HN1 BIDMAS

There are two sets of brackets, so our first step should be to evaluate them both. Which one we do first doesn't matter, so here we'll choose the left one first. It only contains one operation, so we get

$$13 + 2 = 15$$

Now, the bracket on the right contains two operations: an index/power, and a division. We do the index first followed by the division:

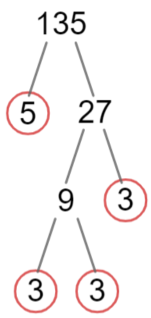
$$36 \div 3^2 = 36 \div 9 = 4$$

Therefore, we only have 1 operation left in our calculation (the multiplication between the two brackets), so we get the answer to be

$$15 \times 4 = 60$$

HN2 Prime Factor Trees

We will use a prime factor tree, first splitting 135 into 5 and 27. 5 is prime, so we circle it, but 27 is not, so we split it into 9 and 3.



3 is prime, so we circle, but 9 is not, so we split it into 3 and 3.
As we established, 3 is prime, so we circle both of these values, and so we have finished our tree. Therefore, the prime factorisation of 135 is

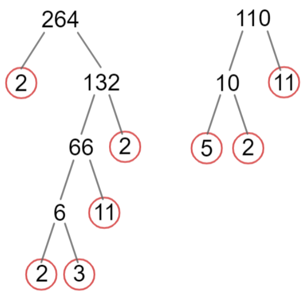
$$135 = 3 \times 3 \times 3 \times 5$$

We can write $3 \times 3 \times 3$ as 3^3, the answer written using index notation is

$$135 = 3^3 \times 5$$

HN3 HCF and LCM

Firstly, we need to find the prime factorisations of 264 and 110. Here, we will do this using prime factor trees.
So, we get that



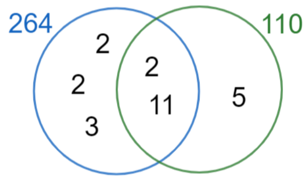
$$264 = 2 \times 2 \times 2 \times 3 \times 11$$

and

$$110 = 2 \times 5 \times 11$$

Then, we need to draw a Venn diagram with one circle for prime factors of 264 and another for prime factors of 110. Then, the first step to filling in this diagram is to look for any prime factors that 264 and 110 have in common. For each shared prime factor, we will cross it off both factor lists, and then write it once in the intersection of the two circles.
After all shared factors are crossed off, write the rest of the prime factors in their appropriate circles. The result should look something like the diagram below.

$$264 = \cancel{2} \times 2 \times 2 \times 3 \times \cancel{11}$$
$$110 = \cancel{2} \times 5 \times \cancel{11}$$



Then, we find the HCF by multiplying the numbers in the intersection:

$$\text{HCF } = 2 \times 11 = 22$$

And find the LCM by multiplying all the numbers in the Venn diagram together:

$$\text{LCM } = 2 \times 2 \times 2 \times 3 \times 5 \times 11 = 1{,}320$$

HN4 Fractions 1: Adding & Subtracting Fractions

To find a common denominator, multiply the denominators: $7 \times 5 = 35$. To make the denominator of the first fraction 35, we need to multiply its top and bottom by 5:

$$\frac{3}{7} = \frac{3 \times 5}{7 \times 5} = \frac{15}{35}$$

To make the denominator of the second fraction 35, we need to multiply its top and bottom by 7:

$$\frac{6}{5} = \frac{6 \times 7}{5 \times 7} = \frac{42}{35}$$

Now we can do the subtraction. We get:

$$\frac{15}{35} - \frac{42}{35} = -\frac{27}{35}$$

HN5 Fractions 2: Multiplying & Dividing Fractions

To divide these fractions, we need to keep the first fraction the same, change the $\div$ to a $\times$, and flip the second fraction upside down. Doing so, we get

$$\frac{12}{7} \div \frac{9}{20} = \frac{12}{7} \times \frac{20}{9}$$

Then, we do the multiplication:

$$\frac{12}{7} \times \frac{20}{9} = \frac{12 \times 20}{7 \times 9} = \frac{240}{63}$$

240 and 63 both have 3 as a factor, so we can simplify our fraction to get

$$\frac{240}{63} = \frac{80}{21}$$

80 and 21 have no common factors, so we are done.

HN6 Fractions 3: Mixed Numbers and Fractions of Amounts

To find a fraction of an amount, first we need to divide by the denominator and then multiply by the numerator.
First we divide by 6

$$96 \div 6 = 16$$

Next we multiply by 5

$$16 \times 5 = 80$$

This gives our final answer as 80.

HN7 Recurring Decimals to Fractions

Firstly, set $x = 4.\dot{3}0\dot{7}$. Then, if we multiply this by 1000, we get

$$1000x = 4,307.\dot{3}0\dot{7}$$

We see that x and $1000x$ are the same after the decimal point. So, if we subtract x from $1000x$, we get

$$1000x - x = 4,307.\dot{3}0\dot{7} - 4.\dot{3}0\dot{7} = 4,303$$

Therefore,

$$999x = 4,303$$

Finally, dividing both sides by 999, we get

$$x = \frac{4,303}{999}$$

HN8 Rounding: Significant Figures & Decimal Places

i) In 7.789, the cut-off digit (the 1st decimal place) is the second 7. The digit after this is an 8, meaning we round the 7, and get the answer: 7.8

ii) In 0.0595, the cut-off digit (the 2nd significant figure) is the 9. The digit after this is a 5, meaning that we round the 9 up to 0, and add an extra 1 to the digit before the 9. Doing so, we get the answer: 0.060. (If you just got 0.06, this is correct)

HN9 Estimations

Firstly, recall that "distance = speed × time". Therefore, rounding both values to 1 significant figure, we get the estimated distance covered to be

$$50 \times 3 = 150 \text{ miles}$$

In this case, both the estimated values are bigger than the actual values. So, since we are multiplying together two bigger values, our result will be bigger. Therefore, this is an overestimate of the distance covered by Mateo during this journey.

HN10 Bounds and Error Intervals

The area of a triangle is $\frac{1}{2}bh$. Since we will be multiplying the side-lengths together, if we want the lower bound for the area, we need the smallest possible values of the side-lengths - i.e., their lower bounds.

The lower bound for the height, when rounded to 2 dp, is 38.965.
The lower bound for the width, when rounded to 2 dp, is 96.875.

Therefore, the lower bound for the area of the triangle is

$$\frac{1}{2} \times 38.965 \times 96.875 = 1,887.3671... = 1,887.37 \text{ mm (2 dp)}$$

HN11 Standard Form

i) We need to see how many times we must move the decimal point to make the number fall between 1 and 10. If we move it 8 times,

$$300,950,000$$

then it becomes 3.0095, which falls between 1 and 10. We are moving the decimal point 8 places to the left, so the power will be positive. (If we move to the right the power is negative)

$$300,950,000 = 3.0095 \times 10^8$$

ii) This is a negative power so want to end up with a small number - we must divide by 10 seven times. So, moving the decimal place 7 spaces to the left, we get

$$1.997 \times 10^{-7} = 0.0000001997$$

HA1 Rules of Indices

First up, the numerator. The term has a power on top of another power, so they should be multiplied to get

$$\left(x^{\frac{2}{3}}\right)^6 = x^{\frac{2}{3}\times 6} = x^{\frac{12}{3}}$$

At this point, we could simplify the fractional power to just be 4, but it will actually make the last step easier if we leave it as a fraction.

Now, the denominator. The terms are being multiplied, so we add the powers. Note: we are going to add a positive number to a negative number in exactly the same we always add a positive number to a negative number. Doing so, we get

$$x^{\frac{5}{3}} \times x^{-\frac{1}{3}} = x^{\frac{5}{3}+\left(-\frac{1}{3}\right)} = x^{\frac{4}{3}}$$

Next, we can treat the whole fraction as a division (meaning we will subtract the powers). So, after having simplified the top and bottom we get

$$\frac{x^{\frac{12}{3}}}{x^{\frac{4}{3}}} = x^{\frac{12}{3}-\frac{4}{3}} = x^{\frac{8}{3}}$$

According to the rules of fractional powers, this can be written like

$$x^{\frac{8}{3}} = \sqrt[3]{x^8}$$

We have found that $a = 3, b = 8$, so we're done.

HA2 Negative Indices

Firstly, simplify 12^{-2}. We need to take the reciprocal of 12, and then make the power positive. Doing so, we get

$$12^{-2} = \frac{1}{12^2} = \frac{1}{144}$$

Next, $36^{\frac{1}{2}}$. A fractional power becomes a root, so we get

$$36^{\frac{1}{2}} = \sqrt{36} = 6$$

Therefore, the original expression is

$$\frac{1}{144} \times 6 = \frac{6}{144}$$

This simplifies to

$$\frac{3}{72}$$

This simplifies further to

$$\frac{1}{24}$$

HA3 Rearranging Formulae

Firstly, we will multiply both sides by 2 to get

$$2v = at^2$$

Then, dividing both sides by a gives us

$$\frac{2v}{a} = t^2$$

We are almost there but not quite - we need to get rid of the power of 2. To do that, we must square root both sides to get

$$\sqrt{\frac{2v}{a}} = t$$

Flipped around (this step is not necessary, but is good practice), this looks like

$$t = \sqrt{\frac{2v}{a}}$$

HA4 Forming & Solving Equations

a) Laline bought two tickets, so that's two lots of £x, and one large drink, which is one lot of £4. Adding these together, we get the expression

$$2x + 4$$

b) The total cost of her trip was £26, and we know the expression above also represents the total cost of her trip, so we can set them equal to each other. Doing so, we get

$$2x + 4 = 26$$

We can now solve this equation. Subtracting 4 from both sides, we get

$$2x = 26 - 4 = £22$$

Then, dividing both sides by 2 gives us the answer

$$x = 22 \div 2 = £11$$

HA5 Algebraic Fractions

Factorising the numerator and denominator of our algebraic fraction, we see that

$$\frac{x^2 + 3x}{4x + 12} = \frac{x(x+3)}{4(x+3)}$$

We see that top and bottom have a common factor $(x + 3)$, so it can cancel. This leaves us with

$$\frac{x}{4}$$

This can not be further simplified, and so we are done.

