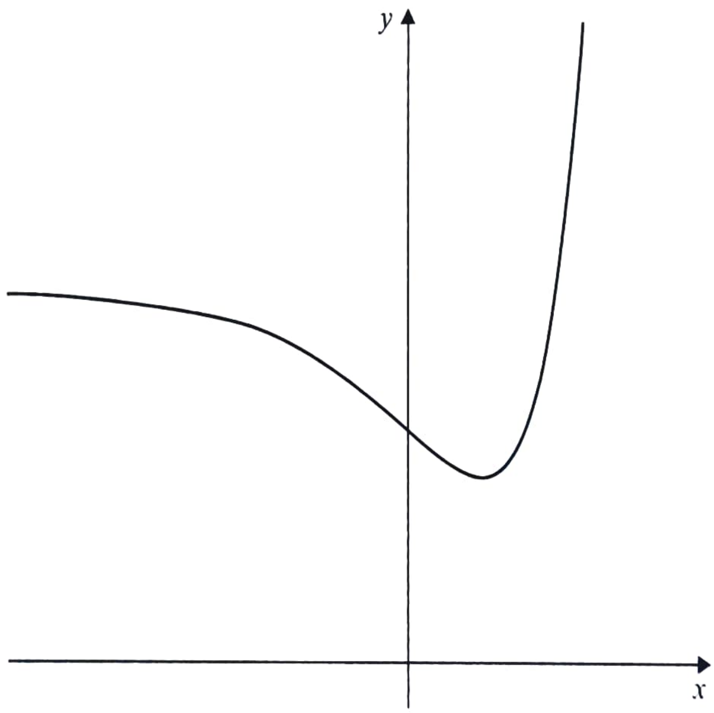
7 A function f has domain $\mathbb{R}$ and range $\{y \in \mathbb{R} : y \geq \mathrm{e}\}$

The graph of $y = \mathrm{f}(x)$ is shown.



The gradient of the curve at the point (x, y) is given by $\frac{\mathrm{d}y}{\mathrm{d}x} = (x - 1)\mathrm{e}^x$

Find an expression for $\mathrm{f}(x)$.

Fully justify your answer.

[8 marks]

$y = \int (x-1)e^x \, dx$

$u = x - 1 \quad , \quad v = e^x$

$u' = 1 \quad , \quad v' = e^x$

$y = (x-1)e^x - \int e^x \, dx$

$= (x-1)e^x - e^x + c$

$= (x-2)e^x + c$

$\frac{dy}{dx} = 0 \Rightarrow x = 1.$

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Curve passes through $(1, e)$.

$\Rightarrow \quad c = 2e$.

$\Rightarrow \quad f(x) = (x-2)e^x + 2e$.

Turn over for the next question

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8 (a) Determine a sequence of transformations which maps the graph of $y = \sin x$ onto the graph of $y = \sqrt{3}\sin x - 3\cos x + 4$

Fully justify your answer.

[7 marks]

$R\sin(x-\alpha) = R\sin x\cos\alpha - R\cos x\sin\alpha$

Let $R\cos\alpha = \sqrt{3}$, $R\sin\alpha = 3.$

~~$\sqrt{3}\sin x - 3\cos x = 2\sqrt{3}\sin(x$~~

$\Rightarrow \tan\alpha = \sqrt{3} \Rightarrow \alpha = \frac{\pi}{3}$

$\Rightarrow R = 2\sqrt{3}.$

$\sqrt{3}\sin x - 3\cos x = 2\sqrt{3}\sin\left(x - \frac{\pi}{3}\right)$

$y = 2\sqrt{3}\sin\left(x-\frac{\pi}{3}\right) + 4.$

- Translate in $\begin{pmatrix}\frac{\pi}{3}\\0\end{pmatrix}$

- Stretch in y-direction with scale factor $2\sqrt{3}$

- Translate in $\begin{pmatrix}0\\4\end{pmatrix}$.