HA6 Surds 1: Basics

We want to find a square number that goes into 72. Here, we will go for 36. So, we can write $\sqrt{72}$ like

$$\sqrt{72} = \sqrt{36 \times 2} = \sqrt{36} \times \sqrt{2}$$

We know $\sqrt{36} = 6$, so we get

$$\sqrt{36} \times \sqrt{2} = 6\sqrt{2}$$

There are no square numbers bigger than 1 that go into 2, so we are done.

HA7 Surds 2: Expanding Brackets & Rationalising Denominator

We need to multiply top and bottom by the surd that's on the bottom: $\sqrt{6}$. Doing so, the numerator becomes

$$8 \times \sqrt{6} = 8\sqrt{6}$$

And the denominator becomes

$$3\sqrt{6} \times \sqrt{6} = 3\sqrt{36} = 3 \times 6 = 18$$

Therefore, the fraction becomes

$$\frac{8\sqrt{6}}{18}$$

We can cancel out a factor of 2 from the top and bottom, meaning the simplified fraction is

$$\frac{4\sqrt{6}}{9}$$

HA8 Expanding Brackets

1) We must multiply the term on the outside, pqr, by all the terms on the inside. To do this, we must use the law of indices that says: when you multiply terms, their powers are added. So, multiplying the terms out, we get

$$\begin{aligned} &= (pqr \times 5pr) + (pqr \times 5r^5) + (pqr \times (-25pqr)) \\ &= 5p^2qr^2 + 5pqr^6 - 25p^2q^2r^2 \end{aligned}$$

2) We will apply FOIL, drawing red lines as we go, and then collect like terms once the expansion is done. So, we get

$$\begin{aligned} (y-2)(y-6) &= (y \times y) + (y \times (-6)) + ((-2) \times y) + ((-2) \times (-6)) \\ &= y^2 - 6y - 2y + 12 \\ &= y^2 - 8y + 12 \end{aligned}$$

HA9 Factorising Quadratics

We want two numbers which multiply to make 10 and add to make -7. In order for them to multiply to make a positive number but add to make a negative one, we need them to both be negative. So, considering factors of 10,

$$10 = (-1) \times (-10) = (-2) \times (-5)$$

We see that $(-2) \times (-5) = 10$ and $-2 + (-5) = -7$, so the two numbers we want are -2 and -5. Therefore, the quadratic factorises to

$$k^2 - 7k + 10 = (k-2)(k-5)$$

HA10 Solving Quadratics by Factorisation

We want two numbers which multiply to make -24 and add to make -2. So, considering factors of -24,

$$-24 = 1 \times (-24) = 2 \times (-12) = 3 \times (-8) = 4 \times (-6)$$

We see that $4 \times (-6) = -24$ and $4 + (-6) = -2$, so the two numbers we want are 4 and -6. Therefore, the equation becomes

$$(z-6)(z+4) = 0$$

Therefore, the two solutions are

$$z = 6 \text{ and } z = -4$$

HA11 The Quadratic Formula

In this case, $a = 2, b = -5$, and $c = 2$. Putting these into the formula, we get

$$t = \frac{-(-5) \pm \sqrt{(-5)^2 - 4 \times 2 \times 2}}{2 \times 2} = \frac{5 \pm \sqrt{25 - 16}}{4}$$

Putting this into a calculator, we get the solutions to be

$$t = 2, \text{ and } t = \frac{1}{2}$$

HA12 Completing the Square

We want to write this quadratic in the form $(z+a)^2 + b$. Firstly, a is half the coefficient of z so $a = -8 \div 2 = -4$. Then, b is 14 takeaway the square of a, so we get

$$\begin{aligned} z^2 - 8z + 14 &= (z-4)^2 + 14 - (-4)^2 \\ &= (z-4)^2 + 14 - 16 \\ &= (z-4)^2 - 2 \end{aligned}$$

Therefore, our equation becomes:

$$(z-4)^2 - 2$$

As we can see this is in the form $(x+a)^2 + b$, so $(z-4)^2 - 2$ is our final answer.

Extra:

Now are have the equation in the form $(x+a)^2 + b$, we can actually solve for x by doing the following.

Adding 2 to both sides and then square rooting, we get

$$\begin{aligned} (z-4)^2 &= 2 \\ z - 4 &= \pm\sqrt{2} \end{aligned}$$

Then, adding 4 to both sides, we get the solution to be

$$z = 4 \pm \sqrt{2}$$

In other words, the 2 solutions are

$$z = 4 + \sqrt{2}, \text{ and } z = 4 - \sqrt{2}$$

HA13 Linear Sequences & the nth Term

The formula must take the form $an+b$. To find a, find the common difference between the terms and confirm that they are the same.

$$1, \quad 5, \quad 9, \quad 13, \quad 17$$
$$+4 \quad +4 \quad +4 \quad +4$$

So, the formula must be $4n+b$. So, we now write out the sequence given by $4n$ (i.e., the 4 times table):

$$4,\ 8,\ 12,\ 16,\ 20$$

Each of these terms is 3 bigger than their respective terms in the original sequence, so we must subtract 3 from them all. Therefore, the nth term is

$$4n-3$$

HA14 The nth Term of a Quadratic Sequence

Firstly, we must find the second differences of the sequence.

$$-1, \quad 4, \quad 15, \quad 32, \quad 55$$
$$+5 \quad +11 \quad +17 \quad +23$$
$$+6 \quad +6 \quad +6$$

The second difference is 6, so we get that $a = 6 \div 2 = 3$. Now, we want to rewrite the sequence, with the values generated by $3n^2$ underneath it. Then, we will subtract the second row from the first to find a sequence of differences.

u_n	-1	4	15	32	55
$3n^2$	3	12	27	48	75
d_n	-4	-8	-12	-16	-20

The sequence, in this case, is

$$-4, -8, -12, -16, -20$$

The common difference of this sequence is -4, so the the nth term for this sequence must begin with $-4n$. In fact, $-4n$ is precisely the nth formula for this sequence - no need to add/subtract anything. Combining this with $3n^2$, we get the nth term formula of the quadratic sequence to be

$$3n^2-4n$$

HA15 Inequalities

We will rearrange to make z the subject. So, adding $4z$ to both sides, we get

$$2 \leq 5z-18$$

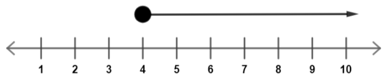
Then, adding 18 to both sides, we get

$$20 \leq 5z$$

Finally, dividing both sides by 5 gives us the solution

$$4 \leq z$$

So now we need to draw a closed circle at 4 on the number line (the inequality is inclusive), and draw an arrow going to the right, away from the circle. This looks like

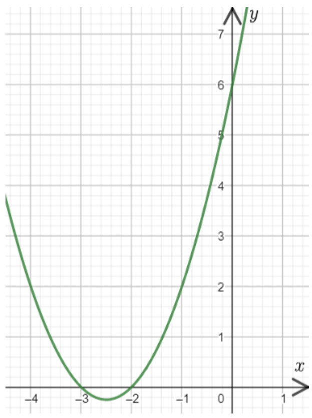


HA16 Quadratic Inequalities

We want to find the two roots of this quadratic in order to draw a sketch of it. So, noticing that $2\times3=6$ and $2+3=5$, we get

$$x^2+5x+6=(x+2)(x+3)$$

Therefore, the two roots of this quadratic are $x=-2$ and $x=-3$. Therefore, the sketch of the graph looks like



Since we are looking for when x^2+5x+6 is less than or equal to zero, we want to see when the graph goes below the x-axis. We can see that that happens between -2 and -3, and so the solution to our inequality is

$$-3 \leq x \leq -2$$

HA17 Iterative Methods

a) There is an x^2 in the end result but no such term in the initial equation. So, our first step is going to be factorising x out of the left-hand side of the equation:

$$x(x^2+2)=4$$

Then, dividing both sides by (x^2+2), we get

$$x=\frac{4}{x^2+2}$$

b) We need to sub in x_0 to the formula to get x_1, then sub in x_1 to get x_2, and so on.

NOTE: we will round each decimal to 3 sf in our answers, but in the calculation, we will use the full decimal expansions given by the calculator. So, we get

$$x_1=\frac{4}{x_0^2+2}=\frac{4}{(1)^2+2}=\frac{4}{3}=1.3333...$$

$$x_2 = \frac{4}{x_1^2+2} = \frac{4}{(1.3333...)^2+2} = \frac{18}{17} = 1.058...$$

$$x_3 = \frac{4}{x_2^2+2} = \frac{4}{(1.058...)^2+2} = 1.281...$$

Therefore, the rounded answers are $x_1 = 1.33, x_2 = 1.06, x_3 = 1.28$.