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8 (b) (i) Show that the least value of $\dfrac{1}{\sqrt{3}\sin x - 3\cos x + 4}$ is $\dfrac{2-\sqrt{3}}{2}$

[2 marks]

Minimum value occurs when

~~sin x =~~ $\sin\left(x - \frac{\pi}{3}\right) = 1$.

$\Rightarrow \quad x = \frac{5\pi}{6}$.

$\Rightarrow \dfrac{1}{2\sqrt{3}+4} = \dfrac{2\sqrt{3}-4}{12-16} = \dfrac{2-\sqrt{3}}{2}$.

8 (b) (ii) Find the greatest value of $\dfrac{1}{\sqrt{3}\sin x - 3\cos x + 4}$

[1 mark]

$\dfrac{1}{4-2\sqrt{3}} = \dfrac{4+2\sqrt{3}}{4} = \dfrac{2+\sqrt{3}}{2}$.

Turn over for the next question

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9 A market trader notices that daily sales are dependent on two variables:

number of hours, t, after the stall opens

total sales, x, in pounds since the stall opened.

The trader models the rate of sales as directly proportional to $\frac{8-t}{x}$

After two hours the rate of sales is £72 per hour and total sales are £336

9 (a) Show that

$$x\frac{\mathrm{d}x}{\mathrm{d}t} = 4032(8-t)$$

[3 marks]

$\frac{dx}{dt} = k\frac{(8-t)}{x}$

$72 = k\frac{8-2}{336} \Rightarrow k = 4032$

$\Rightarrow \frac{dx}{dt} = 4032\frac{8-t}{x}$

$\Rightarrow x\frac{dx}{dt} = 4032(8-t)$

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9 (b) Hence, show that

$$x^2 = 4032t(16 - t)$$

[3 marks]

$\int x \, dx = \int 4032(8-t) \, dt$

$\frac{1}{2}x^2 = 4032\left(8t - \frac{1}{2}t^2\right) + c$

$\frac{1}{2}(336)^2 = 4032(16-2) + c$

$\Rightarrow c = 0.$

$\Rightarrow x^2 = 4032t(16 - t)$

Question 9 continues on the next page

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9 (c) The stall opens at 09.30.

9 (c) (i) The trader closes the stall when the rate of sales falls below £24 per hour.

Using the results in parts **(a)** and **(b)**, calculate the earliest time that the trader closes the stall.

[6 marks]

$$\frac{dx}{dt} = 24.$$

$$24x = 4032(8-t)$$

$$\Rightarrow \quad x = 168(8-t)$$

$$(168(8-t))^2 = 4032t(16-t)$$

$$\Rightarrow \quad t^2 - 16t + 56 = 0$$

$$\Rightarrow \quad t = 5.171\ldots$$

$\Rightarrow$ 5 hours, 10 minutes

Earliest time is 14:40.

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9 (c) (ii) Explain why the model used by the trader is not valid at 09.30.

[2 marks]

When the stall opens, sales, x, will be zero.

The model is undefined at $t=0$.

Turn over for Section B

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Section B

Answer **all** questions in the spaces provided.

10 A garden snail moves in a straight line from rest to $1.28\,\text{cm}\,\text{s}^{-1}$, with a constant acceleration in 1.8 seconds.

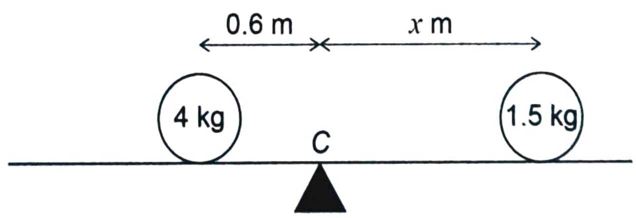
Find the acceleration of the snail.

Circle your answer.