HA18 Simultaneous Equations

To make the coefficients of a the same, we will multiply the first equation by 2. Doing so, we get

$$4a - 8b = -2$$

Then, we will subtract the second equation from this new equation we just obtained. This looks like

$$\begin{array}{r} 4a - 8b = -2 \\ 4a + 6b = 12 \\ \hline -14b = -14 \end{array}$$

Thus, we get the equation $-14b = -14$. If we divide both sides by -14, we get

$$b = \frac{-14}{-14} = 1$$

Then, substituting $b = 1$ into our very first equation, we get

$$2a - 4 = -1$$

Add 4 to both sides:

$$2a = 3$$

Then, divide both sides by 2 to get

$$a = \frac{3}{2}$$

We have found both a and b, so we're done.

NOTE: if your first step was to divide the second equation given in the question by 2, then that is perfectly valid. If you got the right answer, then that method (or any other way of multiply/dividing equations to make the elimination step work) is worth full marks.

HA19 Quadratic Simultaneous Equations

We want to eliminate a variable by using substitution. So, we're going to replace the y in the first equation with $2x-1$, as the second equation tells us that y is equal to $2x-1$. Doing this, we get

$$x^2 + (2x-1)^2 = 2$$

Expanding the brackets and collecting like terms, we get

$$x^2 + 4x^2 - 2x - 2x + 1 = 2$$
$$5x^2 - 4x + 1 = 2$$
$$5x^2 - 4x - 1 = 0$$

This is a quadratic we can solve. This can be done using whichever method you prefer, here we will use the quadratic formula. Doing so, we get

$$x = \frac{-(-4) \pm \sqrt{(-4)^2 - 4\times 5 \times (-1)}}{2\times 5}$$

$$x = \frac{4 \pm \sqrt{16+20}}{10}$$

Putting it all into a calculator, we get the solutions to be

$$x = 1, \quad x = -\frac{1}{5}$$

Then, we find the solutions for y by substituting these values into the original equation. Doing so, we get

$$x = 1 \text{ gives } y = 2(1) - 1 = 1$$

$$x = -\frac{1}{5} \text{ gives } y = 2\left(-\frac{1}{5}\right) - 1 = -\frac{7}{5}$$

Therefore, the two solution pairs are

$$x = 1, y = 1 \text{ and } x = -\frac{1}{5}, y = -\frac{7}{5}$$

HA20 Functions, Inverse and Composite

a) i) We want to rearrange $y = 4x^3$ to make x the subject. So, dividing both sides by 4 we get

$$\frac{y}{4} = x^3$$

Then, cube rooting both sides, we get

$$x = \sqrt[3]{\frac{y}{4}}$$

Finally, switching any y with an x, we get that the inverse function is

$$f^{-1}(x) = \sqrt[3]{\frac{x}{4}}$$

ii) We want to sub 32 into $f^{-1}(x)$. Doing so, we get

$$f^{-1}(32) = \sqrt[3]{\frac{32}{4}} = \sqrt[3]{8} = 2$$

b) So, firstly we get

$$\begin{aligned} fg(0) = f\left(\frac{0-1}{5}\right) &= f\left(\frac{-1}{5}\right) \\ &= 4\left(\frac{-1}{5}\right)^3 \\ &= -0.032 \end{aligned}$$

And then, we get

$$\begin{aligned} gf(0) = g\left(4(0)^3\right) &= g(0) \\ &= \frac{0-1}{5} \\ &= -0.2 \end{aligned}$$

$-0.032 > -0.2$, therefore in this case, $fg(0) > gf(0)$, so it must not always be true that $gf(x) > fg(x)$.

HA21 Proof

To do this, we need to expand the brackets and collect like terms. So, the first bracket:

$$\begin{aligned}(2m+3)^2 &= 4m^2+6m+6m+9\\ &= 4m^2+12m+9\end{aligned}$$

Next, the second bracket:

$$\begin{aligned}(2m-3)^2 &= 4m^2-6m-6m+9\\ &= 4m^2-12m+9\end{aligned}$$

Therefore, subtracting one from another, we get

$$\begin{aligned}(2m+3)^2-(2m-3)^2 &= 4m^2+12m+9-(4m^2-12m+9)\\ &= 12m-(-12m) = 24m\end{aligned}$$

So, the expression simplifies to $24m$. 24 is a multiple of 6, so we can write

$$24m = 6\times 4m$$

Since $4m$ must be a whole number, we can conclude that $24m$, and therefore the original expression, must be a multiple of 6.

HG1 Gradient and $y = mx + c$

Remember we just need to find two numbers to describe our line: the gradient (m) and the y-axis intercept (c).
It's easier if we start with the gradient.

$$m = \frac{\text{change in y}}{\text{change in x}} = \frac{y_2 - y_1}{x_2 - x_1} = \frac{8 - -7}{2 - -3} = \frac{15}{5} = 3$$

So our equation so far is $y = 3x + c$. To find c, we substitute any coordinates that the line passes through for x and y in this equation let's use the point (2,8).

$$8 = 3 \times 2 + c$$

Now we just rearrange this equation to find c.

$$8 = 6 + c$$
$$c = 2$$

So our completed equation is $y = 3x + 2$

HG2 Coordinates and Midpoints

Point A has coordinates $(-2, -2)$.
Point B has coordinates $(0, 3)$.
By taking the average of the x coordinates of A and B, the x coordinate of the midpoint is:

$$\frac{-2 + 0}{2} = -1$$

By taking the average of the y coordinates of A and B, the y coordinate of the midpoint is:

$$\frac{-2 + 3}{2} = \frac{1}{2}$$

Therefore, the coordinates of the midpoint are $\left(-1, \frac{1}{2}\right)$.

HG3 Parallel and Perpendicular Lines

Given that L_1 and L_2 are perpendicular, their gradients must follow the relationship $m_1 \times m_2 = -1$. Rearranging this equation tells us that the gradient of L_2 is given by $m_2 = \frac{-1}{m_1} = \frac{-1}{-3} = \frac{1}{3}$.
So our equation for L_2 takes the form $y = \frac{1}{3}x + c$. To find c, we substitute the coordinates of any point that the line passes through into this equation. We're told that the lines intersect at $(-2, 3)$, so we can substitute $y = 3$ and $x = -2$ into this equation and solve for c:

$$3 = \frac{1}{3} \times -2 + c$$
$$3 = -\frac{2}{3} + c$$
$$c = 3\frac{2}{3} = \frac{11}{3}$$

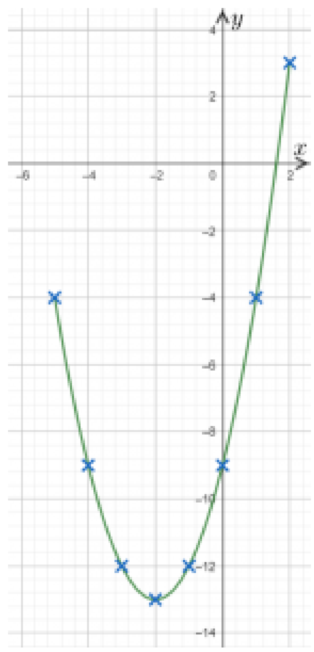
So our equation for L_2 is $y = \frac{1}{3}x + \frac{11}{3}$

HG4 Quadratic Graphs

The coefficient of x^2 is positive, so the graph will be a U-shaped curve. To plot the graph, we need to make a table of coordinates. First, we pick some x coordinates (e.g. -5 to 2), then we use the equation of the graph to work out the y-coordinates. For example, when x is -5, the y coordinate is given by $y = (-5)^2 + 4 \times (-5) - 9 = 25 - 20 - 9 = -4$. We then do this with the other x coordinates until we have the following table:

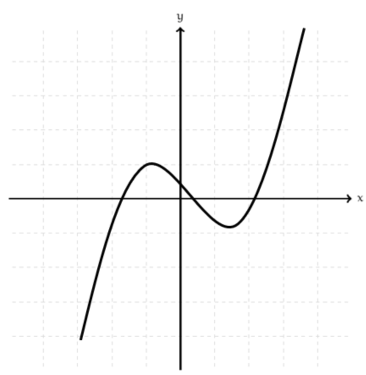
x	-5	-4	-3	-2	-1	0	1	2
y	-4	-9	-12	-13	-12	-9	-4	3

Then we plot the coordinates $(-5, -4), (-4, -9)$ etc. to get the following graph.



HG5 Cubic, Reciprocal, and Exponential Graphs

The name of this function is cubic.
The sketch should be any general cubic sketch, like the one shown below.



HG6 Turning Points of Quadratic Graphs

Firstly, we must find the roots of this quadratic by factorising it and setting it equal to zero. Observing that $2 \times 3 = 6$ and $2 + 3 = 5$, we get that

$$x^2 + 5x + 6 = (x+2)(x+3)$$

Therefore, to find the roots we set

$$(x+2)(x+3) = 0$$

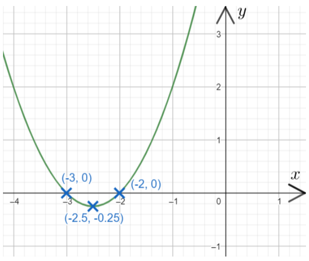
This clearly gives two roots: $x = -2$ and $x = -3$. Then, to find the co-ordinates of the turning, we need the halfway point between the roots, which is

$$\frac{-2+(-3)}{2} = -2.5$$

This is the x coordinate of the turning point. To find the y coordinate, we put this value back into the equation to get

$$y = (-2.5)^2 + 5(-2.5) + 6 = -0.25$$

Then, the resulting sketch of the graph should look like

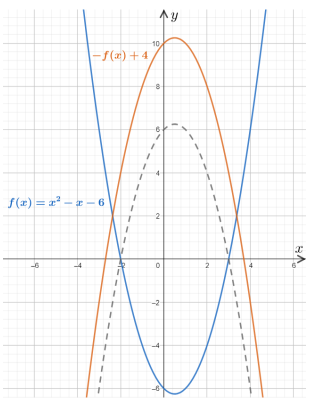


NOTE: because the question asked you for a sketch, it doesn't have to be a perfect drawing, it just has to have the correct shape and correctly identified labelled roots and turning points.