[1 mark]

$2.30\,\text{m}\,\text{s}^{-2}$ $\quad$ $0.71\,\text{m}\,\text{s}^{-2}$ $\quad$ $0.0071\,\text{m}\,\text{s}^{-2}$ $\quad$ $0.023\,\text{m}\,\text{s}^{-2}$

11 A uniform rod, AB, has length 4 metres.
The rod is resting on a support at its midpoint C.
A particle of mass 4 kg is placed 0.6 metres to the left of C.
Another particle of mass 1.5 kg is placed x metres to the right of C, as shown.



The rod is balanced in equilibrium at C.

Find x.

Circle your answer.

[1 mark]

1.8 m $\quad$ 1.5 m $\quad$ 1.75 m $\quad$ 1.6 m

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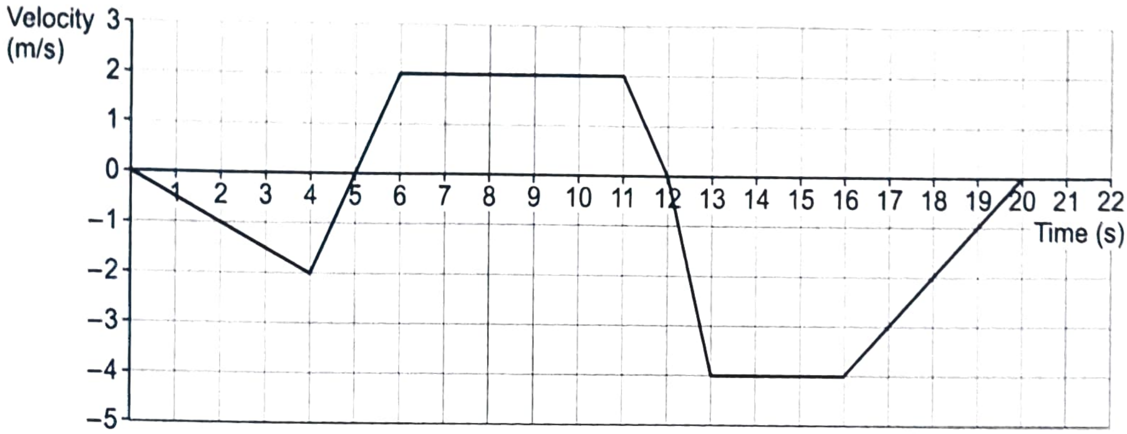
12 The graph below shows the velocity of an object moving in a straight line over a 20 second journey.



12 (a) Find the maximum magnitude of the acceleration of the object.

[1 mark]

Steepest gradient at $12 \leq t \leq 13$, the magnitude of acceleration is $4 ms^{-2}$.

12 (b) The object is at its starting position at times 0, t_1 and t_2 seconds.

Find t_1 and t_2

[4 marks]

$0 \leq t \leq 4 : -4m$

$4 \leq t \leq 5 : -1m$

$5 \leq t \leq 6 : +1m$

$6 \leq t \leq 11 : +10m$

$11 \leq t \leq 12 : +1m$

$12 \leq t \leq 13 : -2m$

$13 \leq t \leq 16 : -12m$

$16 \leq t \leq 20 : -8m$.

Gain 4m after 6s : 8s.

After 8s, +6m.

After 12s, +7m

After 13s, +5m.

Gain −5m after 13s : 14.25s.

$t_1 = 8s$, $t_2 = 14.25s$.

Turn over ▶

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13 **In this question use $g = 9.8\,\mathrm{m\,s^{-2}}$**

A boy attempts to move a wooden crate of mass 20 kg along horizontal ground. The coefficient of friction between the crate and the ground is 0.85

13 (a) The boy applies a horizontal force of 150 N. Show that the crate remains stationary.
[3 marks]

$F_{max} = \mu m g.$

$= 0.85 \times 20\,kg \times 9.8\,ms^{-2}$

$= 166.6\,N$

166.6N > 150N, so the box remains stationary.