HG7 Graph Transformations

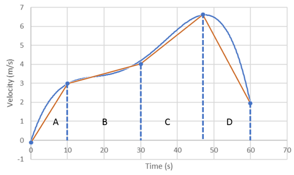
Firstly, $x^2 - x - 6$ factorises to $(x-3)(x+2)$, so it is a positive quadratic with roots at 3 and -2.

Now, $-f(x)+4$ is both a reflection in the x-axis and a translation of 4 in the positive y-direction. We should do the reflection first and the translation second - it often helps to sketch the intermediate step to help you, and you can always rub it out afterwards. Here, the dotted line will be the intermediate step (the reflection before the translation). So, we get



HG8 Area Under a Graph

First, we can draw a straight line that approximately follows the graph. Then we can divide this into simple shapes in this case, we can split it up into 1 triangle and 3 trapeziums.



Next we work out the areas of each shape using the relevant shape area formulas.

A:

$$A = \frac{1}{2} a \times b = \frac{1}{2} 10 \times 3 = 15$$

B:

$$A = \frac{1}{2}(a+b) \times h_v = \frac{1}{2}(3+4) \times 20 = 70$$

C:

$$A = \frac{1}{2}(a+b) \times h_v = \frac{1}{2}(4+6.5) \times 17 = 89.25$$

D:

$$A = \frac{1}{2}(a+b) \times h_v = \frac{1}{2}(6.5+2) \times 13 = 55.25$$

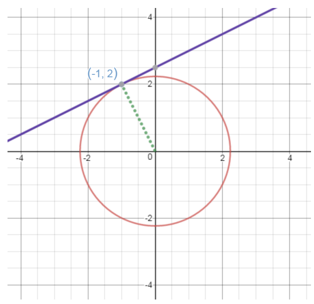
Adding all of these together gives,

$$\text{Total area } = 15 + 70 + 89.25 + 55.25 = 229.5\ (\pm 10)$$

HG9 Equations of Circles & Finding Tangents to Circles

Remember the form of the equation of a circle is $x^2 + y^2 = r^2$, where r is the radius. The radius is $\sqrt{(5)}$, so $r^2 = (\sqrt{(5)})^2 = 5$, so the equation of our circle is $x^2 + y^2 = 5$.

To find the equation of the tangent, we first think about the radius that meets the tangent at (-1,2):



We know that the line perpendicular to the tangent passes through (0,0) and (-1,2), so we can work out the gradient: $(2-0)/(-1-0) = 2/(-1) = -2$. We know that the tangent is perpendicular to this, so the gradient of the tangent must be $\frac{1}{2}$ (because the tangents of perpendicular lines multiply to make -1, and $\frac{1}{2} \times -2 = -1$.

So the equation of the tangent so far is $y = \frac{1}{2}x + c$.

To find c, we substitute the coordinates (-1,2) into the equation:

$$2 = \frac{1}{2} \times -1 + c$$

Then solve to get $c = 2.5$

So the complete equation of the tangent is $y = \frac{1}{2}x + 2.5$

HG10 Distance-Time Graphs

First we need to work out how far Neil travels between each time period:
- 12:00 - 12:30, he travels from 0 km away to 12 km away; remembering that speed = the gradient at the time.

$$\text{Gradient} = \frac{12}{0.5} = 24 \text{ km/h}$$

- 12:30 - 13:30, he travels from 12 km away to 44 km away;

$$\text{Gradient} = \frac{44-12}{1} = 32 \text{ km/h}$$

- 13:30 - 16:30, he stays in one place;

$$\text{Gradient} = \frac{0}{1} = 0 \text{ km/h}$$

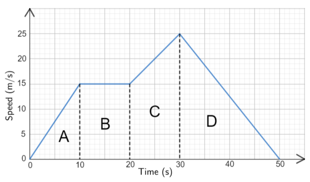
- 16:30 - 18:30, he travels from 44 km away to 0 km away.

$$\text{Gradient} = \frac{-44}{2} = 22 \text{ km/h}$$

From this we can see that the greatest speed is between 12:30 - 13:30.

HG11 Velocity-Time Graphs

Before we get stuck in to answering this, let's go through the journey described by the graph. Firstly, the car accelerated from 0 to 15 m/s over the first 10 seconds (because the line is straight, the acceleration is constant). Then, the line is flat, meaning the car's speed was not changing for 10 seconds - it was moving at constant speed. Next, the car accelerated up to 25 m/s over the next 10 seconds, and finally it spent the last 20 seconds decelerating back down to 0 m/s.
Now, we want to work out the distance travelled. On a speed-time graph, the distance travelled is the area under the graph. To work out the area under this graph, we will break it into 4 shapes: A, B, C, and D - two triangles, a rectangle, and a trapezium are all shapes that we can work out the area of. So, we get



$$A = \frac{1}{2} \times 10 \times 15 = 75\text{m}$$

$$B = 10 \times 15 = 150\text{m}$$

$$C = \frac{1}{2}(15+25) \times 10 = 200\text{m}$$

$$D = \frac{1}{2} \times 20 \times 25 = 250\text{m}$$

Therefore, the total distance travelled is:

$$75 + 150 + 200 + 250 = 675 \text{ m}$$

Next up, the acceleration. To do this, consider that acceleration is a measure of how quickly something's speed is increasing. Therefore, given that the gradient is a rate of change of y (speed) with respect to x (time), we can work out the acceleration by finding the gradient of the graph.
Now, the question asks for "maximum acceleration", so we can rule out certain parts of the journey. Specifically, the part where the graph is flat - there is no acceleration here - and the part where the graph slopes downward - it is decelerating here, so can't be the maximum acceleration.
It might be obvious to you which of the two sections of the graph is steeper, but it isn't always, so we'll work out both just to be sure. Firstly, the first 10 seconds: the car's speed goes from 0 to 15 m/s and it takes 10 seconds, so we get

$$\text{acceleration between 0 s and 10 s} = \text{gradient} = \frac{15-0}{10-0} = 1.5 \text{ m/s}^2$$

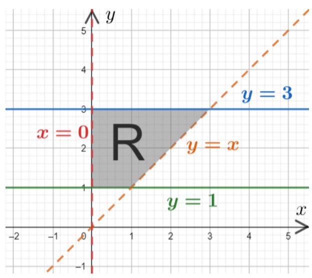
Then, the other portion we're interested in is between 20 and 30 seconds. During this period, the speed increases from 15 to 25, so we get

$$\text{acceleration between 20s and 30s} = \text{gradient} = \frac{25-15}{30-20} = 1 \text{ m/s}^2$$

The first one is larger, so the maximum acceleration is 1.5 m/s^2.

HG12 Graphs of Inequalities

We're going to pretend that the inequalities are equations and plot them as straight lines. The first one will be the solid plot of the line $y = 1$, the second will be a solid plot of the line $y = 3$, the third will be a dashed plot of the line $x = 0$, and the fourth will be a dashed plot of the line $y = x$.
Now, we want to shade the area that is above the line $y = 1$, below the line $y = 3$, to the right of the line $x = 0$, and above the line $y = x$.
The resulting graph looks like:



Labelling the lines is not necessary but you may find it be a helpful intermediate step. Or you might not, which is totally fine - be your own person.

HR1 Ratio

We know that difference between Deborah's and Kemah's ages is 21. Looking at the ratio, Kemah has 1 part and Deborah has 4, meaning that the difference between them (21 years) constitutes 3 parts in the ratio. Therefore, we get that

$$1 \text{ part } = 21 \div 3 = 7$$

Kemah, Bob, and Deborah have 1, 2, and 4 parts in the ratio respectively. So, we get that
Kemah's age $= 1 \times 7 = 7$
Bob's age $= 2 \times 7 = 14$
Deborah's age $= 4 \times 7 = 28$

HR2 Direct Proportion

a) If F is directly proportional to a then we can write

$$F \propto a$$

which becomes the equation:

$$F = ka$$

Now, to find k, we will substitute in the values given in the question. Doing so, we get

$$7,500 = k \times 6$$

Dividing both sides by 6, we get

$$k = 7,500 \div 6 = 1,250$$

Therefore, our equation for F in terms of a is

$$F = 1,250a$$

b) To find the acceleration when $F = 12,000$, we must substitute this into the equation:

$$12,000 = 1,250a$$

Then, dividing both sides by 1,250, we get the acceleration to be

$$a = 12,000 \div 1,250 = 9.6 \text{ m/s}^2$$

HR3 Inverse Proportion

a) F is inversely proportional to the square of d, so we get

$$F \propto \frac{1}{d^2}$$

which becomes the equation

$$F = \frac{k}{d^2}$$

Now, to find k, we will substitute in the values given in the question. Doing so, we get

$$4,500 = \frac{k}{(200)^2} = \frac{k}{40,000}$$

Multiplying both sides by 40,000, we get

$$k = 4,500 \times 40,000 = 180,000,000$$

Therefore, our equation for F in terms of d is

$$F = \frac{180,000,000}{d^2}$$

b) To find the distance when $F = 2,000$, we must substitute this into the equation:

$$2,000 = \frac{180,000,000}{d^2}$$

Firstly, multiplying both sides by d^2 gives us

$$2,000d^2 = 180,000,000$$

Then, divide by 2,000 to get

$$d^2 = 180,000,000 \div 2,000 = 90,000$$

Finally, we get the answer, by square rooting both sides, to be

$$d = \sqrt{90,000} = 300 \text{ km}$$

HR4 Percentages

This is a 32% decrease, so the multiplier for a 32% decrease is

$$1 - \frac{32}{100} = 0.68$$

Therefore, multiplying this by the original value Matt bought the TV for, we get the price that Dave purchased it for to be

$$550 \times 0.68 = £374$$

HR5 Reverse Percentage

We need to consider how we would calculate a 4% increase. We know that 4% = 0.04, so we get the multiplier for a 4% increase to be

$$1 + 0.04 = 1.04$$

Let H be Tom's height from two years ago. We know that the result of multiplying H by 1.04 must be 182. We can write this as an equation:

$$H \times 1.04 = 182$$

Then, if we divide both sides by 1.04 we get

$$H = 182 \div 1.04 = 175$$

So, Tom's height two years ago was 175 cm.

HR6 Growth & Decay

This is a case of compound growth. Firstly, the multiplier for a 40% increase is

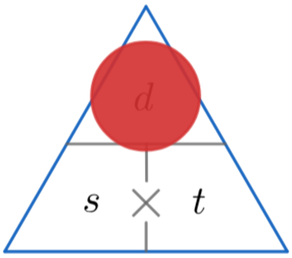
$$1 + 0.40 = 1.4$$

We are looking at the number of bacteria after one week, which means SEVEN 40% increases. Therefore, our calculation is

$$480 \times (1.4)^7 = 5,060 \text{ (to nearest whole number)}$$

5,060 is clearly bigger than 5,000, so the scientist's estimate is correct.

HR7 Speed, Distance, Time



If we cover up d in the triangle, we see that we will have to multiply s by t to get our answer. However, the units don't match up - we need to convert the minutes to hours, which we will do by dividing it by 60.

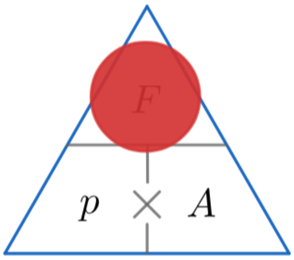
$$t = 414 \div 60 = 6.9 \text{ hours}$$

Now, we can do the multiplication:

$$\text{distance covered } = d = 230 \times 6.9 = 1,587 \text{ km}$$

HR8 Pressure & Density

Covering up F on the triangle,



we see that we need to multiply p (pressure) by A (area). To do this, we need to find the area of the triangle. Before that, however, notice that the sides of the triangle are measured in centimetres which doesn't match up with the "Newtons per metres squared" in the question. So, we will convert the dimensions of the triangle into metres by dividing by 100:

$$\text{height } = 80 \div 100 = 0.8 \text{ m}$$

$$\text{base } = 150 \div 100 = 1.5 \text{ m}$$

Now we can calculate the area of the triangle:

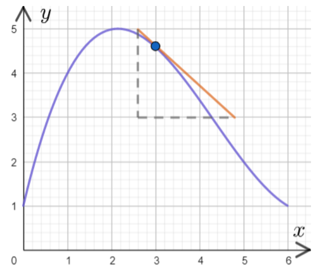
$$\text{area } = \frac{1}{2}bh = \frac{1}{2} \times 0.8 \times 1.5 = 0.6 \text{ m}^2$$

Therefore, we get that the force being applied is

$$F = p \times A = 40 \times 0.6 = 24 \text{ N}$$

HR9 Rates of Change

We must draw a tangent to the graph at the point $x = 3$ and find its gradient. Firstly, that tangent looks like



Calculating the gradient of the tangent line (orange), we get

$$\text{gradient } = \frac{\text{change in } y}{\text{change in } x} = \frac{3-5}{4.8-2.6} = -0.91 \text{ (2 dp)}$$

Therefore, the instantaneous rate of change of y with respect to x at $x = 3$ is -0.91, to 2 dp.

NOTE: your answer may vary slightly - anything between -0.8 and -1 is acceptable.

HM1 Angles in Parallel Lines

Firstly, because angles BEF and EHJ are corresponding angles, we get

$$\text{angle EHJ } = 39°$$

Next, because angles EDH and DHG are alternate angles, we get

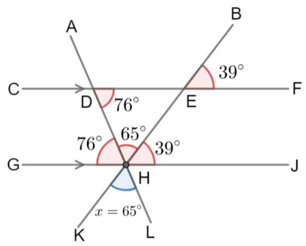
$$\text{angle DHG } = 76°$$

Then, because angles DHG, DHE, and EHJ are angles on a straight line and angles on a straight line add to 180, we get

$$\text{angle DHE } = 180 - 76 - 39 = 65°$$

Finally, because angle DHE and angle x are vertically opposite angles, we get

$$x = 65°$$



HM2 Interior & Exterior Angles

This is a 4-sided shape, which we now know has interior angles that add up to $180 \times 2 = 360$. So, if we know all the interior angles other than x, then we can find x.
Currently we don't know them all, however we do have an exterior angle, and we know that exterior angles form a straight line with their associated interior angles, we get the interior angle at D to be $180 - 121 = 59°$.
Now we know all 4 interior angles, we get that

$$x = 360 - 84 - 100 - 59 = 117°$$

HM3 Areas of Shapes

As we need to find a missing side-length rather than the area, we're going to have to set up an equation and rearrange it to find x. The formula for the area we'll need here is

$$\frac{1}{2}ab\sin(C)$$

so, our equation is

$$\frac{1}{2} \times 2.15 \times x \times \sin(26) = 1.47$$

Simplifying the left-hand-side, we get

$$1.075\sin(26) \times x = 1.47$$

Finally, dividing through by $1.075\sin(26)$, and putting it into a calculator, we get

$$x = \frac{1.47}{1.075\sin(26)} = 3.12 \text{ m (2 dp)}$$

HM4 Area and Circumference of a Circle

When the question asks us for an "exact" answer, we have to make sure we're not rounding anything. So we will need to express our answer in terms of pi where appropriate.
For the first part of the question we want to find the radius of the circle. We know the formula that relates radius and area: $A = \pi r^2$. We need to rearrange this to make r the subject. First we divide both sides of the equation by pi:

$$\frac{A}{\pi} = r^2$$

Then we square root both sides to give us r:

$$\sqrt{\left(\frac{A}{\pi}\right)} = r$$

Substituting the given area for A:

$$\sqrt{\left(\frac{25\pi}{\pi}\right)} = r$$

The pi's cancel out:

$$\sqrt{25} = r$$

So we have $r = 5$.

Next, to find the circumference we just use the formula $c = 2\pi r$.
We could convert this answer into a decimal, which would be 31.4 to 1 dp. But the question asks for an exact answer, so we can't round it. The only way to represent the answer exactly is to leave it in terms of pi.

HM5 Sectors of Circles

The question asks for the total perimeter of the shape. We know that one side is 14 mm but the other two are missing. Immediately we can identify that the other straight side is also a radius of the circle and so will also be 14 mm long. Then, all that remains is to calculate the arc length and add up our answers.
The angle in this sector is 165 degrees, meaning that the arc length will be equal to $\frac{165}{360}$ of the total circumference. The formula for the circumference is πd, or alternatively (and more helpfully in this case), $2\pi r$. So, we get:

$$\text{Arc length } = \frac{165}{360} \times 2\pi \times 14 = 40.3 \text{ mm}$$

Therefore, total perimeter $= 14 + 14 + 40.3 = 68.3$ mm

HM6 Circle Theorems 1

When a question like this tells you to show our workings, you must state what circle theorem/geometry fact you use when you use it.
There's no way for us to immediately find the angle we want, so we're going to try to find the other angles in the quadrilateral ABCD. The first circle theorem we're going to use here is: the angle at the centre is twice the angle at the circumference. The angle at the centre is 126°, so angle BAD $= 126 \div 2 = 63°$.
We now know two out of the four angles inside ABCD. To find a third, simply observe that angles around a point sum to 360, then we get that the angle at point C (the one inside ABCD) is $360 - 126 = 234°$. Since

the angles in a quadrilateral sum to 360°, if we subtract the ones we know from 360 then find the angle we're looking for.

$$\text{Angle ABC } = 360 - 33 - 63 - 234 = 30^\circ$$

HM7 Circle Theorems 2

Our first circle theorem here will be: tangents to a circle from the same point are equal, which in this case tells us that AB and BD are equal in length.

This means that ABD must be an isosceles triangle, and so the two angles at the base must be equal. In this case those two angles are angles BAD and ADB, neither of which know. Let the size of one of these angles be x, then using the fact that angles in a triangle add to 180°, we get

$$x + x + 42 = 180$$

Then, subtract 42 from both sides to get

$$2x = 180 - 42 = 138$$

and divide both sides by 2 to get

$$x = 69^\circ$$

Now we can use our second circle theorem, this time the alternate segment theorem. This tells us that the angle between the tangent and the side of the triangle is equal to the opposite interior angle. Given that angle ADB, which is 69°, is the angle between the side of the triangle and the tangent, then the alternate segment theorem immediately gives us that the opposite interior angle, angle AED (the one we're looking for), is also 69°.

HM8 Congruence

Let's check each shape individually.

Shape B: it has two angles in common with A, but the side is a different length.

Shape C: this has two angles and a side-length in common with A, but to pass the ASA test the side-length needs to be between the two angles, which in C's case it isn't.