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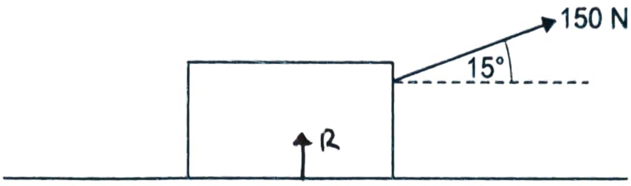
13 (b) Instead, the boy uses a handle to pull the crate forward. He exerts a force of 150 N, at an angle of 15° above the horizontal, as shown in the diagram.



Determine whether the crate remains stationary.

Fully justify your answer.

[5 marks]

$20g = R + 150 \sin 15^\circ$

$\Rightarrow R = 157.177\text{N}$

$F_{max} = 0.85 \times 157.177 = 133.6\text{N}$

$150 \cos 15^\circ = 145\text{N}$

$145\text{N} > 133.6\text{N}$

So the crate will begin to move.

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14 A quadrilateral has vertices A, B, C and D with position vectors given by

$$\overrightarrow{OA} = \begin{bmatrix} 3 \\ 5 \\ 1 \end{bmatrix}, \overrightarrow{OB} = \begin{bmatrix} -1 \\ 2 \\ 7 \end{bmatrix}, \overrightarrow{OC} = \begin{bmatrix} 0 \\ 7 \\ 6 \end{bmatrix} \text{ and } \overrightarrow{OD} = \begin{bmatrix} 4 \\ 10 \\ 0 \end{bmatrix}$$

14 (a) Write down the vector $\overrightarrow{AB}$

[1 mark]

$\overrightarrow{OB} - \overrightarrow{OA} = \overrightarrow{AB} = \begin{pmatrix} -4 \\ -3 \\ 6 \end{pmatrix}$

14 (b) Show that $ABCD$ is a parallelogram, but not a rhombus.

[5 marks]

$\overrightarrow{BC} = \begin{pmatrix} 1 \\ 5 \\ -1 \end{pmatrix}$, $\overrightarrow{AD} = \begin{pmatrix} 1 \\ 5 \\ -1 \end{pmatrix}$, $\overrightarrow{DC} = \begin{pmatrix} -4 \\ -3 \\ 6 \end{pmatrix}$

$|BC| = \sqrt{1^2 + 5^2 + (-1)^2} = 3\sqrt{3}$

$|DC| = \sqrt{(-4)^2 + (-3)^2 + 6^2} = \sqrt{61}$

$\sqrt{61} \neq 3\sqrt{3}$, so we have two pairs of parallel sides ~~w~~ with two distinct lengths for each pair.

$\Rightarrow$ ABCD is a parallelogram, not a rhombus.

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15 A driver is road-testing two minibuses, A and B, for a taxi company.

The performance of each minibus along a straight track is compared.

A flag is dropped to indicate the start of the test.

Each minibus starts from rest.

The acceleration in $\mathrm{m\,s^{-2}}$ of each minibus is modelled as a function of time, t seconds, after the flag is dropped:

$$\text{The acceleration of A} = 0.138\,t^2$$
$$\text{The acceleration of B} = 0.024\,t^3$$

15 (a) Find the time taken for A to travel 100 metres.

Give your answer to four significant figures.

[4 marks]

$v = \int 0.138t^2 \, dt$

$= 0.046t^3 + c$

$t=0,\ v=0 \Rightarrow c=0.$

$s = \int 0.046t^3 \, dt$

$= 0.0115t^4 + k$

$t=0,\ s=0 \Rightarrow k=0.$

$0.0115t^4 = 100$

$\Rightarrow t = 9.657\text{s}.$

Question 15 continues on the next page

Turn over ▶

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15 (b) The company decides to buy the minibus which travels 100 metres in the shortest time.

Determine which minibus should be bought.

[4 marks]

$\int 0.024t^3 \, dt$

$= 0.006t^4 + c$

$t=0, \ v=0 \Rightarrow c=0.$

$s = \int 0.006t^4 \, dt$

$= 0.0012t^5 + k$

$t=0, S=0 \Rightarrow k=0.$

$0.0012t^5 = 100$

$\Rightarrow t = 9.642$

$\Rightarrow$ Minibus B is chosen.