Shape D: this does what shape C didn't - all the numbers match, and the side we know is between the two angles which means that shape D is congruent to A by the ASA criteria.

The real value in being able to spot when two triangles are congruent like this is that we suddenly know that all the other angles and side-lengths must also be the same. This is useful in making quick leaps towards solving bigger problems, for example in circle theorems, so keep the definition of congruence as well as the 4 tests for congruent triangles in mind when solving all kinds of geometry problems.

HM9 Similar Shapes

Firstly, we will determine the scale factor that relates the side-lengths by dividing the side-length of the bigger shape by that of the smaller shape: $SF = 28 \div 7 = 4$.

Now, if the scale factor for the side-lengths is 4, then that means that the scale factor for the areas is: $SF_A = 4^2 = 16$.

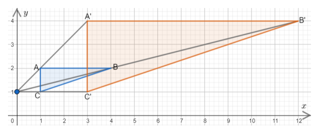
Therefore, to find the area of the smaller shape, we need to divide the area of the bigger shape by the area scale factor: 16. Doing so, we get

$$\text{Area of A } = 320 \div 16 = 20 \text{ cm}^2$$

HM10 Translations and Reflections

Firstly, we must draw the line $y = 1$ onto the graph. Then, you can either choose to use tracing paper or, if you're confident without it, just go right into the reflection.

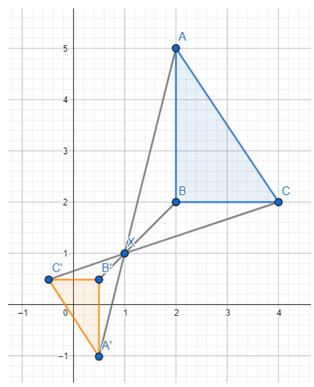
Then, the trace of the shape is the result of the reflection. Draw that shape onto the original axes, mark it with a C and you should get the resulting picture below.



HM11 Rotations and Enlargements

We need to draw lines from the point $(0, 1)$ to all corners of this shape. Then, since this is scale factor 3 enlargement, we need to extend these lines until they are 3 times longer. For example, the line from $(0, 1)$ to A goes 1 space to the right and 1 up. So, once we've extended it, the resulting line should go 3 spaces to right and 3 spaces up.

Then, once all these lines have been drawn, their ends will be the corners of the enlarged shape. Joining these corners up, we get the completed shape, as seen below.

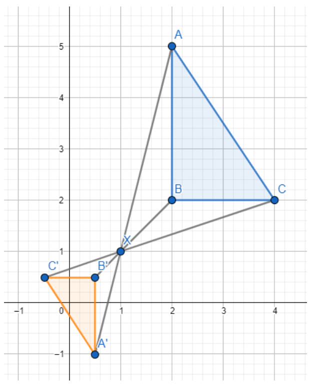


HM12 Negative Enlargements

This question has it all going on: the enlargement factor is negative and it's less than one. Don't panic we start this in the same way we start any enlargement question: by drawing lines from the corners of the shape to the centre of enlargement.

Now we think about the new lines we're going to draw. The fact the enlargement factor is negative tells us that A' (A' is just the new position of A) is going to be in the opposite direction than A, relative to the centre of enlargement. The fact that the size of the enlargement factor is 0.5 tells us that the line to OA' will be half the length of the line to OA.

So, we should get something like this (where X is the centre of enlargement).



HM13 Volumes of 3D Shapes

So, to work out the volume of a prism we must multiply the area of the cross section by the length. In this case, the cross section is a trapezium, and the area of the trapezium is

$$\text{Area of cross section } = \frac{1}{2} \times (45+60) \times 20 = 1,050 \text{ cm}^2$$

The length of the prism is 80 cm, so we get

$$\text{Volume of prism } = 1,050 \times 80 = 84,000 \text{ cm}^3$$

HM14 Volumes of 3D Shapes 2

Firstly, the cylinder is a type of prism, so we know we need to multiply the area of the cross section by its length. Here, the cross section is a circle with radius 4 mm, and the length of the cylinder is 3 mm, so we get

$$\text{Volume of cylinder} = \pi \times 4^2 \times 3 = 48\pi$$

We'll worry about the rounding at the end. Next, we have to work out the volume of the cone part, and fortunately the formula for this is given in exams!
h is the height of the cone, which we know to be 5.5, and r is the radius of the cone, which is the same as that of the base: 4mm. Therefore, we get

$$\text{Volume of cone } = \frac{1}{3}\pi \times 4^2 \times 5.5 = \frac{88}{3}\pi$$

Then, the volume of the shape is the sum of these two answers:

$$\text{Volume of whole shape } = 48\pi + \frac{88}{3}\pi = 242.9498... = 242.9 \text{ mm}^3 \text{ (1 dp)}$$

HM15 Surface Areas of 3D Shapes

We know the whole surface area is 120 cm^2 and we also know the radius. To work out the slant height, we need to first work out what the curved surface area is. In other words, we need to subtract the surface area of the base of the cone (since that's the only other face) from 120 to get the curved surface area. The base is a circle, so its area is

$$\pi \times 3^2 = 9\pi \text{ cm}^2$$

Subtracting this from the total we have $120 - 9\pi$.

Now, this must be the area of the curved face, and the formula for the area of the curved face is given to us: $\pi r l$. So, setting this formula equal to the value we worked out, we get

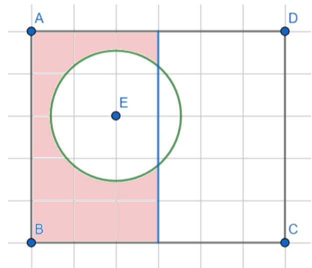
$$\pi r l = 120 - 9\pi$$

Then, to find the slant height, we will divide both sides by πr to get

$$l = \frac{120 - 9\pi}{3\pi} = 9.7 \text{ cm (1 dp)}$$

HM16 Loci and Constructions

For the fountain to be at least 3 m away from his house along CD, we need to only consider the area to the left of the straight line which is parallel to CD and 3 cm away from it.
Then, the locus of points which are 1.5 m away from the tree at E will be a circle of radius 1.5 cm - for the fountain to be at least 1.5 m away, it must be outside this circle.
So, the locus of points where he could place the fountain is to the left of the (blue) line 3 m away from the house, and outside the (green) circle which is 1.5 m away from the tree. The correct region is shaded red on the picture below.

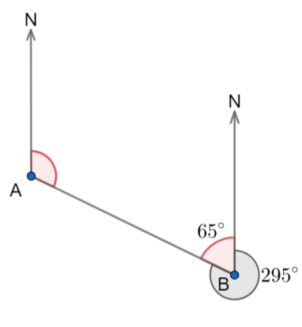


HM17 Bearings

We can't simply measure the angle, since the picture is not drawn accurately. Instead, we will use the fact that the two North lines are parallel to one another.
Firstly, recognise that we can find the other angle around the point B by subtracting 295 from 360.

$$360 - 295 = 65°$$

Then, because the two north lines are parallel, we can say that the bearing of B from A and the 65° angle we just found are interior (sometimes co-interior, or allied, depending on what your teacher likes). From our facts about angles in parallel lines, we know the two angles (marked with red below) must add to 180.



So, we get:

$$\text{Bearing of B from A } = 180 - 65 = 115°$$

HT1 Pythagoras

To do this we will need to use Pythagoras theorem $a^2 + b^2 = c^2$
We then add the numbers we know into the equation. This gives us

$$9^2 + x^2 = 13^2$$

Next, we need to rearrange to make x the subject.

$$x^2 = 13^2 - 9^2$$

Next if we then calculate the right-hand side:

$$x^2 = 169 - 81$$
$$x^2 = 88$$

Finally, we square root both sides

$$x = \sqrt{88} = 2\sqrt{22}$$

HT2 Trigonometry

The two sides we're concerned with are the hypotenuse and the opposite (to the angle) - O and H. Therefore, we want the 'SOH' part of 'SOHCAHTOA', so will be using sin. We have $O = 13, H = 15$, and the angle is q, so we get

$$\sin(q) = \frac{O}{H} = \frac{13}{15}$$

Then, to get q, we have to apply the inverse sin function: $\sin^{-1}$ to both sides. It cancels out the sin on the left-hand side, and we get

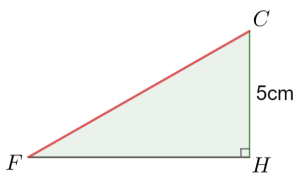
$$q = \sin^{-1}\left(\frac{13}{15}\right)$$

Finally, putting this into the calculator we get

$$q = 60.0735... = 60.1^\circ \text{ (1 dp)}$$

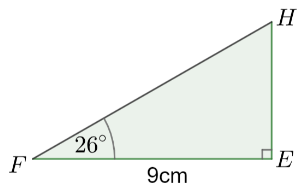
HT3 Trigonometry & Pythagoras in 3D

Firstly, the shape is a cuboid, which means it is full of right-angles. Every corner is a right-angle, in fact, which means our rules mentioned on the card will come in useful.
So, we need to find FC. Let's first we consider the triangle FCH on its own.



Because we're in a cuboid, this is a right-angled triangle. We know that side CH is 5cm, but otherwise we have no information about this triangle.
We need an extra side-length or angle for us to be able to find the length of FC. If we find the length of FH, for example, then we can use Pythagoras to find FC.
It is very useful then to notice that FH is also a hypotenuse of a different triangle

To find side-length FH, we're going to now consider the triangle FEH. Fortunately, we do know a little more about this triangle.