15 (c) The models assume that both minibuses start moving immediately when $t = 0$

In light of this, explain why the company may, in reality, make the wrong decision.

[1 mark]

The drivers' reaction times may not be the same.

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16 **In this question use $g = 9.81\,\text{m}\,\text{s}^{-2}$**

A particle is projected with an initial speed u, at an angle of 35° above the horizontal.

It lands at a point 10 metres vertically below its starting position.

The particle takes 1.5 seconds to reach the highest point of its trajectory.

16 (a) Find u.

[3 marks]

$V_{vert} = u_{vert} + at$.

$v = 0, \quad u_{vert} = u\sin 35°$

$0 = u\sin 35° - (9.81 \times 1.5)$

$\Rightarrow \quad u = 25.7\,\text{ms}^{-1}$

16 (b) Find the total time that the particle is in flight.

[3 marks]

$S = ut + \frac{1}{2}at^2$

$S = -10, \quad u = 25.7\sin 35°, \quad a = 9.81$

$-10 = (25.7\sin 35)t + (\frac{1}{2} \times 9.81t^2)$

$4.905t^2 + 14.74t + 10 = 0$.

$t = \dfrac{-14.74 \pm \sqrt{14.74^2 - (4 \times 4.905 \times 10)}}{9.81} = 3.57\text{s}.$

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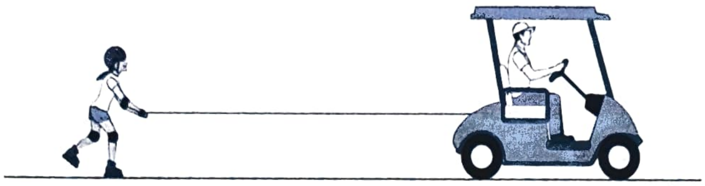
17 A buggy is pulling a roller-skater, in a straight line along a horizontal road, by means of a connecting rope as shown in the diagram.



The combined mass of the buggy and driver is 410 kg
A driving force of 300 N and a total resistance force of 140 N act on the buggy.

The mass of the roller-skater is 72 kg
A total resistance force of R newtons acts on the roller-skater.

The buggy and the roller-skater have an acceleration of $0.2\,\text{m}\,\text{s}^{-2}$

17 (a) (i) Find R.

[3 marks]

$(410 + 72)$

$300\text{N} - 140\text{N} - R = 482\text{kg} \times 0.2\text{ms}^{-2}$

$\Rightarrow R = 63.6\text{N}.$

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17 (a) (ii) Find the tension in the rope.

[3 marks]

$T - R = F = ma$

$T - 63.6N = 72 \times 0.2$

$\Rightarrow T = 78N$

17 (b) State a necessary assumption that you have made.

[1 mark]

Rope is horizontal, no force is used vertically.

Question 17 continues on the next page

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17 (c) The roller-skater releases the rope at a point A, when she reaches a speed of $6\,\text{m}\,\text{s}^{-1}$

She continues to move forward, experiencing the same resistance force.

The driver notices a change in motion of the buggy, and brings it to rest at a distance of 20 m from A.

17 (c) (i) Determine whether the roller-skater will stop before reaching the stationary buggy.

Fully justify your answer.

[5 marks]

$-R = ma$

$-63.6\,\text{N} = 72\,\text{kg} \times a$.

$\Rightarrow \quad a = -0.883\ldots\,\text{ms}^{-2}$

$v^2 = u^2 + 2as$

$u = 6,\ v = 0,\ a = -0.883$.

$S = \dfrac{-36}{2 \times -0.883} = 20.4\,\text{m}$.

$20.4 > 20$,

therefore, the skater will hit the buggy.

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17 (c) (ii) Explain the change in motion that the driver noticed.

[2 marks]

There is no tension from the skater holding the rope, so the driver will notice an increase in acceleration.

END OF QUESTIONS

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