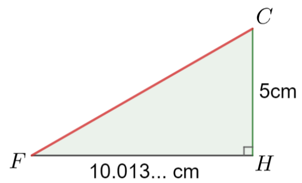
As before, this is also a right-angled triangle, but the difference is we know one of the side-lengths as well as an angle. This is enough information for us to use SOHCAHTOA to find the length of FH.
FE is adjacent to the angle, and we're looking for the hypotenuse FH, so we will use cos.

$$\cos(26) = \frac{\text{Adjacent}}{\text{Hypotenuse}} = \frac{9}{FH}$$

$$FH \times \cos(26) = 9$$

$$FH = 9 \div \cos(26) = 10.013 \text{ cm}$$

Now we know FH, our first triangle, FCH, looks like this:



Firstly, note that the length of FH is expressed as 10.013 cm because when you're putting it into your calculator, you should use the exact answer (by utilising the ANS button) and not a rounded-off decimal.
Now, we know two side-lengths of this triangle, we can use Pythagoras' theorem to find the third, FC, which is the answer to the whole question.

$$(FC)^2 = 5^2 + (10.013)^2$$
$$FC = \sqrt{5^2 + (10.013)^2} = 11.2 \text{ cm (3 sf)}$$

HT4 The Sine Rule

Here we can use the 'flipped-over' version of the sine rule to make things easier:

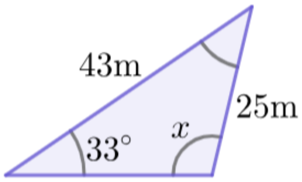
$$\frac{\sin A}{a} = \frac{\sin B}{b} = \frac{\sin C}{c}$$

Then, as usual, we label the triangle. Making sure to label A as the unknown, so $A = x$, and so the side opposite to it is 43 m and so we get $a = 43$.
Then, the remaining information is a pair, and we will let $b = 25$ and its opposite angle $B = 33^\circ$. Subbing these values into the formula, we get

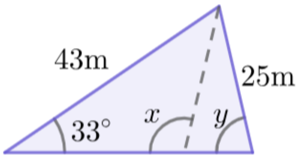
$$\frac{\sin x}{43} = \frac{\sin(33)}{25}$$

Multiply both sides by 43 to get

$$\sin x = \frac{43 \sin(33)}{25}$$



Then, taking $\sin^{-1}$ of both sides, and putting it into the calculator, we get $x = 70°$ to 2 sf. However, the question asked for an obtuse angle, but we got an acute answer - why? It's because we can draw two different (but both correct) triangles using the information we were given at the start.



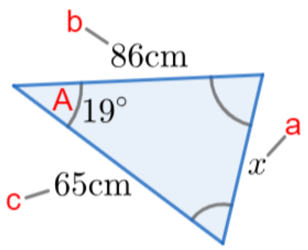
This triangle on the left also has 2 sides of 43 m and 25 m and an angle of 33°, all in the same positions as the original triangle. However, as you can see, it has a different angle, y, and this angle is acute. This is the ambiguous case of the sine rule, and it occurs when you have 2 sides and an angle that doesn't lie between them. To find the obtuse angle, simply subtract the acute angle from 180, so the answer here is $180 - 70 = 110°$.

NOTE: if the sum obtuse answer and original angle is above 180, then it is not ambiguous. Angles in a triangle cannot go above 180, so the acute answer must be the only correct one.

HT5 The Cosine Rule

Firstly, we need appropriately label the sides of this triangle. Firstly, we set $a = x$, and therefore we get that $A = 19$, since it is the angle opposite. It doesn't matter how we label the other two sides, so here we'll let $b = 86$ and $c = 65$.

$$a^2 = b^2 + c^2 - 2bc\cos A$$



Now, subbing these values into the cosine rule equation, we get

$$x^2 = 86^2 + 65^2 - (2 \times 86 \times 65 \times \cos(19)) = 7,396 + 4,225 - 11,180\cos(19)$$

Then, taking the square root, and putting it into the calculator, we get

$$x = \sqrt{7,396 + 4,225 - 11,180\cos(19)} = 32 \text{ cm (2 sf)}$$

HT6 Trigonometry Common Values

This question requires a bit less work but a bit more thought.

Since two sides of this triangle are the same length, it must be an isosceles triangle. In an isosceles triangle, we must have two angles the same - specifically the two angles that aren't given to us (we can't have one angle be the same as the right-angle, because then the sum of the angles in the triangle would go above 180°).
If those two angles are the same, the other one must also be w. Then, because angles in a triangle sum to 180°, we get

$$w + w + 90 = 180$$

Subtract 90 from both sides to get

$$2w = 90$$

Then, dividing both sides by 2 gives us: $w = 45$. Now we know the size of w, from the values of sin that we memorised we get that

$$\sin(w) = \sin(45) = \frac{1}{\sqrt{2}}$$

ALTERNATIVE METHOD: According to SOHCAHTOA, the sin of w must be equal to the opposite side divided by the hypotenuse. The opposite side is given to us: 2, but the hypotenuse is not. However, we can find it using Pythagoras, since this is a right-angled triangle.
The hypotenuse is c, and then a and b are both 2, so the equation $a^2 + b^2 = c^2$ becomes

$$c^2 = 2^2 + 2^2 = 4 + 4 = 8$$

Square rooting both sides, we get

$$c = \sqrt{8}$$

At this point you can simplify the surd and make it into $2\sqrt{2}$, but you don't have to. Then, now we know the hypotenuse, we get

$$\sin(w) = \frac{2}{\sqrt{8}} = \frac{2}{2\sqrt{2}} = \frac{1}{\sqrt{2}}$$

HT7 Graphs of Trigonometric Functions

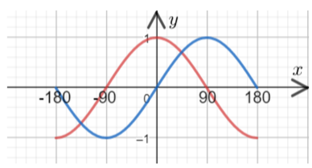
If you can't remember their shapes, check a few points. So, we have that:

$$\cos(0) = 1, \text{ and } \cos(90) = 0$$

Which is enough to start of the pattern of the cos graph. Similarly, we have:

$$\sin(0) = 0, \text{ and } \sin(90) = 1$$

Which is enough to start the pattern of the sin graph. If you aren't sure, just try more values. The resulting graph looks like:



HT8 Vectors

There are a lot of steps here, so take your time to read through it and make sure you understand.
We will find $\overrightarrow{EB}$ by doing

$$\overrightarrow{EB} = \overrightarrow{EA} + \overrightarrow{AB}$$

The first vector is straightforward, because we know $\overrightarrow{AE}$, and that is just the same vector in the opposite direction. So, we get

$$\overrightarrow{EA} = -\overrightarrow{AE} = -(3\mathbf{a} - 2\mathbf{b}) = -3\mathbf{a} + 2\mathbf{b}$$

Now we need $\overrightarrow{AB}$. Since B is the midpoint of AC (given in the question), we must have that $\overrightarrow{AB} = \frac{1}{2}\overrightarrow{AC}$. Therefore, looking at the diagram, we get that

$$\overrightarrow{AB} = \frac{1}{2}\overrightarrow{AC} = \frac{1}{2}\left(\overrightarrow{AD} + \overrightarrow{DC}\right)$$

We're given the second part of this, $\overrightarrow{DC} = 2\mathbf{a} + 4\mathbf{b}$, and since E is the midpoint of AD, we can also work out the first part:

$$\overrightarrow{AD} = 2\overrightarrow{AE} = 2(3\mathbf{a} - 2\mathbf{b}) = 6\mathbf{a} - 4\mathbf{b}$$

Now, at long last, we have everything we need and can go back through our work, filling in the gaps. Now we have $\overrightarrow{AD}$, we get that

$$\overrightarrow{AB} = \frac{1}{2}(6\mathbf{a} - 4\mathbf{b} + 2\mathbf{a} + 4\mathbf{b}) = \frac{1}{2}(8\mathbf{a}) = 4\mathbf{a}$$

Therefore, finally we have that

$$\overrightarrow{EB} = \overrightarrow{EA} + \overrightarrow{AB} = -3\mathbf{a} + 2\mathbf{b} + 4\mathbf{a} = \mathbf{a} + 2\mathbf{b}$$

If $\overrightarrow{EB}$ and $\overrightarrow{DC}$ are parallel, then one must be a multiple of the other. Well, if we multiply $\overrightarrow{EB}$ by 2 then we get

$$2 \times \overrightarrow{EB} = 2(\mathbf{a} + 2\mathbf{b}) = 2\mathbf{a} + 4\mathbf{b} = \overrightarrow{DC}$$

Therefore, we've shown that $2\overrightarrow{EB} = \overrightarrow{DC}$, and thus the two lines must be parallel.

HP1 Probability Basics

We know their probabilities must add up to 1 to make Amira's statement true. To add these values together, we must make them all share the same format. Here, we're going to convert them all to percentages. Firstly, we get that

$$0.35 = 35\%$$

Then, we get

$$\frac{1}{4} = 1 \div 4 = 0.25 = 25\%$$

Now, we can add the three probabilities together:

$$40\% + 25\% + 35\% = 100\% = 1$$

They all add to 1, so Amira's statement is correct.

HP2 Tree Diagrams

a) Firstly, we know she either wears a jumper or doesn't. Therefore, to fill in the gap at the top (after she has chosen trousers), we simply subtract the probability of her wearing a jumper from 1, to get

$$\text{P(N)} = 1 - \text{P(J)} = 1 - 0.85 = 0.15$$

Next, we know that the probability of her wearing shorts and a jumper is 0.144. This means that 0.144 must be the result of multiplying along the SJ branch, so in other words

$$0.45 \times x = 0.144$$

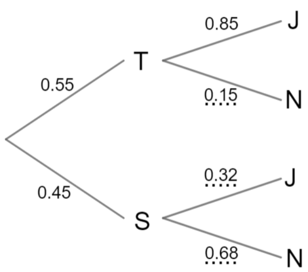
Thus, if we divide by 0.45, we get

$$x = 0.144 \div 0.45 = 0.32$$

Then, for the final gap, we subtract 0.32 from 1 to get

$$1 - 0.32 = 0.68$$

So, the completed tree diagram looks like



b) The two circumstances in which Heloise wears a jumper are: she wears trousers and a jumper, or she wears shorts and a jumper. Multiplying along the branch, we get

$$\text{P(T and J)} = 0.55 \times 0.85 = 0.4675$$

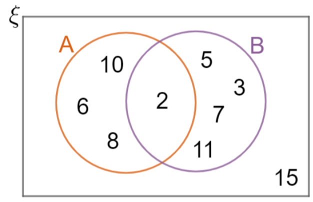
We already know the probability of her wearing shorts and a jumper: 0.144. This is an 'or' situation (since in either circumstance, she's wearing a jumper), so we must add these probabilities to get

$$\text{P(Jumper)} = 0.144 + 0.4675 = 0.6115$$

c) In this case, we know that she has chosen to wear trousers, so that means we're limited to the options at the end of the 'T' branch. At that point, we can read off the probability of her wearing a jumper, and so our answer is: 0.85.

HP3 Venn Diagrams

a) Firstly, let's consider any number that are both even and prime. There is one: 2. This is the only number that will go in the section where the two circles cross over.
Then, the rest of the even numbers: 6, 8, and 10, will go in the section of the A circle that doesn't cross over with B. Next, the rest of the prime numbers: 3, 5, 7, and 11, will go in the section of the B circle that doesn't cross over with A.
Finally, the one number that is neither even nor prime is 15, so that goes outside the circles. The completed Venn diagram looks like the one below.



b) $A \cap B$ refers to "A and B". There is only one number in both A and B, so the answer is 1.

c) $A \cup B$ refers to "A or B". There are 8 numbers that are contained in circle A and/or circle B, and there are 9 numbers in total, so we get

$$P(A \cup B) = \frac{8}{9}$$

HP4 Averages and Spread

a) We must add up all the values and divide by 10.

$$\text{mean} = \frac{181 + 182 + 175 + 176 + 210 + 169 + 175 + 184 + 167 + 175}{10}$$
$$= 179.4 \text{ cm}$$

b) To find the median, we must first put the values in ascending order:

$$167, 169, 175, 175, 175, 176, 181, 182, 184, 210$$

Then, if you cross off alternating biggest and smallest values, you'll be left with two numbers: 175 and 176. Therefore, the median is 175.5 cm, (the halfway point).

c) In this case, the man who is 210 cm tall is significantly taller than the other men. Therefore, when we calculate the mean, the 210 value is going to make the mean much higher than otherwise, and it might not be representative of the data (try calculating the mean without 210 and see what happens). The median, however, is not affected by the value of 210, so it might be a better measure of average in this case.

HP5 Estimating the Mean

Firstly, we need to find the midpoints of each class and write them in a new column attached to the one given in the question.

Distance, d (m)	Frequency	Midpoint
$0 < d \leq 20$	21	10
$20 < d \leq 40$	43	30
$40 < d \leq 70$	12	55
$70 < d \leq 110$	4	90

Then, treat the midpoints as the actual values and find the sum of all the midpoints. To make this quicker, we multiply each midpoint by its frequency, and sum all the results. Then, since we are estimating the mean, we divide by the total number of people in the experiment: 80. Doing this, we get

$$\begin{aligned}\text{estimated mean } &= \frac{(21 \times 10) + (43 \times 30) + (12 \times 55) + (4 \times 90)}{80} \\ &= \frac{2,520}{80} = 31.5\end{aligned}$$

HP6 Scatter Graphs and Correlation

a) We can see that these points following a straight-line pattern fairly closely, and we can see that as the x value increases, so does the y value. Therefore, this graph displays moderate positive correlation.

b) There appears to be no relationship followed by the points on this graph. Therefore, it displays no correlation.

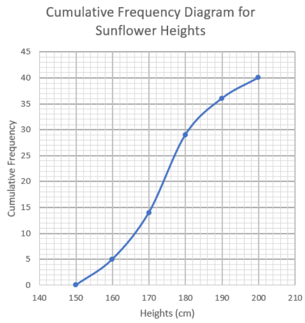
c) We can see that these points following a straight-line pattern very closely, and we can see that as the x value increases, the y value decreases. Therefore, this graph displays strong negative correlation.

HP7 Cumulative Frequency

Obtaining cumulative frequency from a frequency table amounts to adding up the values as we go along, using the upper limit of each class as our new upper limit at each step. So, the first value is 5, then the second is $5 + 9 = 14$, then the third is $5 + 9 + 15 = 29$. Continuing this, the completed table looks like

Height, h (cm)	Frequency	Cumulative Frequency
$150 < h \leq 160$	5	5
$160 < h \leq 170$	9	14
$170 < h \leq 180$	15	29
$180 < h \leq 190$	7	36
$190 < h \leq 200$	4	40

Then, plotting each of these cumulative frequency values against each of the upper limits of the classes, and joining them all together with a smooth curve, we get the graph shown below.



HP8 Boxplots

We need the smallest value, largest value, lower quartile, upper quartile, and median. Given that the range is the largest value take away the smallest, if we add the range to the smallest value it will give us the largest value:

$$\text{largest value } = 10 + 38 = 48$$

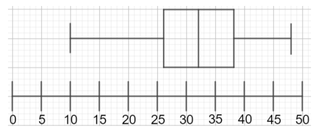
Similarly, as the interquartile range is the upper quartile take away the lower quartile, if we take away the interquartile range from the upper quartile, it will give us the lower quartile:

$$\text{lower quartile } = 38 - 12 = 26$$

Lastly, the median is halfway between the two quartiles. So, we get

$$\text{median } = \frac{26 + 38}{2} = 32$$

Now, we have all the information we need, and the resulting boxplot looks like



HP9 Histograms

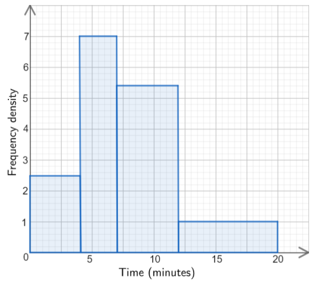
We need to add a third column to the table containing the frequency densities, which we calculate by dividing each frequency by its class width. So, for the first one we'd get:

$$\text{frequency density } = 10 \div 4 = 2.5$$

Continuing this, our table with a completed frequency density column looks like:

Time, t (minutes)	Frequency	Frequency Density
$0 < t \leq 4$	10	2.5
$4 < t \leq 7$	21	7
$7 < t \leq 12$	27	5.4
$12 < t \leq 20$	8	1

Now we can plot the histogram. With each bar having the width of its class interval and the height of its frequency density, our resulting histogram looks like:



HP10 Reading Histograms

We need to determine what the missing frequency density scale should be. We know that frequency density is frequency divided by class width, so for the 0 to 18 kg class, we get

$$\text{frequency density } = 81 \div 18 = 4.5$$

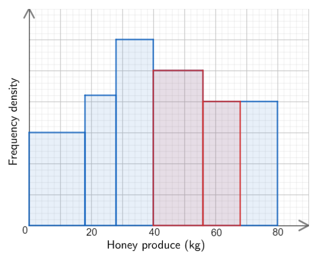
Counting squares, we can see that the 0 to 18 kg bar is 15 small squares high, therefore one square on the y-axis is worth

$$4.5 \div 15 = 0.3$$

Now, to find the estimate for the number of beekeepers between 40 and 68kg, we need to work out how many beekeepers were in the 40 to 56 class first. This bar is 25 small squares high, so its height on the y-axis is $25 \times 0.3 = 7.5$. The frequency is given by the area of the bar, and its width is 16, so we get

$$\text{beekeepers in "40 to 56" class } = 7.5 \times 16 = 120$$

Next, we need to estimate how many beekeepers collected between 56 and 68kg of honey. Looking at the histogram, we can see that 56 to 68 kg is half the width of the last bar, so we will "cut" the bar in half and find the frequency of one of these new halves. (The red section below highlights how we are considering the "40 to 56" group to look)



This bar is 20 small squares high, so its height on y-axis is $20 \times 0.3 = 6$. As before, the frequency is given by the area of the bar, and its width is 12 (half the width of the full bar), so we get

$$\text{estimate for beekeepers in "56 to 68" range } = 12 \times 6 = 72$$

Summing the two values together, our estimate of the total frequency of beekeepers who collected between 40 and 68 kg of honey (the red section) is
$120 + 72 = 192$

HP11 Pie Charts

We know that the formula for finding the angle is

$$\text{angle } = \frac{\text{number in one category}}{\text{sum of all categories}} \times 360$$

This time we know the angle (224), and the sum of all categories (1,260). So, the equation becomes

$$224° = \frac{\text{paperbacks sold}}{1,260} \times 360$$

Divide by 360 and then multiply by 1,260 to get

$$\text{paperbacks sold} = \frac{224}{360} \times 1,260 = 784$$

The number of paperbacks sold is 784, so the number of other books sold is $1,260 - 784 = 476$. The ratio of hardbacks:audiobooks is 3:1, so audiobooks constitute 1 part out of 4 in the ratio.
Therefore, we get

$$\text{audiobooks sold } = \frac{476}{4} = 119$$